NATIONAL ACADEMY OF SCIENCES

VALENTINE BARGMANN

1908—1989

A Biographical Memoir by

JOHN R. KLAUDER

Biographical Memoir

NATIONAL ACADEMIES PRESS
WASHINGTON D.C.

Courtesy of Princeton University Libraries, Princeton, N.J.

Valentine Bargmann

VALENTINE BARGMANN

April 6, 1908–July 20, 1989

BY JOHN R. KLAUDER

"LET'S ASK BARGMANN!" With that phrase—addressed to me by my Princeton thesis advisor—I was led to my first real encounter with Valentine Bargmann. Our question pertained to a mathematical fine point dealing with quantum mechanical Hamiltonians expressed as differential operators, and we got a prompt, clear, and definitive answer. Valya—the common nickname for Valentine Bargmann—was already an established and justly renowned mathematical physicist in the best sense of the term, and his advice was widely sought by beginners and experts alike. It was, of course, a thorough preparation that brought Valya to his well-deserved reputation.

Valya Bargmann was born on April 6, 1908, in Berlin, and studied at the University of Berlin from 1925 to 1933. As National Socialism began to grow in Germany, he moved to Switzerland, where he received his Ph.D. in physics at the University of Zürich under the guidance of Gregor Wentzel. Soon thereafter he emigrated to the United States. It is noteworthy that his passport, which would have been revoked in Germany at that time, had but two days left to its validity when he was accepted for immigration into the United States. He soon joined the Institute for Advanced

Study in Princeton, and in time was accepted as an assistant to Albert Einstein.

Along with Peter Bergmann, Bargmann analyzed five-dimensional theories combining gravity and electromagnetism at a classical level. During World War II, Bargmann worked on shock wave studies with John von Neumann and on the inversion of matrices of large dimension with von Neumann and Deane Montgomery. Bargmann taught informally at Princeton beginning in 1941. He received a regular appointment as a lecturer in physics in 1946 and remained at Princeton essentially for the rest of his career.

Bargmann worked with Eugene Wigner on relativistic wave equations and together they developed the justly famous Bargmann-Wigner equations for elementary particles of arbitrary spin. In 1978 Bargmann and Wigner jointly received the first Wigner Medal, an award of the Group Theory and Fundamental Physics Foundation. Besides this honor, Bargmann was elected to the National Academy of Sciences in 1979 and won the Max Planck Medal of the German Physical Society in 1988.

Valya was a gentle and modest person—and he was a talented pianist. At social occasions it was not uncommon for Valya to perform solo or accompany other musicians.

His lectures were renowned for their clarity and polish. Among the prized series of lectures were those on his acknowledged specialties, such as group theory (e.g., the Lorentz group and its representations, and ray representations of Lie groups), as well as second quantization. In mathematics, his most influential work was on the irreducible representations of the Lorentz group. This work has served as a paradigm for representation theory ever since its appearance. Bargmann also made important contributions to several aspects of quantum theory. He was a stellar example of the European tradition in mathematical physics in the spirit of Hermann Weyl, von Neumann, and Wigner. A book in Bargmann's honor offers expert comments on a number of topics dear to the heart of Valya.[1]

Bargmann had his less serious side as well. No better example of that can be given than the story told by Gérard G. Emch, who arrived at Princeton in 1964 to begin a postdoctoral year with Valya. Emch also arrived with a newly minted "theorem," which he proudly presented to the master. No sooner had the theorem been laid out than Bargmann was ready with a counterexample. Sorely disappointed, Emch retreated for home that day and continued to study the matter. At 3 a.m. Emch's phone range. The caller, Bargmann, heartily laughed when Emch quickly picked up the phone. He then said, "I thought you would still be up. Go to bed and get some sleep. I have found an error in my counterexample. We can discuss it tomorrow!"

Valya Bargmann published a modest number of papers by contemporary standards, but he nevertheless was instrumental in opening several distinct fields of investigation. His paper on establishing a limit on the number of bound states to which an attractive quantum mechanical potential may lead has spawned a minor industry in the research on such issues. His paper dealing with distinct potentials that exhibit identical scattering phase shifts redirected research in inverse scattering theory in which it had been previously assumed that the scattering phase shifts would uniquely characterize the potential. His study of the unitary irreducible representations of the noncompact group SL(2,R) have proved not only invaluable in their own right but have served as a model of how such representations are to be sought for more general noncompact Lie groups. Shortly after completing the work on SL(2,R) he also completed a manuscript on the related group SL(2,C). This work, however, was never published because independent work by Israel

Gel'fand and Mark Naimark covering the same ground reached the publisher ahead of Bargmann's planned submission.

