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LIPMAN BERS
1914–1993

A Biographical Memoir by

IRWIN KRA AND HYMAN BASS

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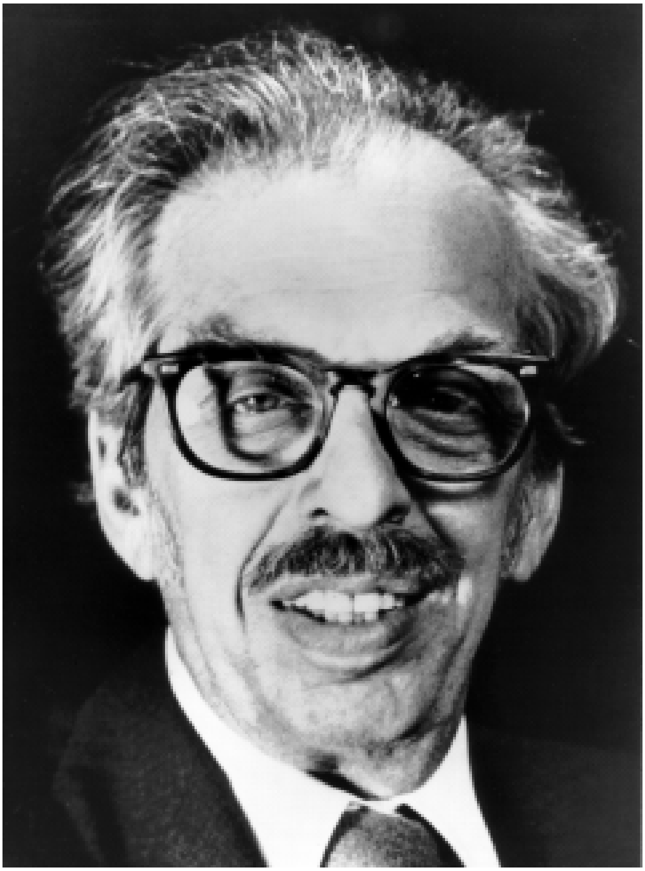

Courtesy of Victor Bers

LIPMAN BERS

May 22, 1914–October 29, 1993

BY IRWIN KRA AND HYMAN BASS

INTRODUCTION

LIPMAN BERS WAS BORN in Riga, Latvia, on 22 May 1914 into a secular intellectual Jewish family. At the time of his death, in New Rochelle, New York on 29 October 1993, he was the focal point of a large extended group of scientists—mostly mathematicians, many former doctoral students with whom he maintained close and continuous ties for decades. His friends and colleagues knew him as "Lipa." His life was a twentieth-century Jewish and intellectual odyssey. He had close encounters with fascism and Stalinism. In an irrational world he approached all issues through his intellect. He was born in Europe on the brink of revolutionary changes. He died in America after several tyrannies had come and gone. He started as a Bundist with strong anti-nationalist leanings, but over the years he grew increasingly fond of Israel.

He opposed nuclear armaments as if there were no cold war; he fought tyranny as if this struggle had no arms control implications. He played an important role in American

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scientific politics. He made important and lasting contributions to both mathematics and the protection of human rights. He was never afraid to take strong moral positions, even from platforms of official leadership, yet he was pragmatic and effective. His optimism and good humor remained unflagging. Above all, he was a mathematician, and in whatever subfields of the subject he worked, he did complex analysis. All that he did, he did with style.

THE EARLY YEARS

The simmering Russian Revolution failed to cloud Lipa's happy memories of his earliest years in Petrograd. He began school in Riga, where his mother was principal of the secular Yiddish-language elementary school, and his father principal of the gymnasium. In the elementary school he met Mary Kagan. He subsequently lived in Berlin, where his mother trained at the Berlin Psychoanalytic Institute. His love for learning and for mathematics, his faith in rationalism, and his worldly awareness were largely molded during the early years in Riga. His entire intellectual outlook was strongly influenced by a school environment that included the classics such as Shakespeare in Yiddish.

After a brief period in Zurich, Lipa returned to the university in Riga, as a politically involved social democrat in an atmosphere of growing political turmoil and violence. He wrote for an underground newspaper that mocked the prevailing dictatorship. With an unerring instinct for survival that, happily, was never compromised by his political courage and boldness, Lipa fled Riga for Prague when an arrest warrant was issued for him. Mary later joined him there, where they married.

At Charles University Lipa worked under the effective, but not overly nurturing, mentorship of Karl Löwner. That this thesis supervision style agreed with Lipa is obvious from his strong affection for Karl and the life-long friendship between the Bers and Löwner families. The deadline for Lipa's dissertation, on potential theory, was dictated less by scientific considerations than by the imminent fall of Czechoslovakia to the Nazis. Stateless, leftist, and Jewish, the Berses were vulnerable, on several accounts, in Nazi-dominated Europe. They fled to Paris, where their application for an American visa resulted in not uncommon frustration.

The brief interlude in Paris produced two short mathematical papers: one on Green's functions, another on integral representations of biharmonic functions; these were partly inspired by Stefan Bergman's then-influential work on kernel functions. It also produced their daughter, Ruth, now a psychoanalyst, and a professor of psychology at John Jay College of the City University of New York.

Ten days before the Nazis arrived in Paris, the Bers family moved south to unoccupied France, at Mary's insistence. There they benefited from the ten thousand American visas for political refugees issued after the personal intervention of Eleanor Roosevelt. Lipa's mother and stepfather, Beno Tumarin—an actor, director, and in the last years of his life a teacher at the Julliard School—were already in New York. Finding no available professional positions, Lipa at first received aid from YIVO, the Institute for Jewish Research, for which he produced a paper in Yiddish about Yiddish mathematics textbooks.

WARTIME MATHEMATICS AND THE SYRACUSE YEARS

In 1942 Bers accepted a position in the program for advanced research and instruction in applied mathematics at Brown University. His was a low-paying entry-level job in the academic sector of the war effort. In Providence he began his investigations of two-dimensional subsonic flows. He initiated a collaboration with Abe Gelbart, which later developed into the theory of pseudoanalytic functions. He also supervised the work of the first three of his forty-eight doctoral students, an impressive list that includes sixteen women. A few of his papers and technical reports on differential equations, subsonic flows, and pseudoanalytic functions appeared during this period; some of his publications in this area appeared as late as 1953, long after he started to work on different problems. These laid the foundation for the work that led to the 1958 monograph *Mathematical Aspects of Subsonic and Transonic Gas Dynamics*, published by John Wiley and Sons (for whom he would later become a consultant), and the 1964 book *Partial Differential Equations*, written jointly with his New York City colleagues F. John and M. Schechter.[1] The Bers's son, Victor, now a professor of classics at Yale University, was born in Providence.

Bers spent the four academic years 1945-49 at Syracuse University, where mathematics then flourished. Its faculty included for a brief period Paul Erdös, Dan Mostow, and Atle Selberg, among others. Bers's interests shifted further toward partial differential equations and Riemann surfaces. A beautiful theorem, which he discussed at the 1950 International Congress of Mathematicians (ICM) and published in the *Annals of Mathematics* the next year, on removable singularities of minimal surfaces, was the result of work initiated during this time. Although the result belongs to the field of partial differential equations, the methods are from complex analysis, hinting at the direction of his most important future work.

QUASICONFORMAL MAPPINGS, TEICHMÜLLER THEORY, AND KLEINIAN GROUPS

Bers's 1948 paper in the *Bulletin of the American Mathematical Society*, "On rings of analytic functions," created a small industry that contributed to an algebraization of questions in analysis, subsequent contributions by S. Kakutani, W. Rudin, and I. Richards, and a dissertation by one of the authors of this article. The two years at the Institute for Advanced Study (1949-51) transformed Bers into a full-fledged member of the "quasi world"—the world of quasiconformal mappings and their diverse applications. His subsequent work on moduli of Riemann surfaces and Kleinian groups would include some of the most important contributions to complex analysis in the second half of the twentieth century. He attributed his entry into the field to its beauty and wide applicability,[2] and to the fact that it was practiced by gifted individuals of unusual generosity of spirit. Most notable among these, for Bers, was Lars Ahlfors, a distinguished mathematician of very different temperament and personality, with whom Bers maintained a close mathematical and social connection for the rest of his life.