To a large extent, Bargmann tended to write either short notes or long, extensive articles. When he felt he really had something to say it seems he would become didactic, thorough, and complete. Thus, his papers on SL(2,R) and the factor representation of groups were both long papers by Bargmann's standards. However, he saved his longest and most sustained study until the 1960s, when he dealt with one of the subjects for which he will long be remembered. It is to this set of papers and a brief sketch of some of their principal novelty that I would like to turn my attention at this point. I have chosen to outline two mathematical arguments, because on the one hand they are relatively simple and on the other hand they are universal and profound.

From 1961 onward, Valya published several papers dealing with the foundations and applications of Hilbert space representations by holomorphic functions now commonly known as Bargmann spaces (or sometimes as Segal-Bargmann spaces in view of an essentially parallel analysis of the main features by Irving Segal). We can outline a few of the principal ideas in such an analysis by first starting with the following background material. The basic kinematical operators in canonical quantum mechanics for a single degree of freedom may be taken as the two Hermitian operators Q and P, which obey the fundamental Heisenberg commutation relation

$$[Q,P] \equiv QP - PQ = i\hbar I, \quad i \equiv \sqrt{-1},$$

where $\hbar$ denotes Planck's constant $h/2\pi$, and where I denotes the unit operator. In the Schrödinger approach to quantum mechanics these operators are represented as

$Q \to x$ and $P \to -i\hbar\partial/\partial x$ acting on complex-valued functions $\psi(x)$ that belong to the Hilbert space $L^2(\mathrm{R}, dx)$ composed of those functions for which

$$\int_{-\infty}^{\infty} \psi(x)^* \psi(x) dx < \infty.$$

Given any two such functions ϕ and ψ, which are sufficiently smooth and vanish at infinity, then the Hermitian character of x and $-i\hbar\partial/\partial x$ follow from the properties that

$$\int_{-\infty}^{\infty} [x\phi(x)]^* \psi(x) dx = \int_{-\infty}^{\infty} \phi(x)^* x\psi(x) dx,$$

and

$$\int_{-\infty}^{\infty} [-i\hbar\partial\phi(x)/\partial x]^* \psi(x) dx$$
$$= i\hbar\phi(x)^* \psi(x) |_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \phi(x)^* (-i\hbar)\partial\psi(x)/\partial x \, dx$$
$$= \int_{-\infty}^{\infty} \phi(x)^* [-i\hbar\partial\psi/\partial x] dx.$$

An elementary yet important alternative combination of the basic operators Q and P is given by

$$A \equiv (Q + iP)/\sqrt{2\hbar}, \quad A^\dagger \equiv (Q - iP)/\sqrt{2\hbar},$$

where $A^\dagger$ denotes the Hermitian adjoint of the operator A. The basic commutation relation between Q and P then leads to

$$[A, A^\dagger] = AA^\dagger - A^\dagger A = I,$$

which is often chosen as an alternative starting point. Indeed, Vladimir Fock in 1928 recognized that this form of the commutation relation may be represented by the expressions $A \rightarrow \partial/\partial z$ and $A^\dagger \rightarrow z$ acting on a space of analytic functions $f(z)$ defined for a complex variable z. While it is true that this representation of the operators A and $A^\dagger$ satisfies the correct commutation relation, it is unclear how z and $\partial/\partial z$ can be considered adjoint operators to one another, especially in light of the fact demonstrated above that $x^\dagger = x$ and $(-i\hbar\partial/\partial x)^\dagger = (-i\hbar\partial/\partial x)$ hold in the Schrödinger representation.