Lipman Bers's most productive mathematical years followed, at New York University (NYU) (1951-64) and at Columbia (1964-82).[3] His research would henceforth lie in the broad field that can described as Riemann surfaces, Kleinian groups and Teichmüller theory. Investigators of these subjects are interested in understanding the different ways one can do complex analysis on a fixed topological surface. One of the important open problems, the moduli problem, left to us from the nineteenth century, was to make rigorous and precise Bernhardt Riemann's claim that the complex analytic structure on a closed surface with $p \geq 2$ handles depends on $3p$–3 complex parameters. What turned out to be the most fundamental result of this period was announced by Bers[4] at the 1958 ICM as a new proof[5] of what is now called the *measurable Riemann mapping theorem*.[6] The new proof yielded much more than the previous methods.[7] It showed that properly normalized homeomorphic solutions of the fundamental Beltrami equation[8] depend holomorphically on parameters. Among the consequences, Bers obtained a solution of Riemann's problem of moduli that significantly extends Ahlfors's earlier solution.[9] Bers continued to work on moduli problems for the rest of his life. He marveled at being paid for doing something that gave him so much joy and pleasure. Teichmüller theory, the brilliant mathematical work, and even more brilliant mathematical insights of O. Teichmüller[10] established the connection between quasiconformal mappings and the metric/topological theory of moduli of Riemann surfaces. Bers's contributions had two surprising consequences. Quasiconformal mappings can also be used to study complex analytic aspects of moduli theory.[11] There is a deep connection between the theory of moduli and univalent functions since a very important class of such functions can be obtained as solutions of Beltrami equations. This last fact led Bers to a number of important conjectures, some of them still open, that influenced complex analysis for more than twenty-five years.

During the first half of this century much progress was made in our understanding of discrete subgroups of PSL (2,R), *Fuchsian* group, in part because of their deep connections to Riemann surfaces and two-dimensional hyperbolic geometry. Poincaré suggested that the study of discrete subgroups of PSL (2,C) *Kleinian* groups, should exploit and be based on their connections to three-dimensional hyperbolic geometry. The passage from the real to the complex universe proved surprisingly difficult. Significant progress had to wait more than fifty years. Little was known about three-dimensional topology/geometry throughout the sixties. The progress in the theory of Kleinian groups during three decades illustrates both Lipa's influence on the field and his strong interactions with Ahlfors. In 1965, Bers published, in the *American Journal of Mathematics*, a new proof of a finiteness theorem for Fuchsian groups. The new ingredient was the use of Eichler cohomology, which had proved important in the solution of a number of problems in number theory. Bers used meromorphic functions to construct cohomology classes. Ahlfors saw that this idea could be greatly expanded. He made a conceptual and a technical advance that were extremely surprising and far-reaching. He used smooth functions to construct cohomology classes from a class of holomorphic functions known as *cusp forms*. He then needed a delicate *approximate identity* to establish that a natural map is injective. The result was the "Ahlfors Finiteness Theorem" (AFT) for Kleinian groups:[12] A finitely generated Kleinian group represents a finite number of surfaces each of which can be compactified by the addition of finitely many points. Bers noticed that Ahlfors's proof had a small gap. He, among others, filled in the gap in 1967.[13] In the same year, he also noticed that Ahlfors's method can be generalized to produce quantitative versions of AFT, now known as the *Bers Area Theorems*. These in turn led Ahlfors and Kra to a study of the structure of the Eichler cohomology groups in 1969. The same ideas were further developed in the eighties to translate questions in the "transcendental" theory of convergence of series of rational functions, *Poincaré series*, to an algorithmic linear algebra theory.

In the seventies a new force appeared in mathematics: William Thurston. Thurston revolutionized practically every field he touched. He brought new insights to old and new problems. His studies in topology/geometry revealed new structures in what we thought were well-understood areas (for example, the structure of self maps of [two-dimensional] surfaces) and led to remarkably rapid and dramatic progress in not so well understood topics (for example, the classification of three-dimensional manifolds). Bers's attempt to understand and translate Thurston's work on topological automorphisms of surfaces to the language of Teichmüller theory, led him to the formulation and solution of an elegant new extremal problem. This extremal problem is both natural and simple. The question should have been asked over thirty years before. It waited for Thurston's vision and Bers's inspired efforts to understand his young colleague's insights to be posed. Its solution, published in *Acta Mathematica* in 1978, provided an alternate classification of self-maps of surfaces, showed the intimate connections of this set of problems to moduli problems, and involved Riemann surfaces with simple singularities, a topic already of interest to Bers for other reasons (to construct analytically the compactification of moduli spaces of nonsingular curves).[14]

Bers claimed that he was very lucky to be surrounded by first-rate colleagues. After retiring from Columbia in 1984, Bers accepted an invitation to join the Graduate Center of the City University of New York. His colleagues there included not only a number of his former students, but also the Einstein Professor, Dennis Sullivan, a mathematician Bers greatly admired, and who admired him as well. Sullivan's interaction with Bers resulted in a broadening of his research interests to include complex dynamics (the "no wandering domains" theorem), to alternate proofs of the AFT, as well as studies of holomorphic motions. His research on this last subject resulted in an important paper, among his final scientific manuscripts, with Halsey L. Royden, published in 1986 by *Acta Mathematica*.

At the time of his death, Lipa had a partially completed manuscript on compactification of moduli spaces. It was a topic he knew well and needed only time to complete. He announced results on this topic as early as 1974. His fertile imagination found so many distractions that, twenty years later, detailed Bers proofs regarding compactified moduli spaces were still missing. The distractions were fortunate for mathematics. Many probably know how to finish this project on moduli. Few, if any, could have produced the new extremal problem or contributed as he has to the programs of Thurston and Sullivan. There were other unfinished projects at the time of his death. Lipa was interested in presenting a first-person account of his life and times. The first chapters of an autobiography are in the hands of the Bers family.

MENTOR AND EXPOSITOR

Lipman Bers's students were immediately admitted to the extended "Ahlfors-Bers family"—a mutually supportive group of mathematicians with common research interests consisting mostly, but not exclusively, of colleagues and former doctoral students of Ahlfors and Bers. Bers treated each of his students with respect, as a future friend and current colleague.[15] His love for his students was, in part, independent of the level of their mathematical achievements and successes. His students returned this love and quite naturally extended it to embrace Mary Bers, whose warmth and genuine interest in them made most of Lipa's students feel that they were part of a wonderful family. Lipa was successful both at NYU and Columbia in attracting large numbers of talented students. For a long period, he and Ahlfors engaged in a gentle and "gentlemanly" competition to determine who had more students at various family gatherings. These occurred roughly every four years beginning with the 1965 Tulane conference. In the eighties this competition was extended to include grandstudents. At NYU Lipa's weekly lunch with "his children" was an event which attracted more than his students. Here and later, more noticeably, at Columbia he partially reverted to his advisor's style (in his unique "deformation" of it), supervising each student's work *only* to the extent necessary. He designed an individual research program (thesis problem) for each student, suited to his or her abilities. At times, the original problem took years to solve—a dissertation resulted from an interesting special case or a related, but probably less central, set of questions. For example, Maskit received his Ph.D. in 1964. His task of extending Klein's work on combination theorems and describing how to construct an arbitrary function group from simpler groups was not completed before 1975. His success can be measured by the fact that the series of results previously known as the "Klein combination theorems" have become the "Klein-Maskit combination theorems."