Bargmann was among the first to show clearly how one can justify the relation $z^\dagger = \partial/\partial z$. To that end Bargmann defined a Hilbert space F of holomorphic functions $f(z)$ restricted so that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(z)^* f(z)e^{-|z|^2}\,dx\,dy < \infty$$

where $|z|^2 = z^*z$, $z = x + iy$, and the domain of integration is over the plane R^2. We observe that a general function of x and y can be viewed as a (related) general function of $z = x + iy$ and $z^* = x - iy$. A holomorphic function $g(z)$ depends on only one such variable, z, or stated otherwise $\partial g(z)/\partial z^* \equiv 0$ along with the complex conjugate relation $\partial g(z)^*/\partial z \equiv 0$. Let f and g be two holomorphic functions and consider the following integral (with the limits implicit)

$$\begin{aligned}
&\int\int [zg(z)]^* f(z)e^{-|z|^2}\,dx\,dy \\
&= \int\int g(z)^* z^* f(z)e^{-|z|^2}\,dx\,dy \\
&= \int\int g(z)^* f(z)(-\partial/\partial z\, e^{-|z|^2})\,dx\,dy \\
&= \int\int e^{-|z|^2}(\partial/\partial z)[g(z)^* f(z)]\,dx\,dy \\
&= \int\int g(z)^*[\partial f(z)/\partial z]e^{-|z|^2}\,dx\,dy,
\end{aligned}$$

which shows that $z^{\dagger} = \partial/\partial z$ as required. With the introduction of the given inner product for two holomorphic functions, Bargmann put the heuristic notion that z and $\partial/\partial z$ were adjoint operators onto a firm mathematical foundation.

All separable and infinite-dimensional Hilbert spaces are isomorphic. The relation between the space $L^2(\mathrm{R}, dx)$ and the space F can be given in the following form. To each $\psi \in L^2$ associate the expansion

$$\psi(x) = \sum_{n=0}^{\infty} a_n h_n(x), \quad a_n = \int_{-\infty}^{\infty} h_n(x)\psi(x)dx$$

in terms of the complete, real, orthonormal set of Hermite functions $\{h_n(x)\}_{n=0}^{\infty}$ each element of which is implicitly defined by the fact that

$$\exp(-s^2 + 2sx - 1/2\,x^2) = p^{1/4} \sum_{n=0}^{\infty} (n!)^{-1/2} (s\sqrt{2})^n h_n(x).$$

Moreover,

$$\sum_{n=0}^{\infty} | a_n |^2 = \int_{-\infty}^{\infty} | \psi(x) |^2 \, dx < \infty.$$

Clearly, $\psi \in L^2$ uniquely determines the sequence $\{a_n\}_{n=0}^{\infty}$. To each such sequence we associate the holomorphic function

$$f(z) = \sum_{n=0}^{\infty} a_n z^n / \sqrt{n!},$$

which converges absolutely for all z. In a decided advance, Bargmann was able to put this association to an expanded use as follows.

The space of tempered test functions consists of those complex functions $u(x)$ such that $u \in C^{\infty}$ and $x^r \, d^s u(x)/dx^s$

$\rightarrow 0$ as $x \rightarrow \pm \infty$ for all nonnegative r and s. Each such function is realized by the expansion

$$u(x) = \sum_{n=0}^{\infty} b_n h_n(x),$$

where $n^r b_n \rightarrow 0$ as $n \rightarrow \infty$ for all r (i.e., b_n falls to zero faster than any inverse power). Dual to the space of tempered test functions is the space of tempered distributions, a special class of generalized functions. If *D(x)* denotes such a generalized function, then D admits the formal expansion

$$D(x) = \sum_{n=0}^{\infty} d_n h_n(x),$$

where $\{d_n\}_{n=0}^{\infty}$ is a sequence of polynomial growth (i.e., $|d_n| \leq R + S^n$ for suitable R and S). Generally, D is only a generalized function (e.g., $D(x) = \delta\,(x - y)$ when $d_n = h_n(y)$, etc.), not an ordinary function, and, although the left-hand side of the relation

$$\sum_{n=0}^{\infty} b_n^* d_n = \int_{-\infty}^{\infty} u(x)^* D(x) dx$$

is well defined, the right-hand side does not exist as a traditional integral.

Bargmann realized, however, that the action of tempered distributions on test functions did indeed possess a genuine integral representation in terms of holomorphic functions. For that purpose let

$$u(z) \equiv \sum_{n=0}^{\infty} b_n z^n / \sqrt{n!}$$

denote the image of *u(x)* in *F*. Furthermore, we define

$$D(z) \equiv \sum_{n=0}^{\infty} d_n z^n / \sqrt{n!}$$

as the image of the generalized function *D(x)*. Since $|d_n| \leq R$

+ S^n it follows that the series defining $D(z)$ converges everywhere and thereby defines a holomorphic function. Moreover, it follows that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(z) * D(z) e^{-|z|^2} \, dx \, dy \, / \, \pi = \sum_{n=0}^{\infty} b_n^* d_n \, .$$

Thus, the holomorphic function representation endowed with the Bargmann inner product provides an explicit integral representation for the action of an arbitrary tempered distribution, a feature entirely unavailable in the usual form of generalized functions and formal integrals.