Mathematics has been regrettably slow to change from its historically male-dominated status. An extraordinarily high proportion—sixteen of forty-eight—of Lipa's doctoral students were female. He was comfortable with women, as with men, and encouraged the inquiring minds of his students and colleagues without reference to gender. He practiced, in a most real and effective way, a policy of "equal opportunity/affirmative action," long before it became a part of public policy. He offered the same care and nurturing to all students who showed an interest or a talent for mathematics, working diligently to bring out the best in each. He was equally proud of the student who, in his opinion, would make first-rate contributions to research, to teaching, or to administration. He even admired would-be deans.

Lipa was a model expositor. His papers are clear and well written; he identified the key elements of the problem and the solution. He believed that before one left a subject, one ought to write a book. His books on gas dynamics and partial differential equations are such examples. His graduate teaching at NYU produced influential sets of lecture notes on pseudoanalytic functions (1952), on topology (1955), on Riemann surfaces (1957), and on several complex variables (1963). He did not need to leave the field to produce the latter, nor the 1964 notes on moduli of Riemann surfaces based on his Zurich ETH lectures. He never wrote a book on moduli of Riemann surfaces since he never left the field. Partly in recognition of his 1972 article on moduli of Riemann surfaces and Kleinian groups in the *Bulletin of the London Mathematical Society,*[16] which reported on his Hardy Lectures,[17] the American Mathematical Society (AMS) awarded him the Steele Prize for exposition in 1974.

POLITICS AND SOCIAL ACTIVISM

When graduate study in Prague initiated his mathematical career, Bers was already a seasoned political and social activist and veteran. His concern for human rights was never diminished by his love for mathematics and his scientific and administrative efforts and achievements. He sought to broaden the social consciousness and elevate the conscience of the institutions that he could influence. A rare combination of personal qualities made his efforts extraordinarily effective. He was broadly cultured, with a rich and insightful knowledge of history; he was eloquent, witty, civil, and always good humored. His deep and passionate moral concerns found a tempered and effective, rather than righteous and polarizing, expression. His convictions were rendered in what he called “the international language of science: heavily accented English.”

Lipa’s experiences during his flight from Nazi-dominated Europe educated him about the excesses of some governments and people, and the generosity and courage of others. In America, the experience and wisdom that he brought to social issues played itself out mainly on three stages: at Columbia during the anti-Vietnam War protests; at the American Mathematical Society, as vice president 1963-65 and president 1975-77; and at the National Academy of Sciences (NAS), as founder and chair of the Committee on Human Rights. In the fifties, before his prominence in the American scientific community extended beyond mathematics, he helped victims of McCarthyism and cold-war politics obtain academic positions.

In opposing the Vietnam War, Lipa felt a strong moral sympathy with the protest movement, but often tried to temper its destructive excesses. Echoes of these same political tensions reverberated during his leadership of the AMS, a historically conservative scholarly society narrowly focused on research-related issues. Bers helped orchestrate a substantial, but not destabilizing, broadening of its focus to include both general professional matters and issues of political and moral concern, but only insofar as they specifically affected professional mathematicians. In particular, he was instrumental in founding the AMS Committee on the Human Rights of Mathematicians, whose initial charge he drafted.

Lipa was poignantly aware of the importance of providing support for oppressed people, in particular scientists, throughout the world. The Human Rights Committee he helped found and lead at the NAS has had a distinguished record of interventions on behalf of persecuted scientists, engineers, and health professionals. Its continuing work is a living tribute to Bers's inspiration and example.

There is no better way to convey Bers's convictions, eloquence, and style than to quote his own words. Appealing to the Council of the NAS to speak out publicly on behalf of Andrei Sakharov, Lipa said, "When Sakharov began speaking out about victims of injustice, he risked everything, and he never knew whether his intervention might help. Should we, living in a free country, do less?" The Council was persuaded.

In a 1984 commencement address at the State University of New York at Stony Brook, on the occasion of receiving an honorary degree, Lipa articulated his moral credo for human rights activism:

> . . . By becoming a human rights activist, as I urge you to do, you do take upon yourself certain difficult obligations. . . . I believe that only a truly even-handed approach can lead to an honest, morally convincing, and effective human rights policy. A human rights activist who hates and fears communism must also care about the human rights of Latin American leftists. A human rights activist who sympathizes with the revolutionary movement in Latin America must also be concerned about human rights abuses in Cuba and Nicaragua. A devout Moslem must also care about the human rights of the Bahai in Iran and of the small Jewish community in Syria, while a Jew devoted to Israel must also worry about the human rights of Palestinian Arabs. And we American citizens must be particularly sensitive to human rights violations for which our government is directly or indirectly responsible, as well as to the human rights violations that occur in our own country, as they do.

Lipa's humane pragmatism was eloquently expressed during a symposium on human rights at the 1987 annual meeting of the NAS. He explained his belief that the Committee on Human Rights should focus on political and civil rights, which he called "negative rights," rather than on a "positive" economic, social, and cultural agenda. Negative rights prohibit government restraint of certain individual actions; for example, associating freely with colleagues from abroad. Positive rights instead require a government action, such as providing food, shelter, medical care, education, employment, and so forth. Lipa spoke thus:

> As an old social democrat—I would say an old Marxist, if the word had not been vulgarized—I certainly recognize the importance of positive rights. Yet, I think there is a good reason why the international human rights movement, of which our committee is a small part, concentrates on negative rights. It makes sense to tell a government, "Stop torturing people." An order by the prime minister or the president to whomever is in charge

could make it happen. It makes sense to tell a foreign ambassador that, "The American scientific community is outraged that you keep Dr. X in jail. Let him out and let him do his work." It requires no planning, no political philosophy, and it can unite people with very different opinions.

Lipa, who was never accused of mincing words, added: ". . . the idea that people of the Third World are somehow less appalled by torture or by government-sponsored murder than citizens of developed nations [is, to me] rank racism." He then added: "It is quite a different matter to tell a foreign government of a developing country, 'You really should give this or that positive right to your people.' If we make such a demand in good faith, it must be accompanied by some plan for implementing this right and by some indication of the cost and of who will pay it and how it will be paid. . . . I think that the basic emphasis on negative rights by the international human rights movement is a reasonable thing." Lipman, ever the politician, continued: "Now, if we want to do things beyond this and participate in organizing a social democratic party in America, I will gladly discuss this later."

In his personal life and in advancing his social agenda, especially on human rights, Lipa was remarkably accurate in foreseeing the future. But he was not as able a prophet on all fronts. During 1966-68, Bers chaired the NAS/National Research Council Committee on Support of Research in the Mathematical Sciences. Its report boasted of the health and promise for mathematical research, and projected that there would be an expanding need for production of Ph.D.'s continuing into the next century. This was a great tribute to his optimism, if not to his appreciation of population dynamics.

COLLEAGUE AND FRIEND

Lipa and Mary were the co-founders of the extended Ahlfors-Bers family that provided a nurturing environment for friends and colleagues. Many, probably most, complex analysts, quasiconformal mappers, and Russian dissidents who passed through the New York City area made at least one trip to the Bers home in New Rochelle. There they were welcomed; their good causes were given moral and, at times, financial support. Lipa's lifelong friendship and respect for Lars Ahlfors[18] helped create a community and a mathematical dynasty. It helped shape the way a whole generation thinks about and does mathematics. Many of his professional colleagues, especially those from NYU and Columbia, were an important part of his social/political entourage. Perhaps most lasting and permanent were his ties to most of his former students. These students did not leave the family upon receiving "the union card." They stayed in touch and relied on his advice on professional, social, and family matters. No one could show more excitement about even the most trivial advance than Lipa. No one can forget his supreme compliment, "you sly dog," when he encountered a clever proof or surprising result by a student.

CONCLUDING REMARKS

Lipa joked that the president of the AMS had one statutory obligation and one traditional privilege. The obligation was to deliver a retiring presidential address. His was published in 1978, in the *Bulletin of the AMS*. The privilege, which disappeared by 1993, was to have an obituary published in the *Bulletin*. It was however quite fitting that the first issue of the new revised[19] *Notices of the AMS* (January 1995) featured a number of articles "Remembering Lipman Bers." In the words of his friend Aryeh Dvoretsky "Like the sun in our sky, he cast a giant light."