The discussion just concluded regarding two topics dealing with holomorphic function spaces illustrates the penetrating simplicity of Bargmann's approach to mathematical physics. Would that we had more like him today; I occasionally miss the opportunity to "ask Bargmann."

THANKS ARE EXTENDED to Gérard G. Emch, Elliott H. Lieb, Barry Simon, and Arthur S. Wightman for their input and/or comments that have found their way into this article.

NOTE

1. E. H. Lieb, B. Simon, and A. S. Wightman, eds. *Studies in Mathematical Physics, Essays in Honor of Valentine Bargmann*. Princeton, N.J.: Princeton University Press, 1976.

SELECTED BIBLIOGRAPHY

1934

Über den Zusammenhang zwischen Semivektoren and Spinoren und die Reduktion der Diracgleichung für Semivektoren. *Helv. Phys. Acta* 7:57-82.

1936

Zur Theorie des Wasserstoffatoms. *Z. Phys.* 99:576-82.

1937

Über die durch Elektronenstrahlen in Kristallen angeregte Lichtemission. *Helv. Phys. Acta* 10:361-86.

1941

With A. Einstein and P. G. Bergmann. On the five-dimensional representation of gravitation and electricity. In *Theodore von Kármán Anniversary Volume*, pp. 212-25. Pasadena: California Institute of Technology.

1944

With A. Einstein. Bivector fields. *Ann. Math.* 45:1-14.

1945

On the glancing reflection of shock waves. Applied Mathematics Panel Report No. 108.

1946

With D. Montgomery and J. von Neumann. Solution of linear systems of high order. Report to the Bureau of Ordinance, U. S. Navy.

1947

Irreducible unitary representations of the Lorentz group. *Ann. Math.* 48:568-640.

1948

With E. P. Wigner. Group theoretical discussion of relativistic wave equations. *Proc. Natl. Acad. Sci. U. S. A.* 34:211-23.

1949

Remarks on the determination of a central field of force from the elastic scattering phase shifts. *Phys. Rev.* 75:301-303.

On the connection between phase shifts and scattering potential. *Rev. Mod. Phys.* 21:488-93.

1952

On the number of bound states in a central field of force. *Proc. Natl. Acad. Sci. U. S. A.* 38:961-66.

1954

On unitary ray representations of continuous groups. *Ann. Math.* 59:1-46.

1959

With L. Michel and V. Telegdi. Precession of the polarization of particles moving in a homogeneous electromagnetic field. *Phys. Rev. Lett.* 2:435-36.

1960

Relativity. In *Theoretical Physics in the Twentieth Century* (Pauli Memorial Volume), eds., M. Fierz and V. F. Weisskopf, pp. 187-98. New York: Interscience Publishers.

With M. Moshinsky. Group theory of harmonic oscillators. I. The collective modes. *Nucl. Phys.* 18:697-712.

1961

With M. Moshinsky. Group theory of harmonic oscillators. II. The integrals of motion for the quadrupole-quadrupole interaction. *Nucl. Phys.* 23:177-99.

On a Hilbert space of analytic functions and an associated integral transform. Part I. *Commun. Pure Appl. Math.* 14:187-214.

1962

On the representations of the rotation group. *Rev. Mod. Phys.* 34:829-45.

1964

Note on Wigner's theorem on symmetry operations. *J. Math. Phys.* 5:862-68.

1967

On a Hilbert space of analytic functions and an associated integral transform. Part II. A family of related function spaces application to distribution theory. *Commun. Pure Appl. Math.* 20:1-101.

1971

With P. Butera, L. Girardello, and J. R. Klauder. On the completeness of the coherent states. *Rep. Math. Phys.* 2:221-28.

1972

Notes on some integral inequalities. *Helv. Phys. Acta* 45:249-57.

1977

With I. T. Todorov. Spaces of analytic functions on a complex cone as carriers for the symmetric tensor representations of SO(n). *J. Math. Phys.* 18:1141-48.

1979

Erinnerungen eines Assistenten Einsteins. Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich, Jahrgang 124, Heft 1, pp. 39-44. Zürich: Druck und Verlag Orell Fussli Graphische Betriebe AG.