NOTES

1. Published by Interscience.

2. He was looking for an inequality that would establish an existence theorem in partial differential equations and suspected that quasiconformal mappings might provide the needed a priori estimate.

3. He served for a period as chairman of the graduate program at NYU and for three years (1972-75) as department chair at Columbia.

4. Lipa's style in scholarship paralleled his teaching method. The low key and modest declaration that it was "merely" a new proof was followed by the dramatic listing of consequences of the result.

5. The paper containing the result, the only joint publication with L. V. Ahlfors, was published in the *Annals of Mathematics* in 1960.

6. Lipa objected to this name—the theorem is not measurable, the data that appear in the hypothesis are measurable, he explained.

7. Due to Lavrentiev in special cases and Morrey in the general case.

8. A generalization of the classical Cauchy-Riemann equations.

9. One of the most important aspects of this problem was reduced to showing that a cover of Riemann's moduli space, called the *Teichmüller space*, has a natural complex structure.

10. That Lipa, a fighter for social justice and against nazism, built his work on that of Teichmüller, an active opponent of racial justice and a proponent of nazism, was an oft-cited irony of his career. One of Lipa's early papers in this area quotes Plutarch (*Pericles* 2.1): ". . . It does not of necessity follow that, if the work delights you with its grace, the one who wrought it is worthy of your esteem."

11. This circle of ideas was completed in 1966 when H. Royden proved that the metric structure of Teichmüller space can be recovered from its complex structure.

12. Although Ahlfors describes Bers's finiteness proof as a model for his finiteness theorem, Ahlfors's paper, also published in the *American Journal of Mathematics*, appeared a year before Bers's.

13. All dates concerning mathematics research results refer to publication dates—the relevant theorem may have "circulated" for two to three years before publication. The "gentlemanly" manner in which mathematics was done in this field is illustrated by Ahlfors's frequent reference to his finiteness theorem as the "theorem finally

established by Bers” because he filled in the above-mentioned gap. Needless to say, Bers always thought of AFT as one of Ahlfors’s greatest accomplishments.

14. Carried out earlier, by others, in the setting of algebraic geometry.

15. As a result of his socialist background or European traditions, a student would be introduced to a visitor as Mr. or Ms. X; the visitor would in turn be introduced as Mr. or Ms. Y no matter what her or his titles might have been.

16. Another first-rate expository article appeared in the *Bulletin of the AMS* in 1981.

17. Bers was the first Hardy Lecturer, an honor bestowed by the London Mathematical Society.

18. Completely reciprocated.

19. And improved.

MOST SIGNIFICANT PUBLICATIONS OF LIPMAN BERS

Bers worked in several areas of mathematics including partial differential equations (he coauthored an important book on the subject), minimal surfaces and complex analysis. It was to the latter field that he devoted his last and best productive scientific years (a span of over 40 years) and to which he made the most significant contributions; in some sense all of his work was in (certainly, preparation for) complex analysis. All the papers referenced here deal with this subject and appear in his selected works, [26]. His paper on minimal surfaces [3] is a perfect example of the role of complex analytic methods in related fields. His only paper on spaces of analytic functions [2] started a whole industry. The most productive period in Bers's scientific life saw fundamental contributions to Teichmüller theory. No papers perhaps have been more influential in this field than [1] and [9]. What appeared to him an oddity, [5], proved to be a key tool for understanding what is today called the *Bers embedding* [7]; other Teichmüller theory papers include [21], [12], [14], [19], [23] and [24]. Results about compactified moduli spaces are announced in [16]. The close relation between quasiconformal mappings and Teichmüller theory are explored in [4], [11] and [18]. Fundamental contributions to the theory of Fuchsian and Kleinian groups are contained in [6], [8], [10] and [15]. The expository papers [13] and [20] introduced a whole generation to subfields of complex analysis. In [22], Teichmüller theoretic methods are used to prove results about iteration of rational maps; [25] is a contribution to the study of holomorphic motions; [17] opens up areas for future contributions by others.

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