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HANS FREDERIK BLICHFELDT

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A Biographical Memoir by

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Hans Frederik Blichfeldt was born on January 9, 1873, in the village of Iller, Denmark. His father was a farmer, who emigrated to the United States in 1888. His mother, Nielsine Marie Schlaper, was a widow of 31 when she married Hans' father. By her first marriage there was one son, by her second, one son, Hans, and two daughters. Till 1881 the family lived on a farm, when it moved to Copenhagen. "Though not actually poverty-stricken," according to Blichfeldt, "the family had to live very frugally; we children had to work to help out during our earlier years." The genealogy of that branch of the Blichfeldt family to which Hans belonged has been traced back from 1910 to 1540. Nearly all the men in the family and some of the women lived to an advanced age. His immediate ancestors were farmers, who had received their education in the Danish public schools. In the Denmark of their day there was but little opportunity for a strong and ambitious young man to advance beyond the hard labor for which he was naturally fitted by a sturdy physique. The brains in the family seem to have come from a long line—about 300 years—of ministers. Emigration to the United States gave young Blichfeldt the opportunity he probably would not have had in his native land, in spite of his demonstrated mathematical ability in the public schools.

Blichfeldt discovered his mathematical talent before he was fifteen, when he found for himself the solutions of the general algebraic equations of the third and fourth degrees. Even with all the advantages of modern notation, this was no small achievement, as anyone familiar with the history of algebra, particularly in the Sixteenth Century, will agree. Throughout his school career, Blichfeldt stood far out in front of his classmates in mathematics, gaining the highest honors in the "general preliminary examination," a Danish State examination held at the University. Part of this success he owed to his father's teaching. In languages he was handicapped by a poor memory for foreign words, aggravated by the fact that he could not afford to buy dictionaries. Later, on coming to the United

States, he mastered English—or American—and spoke with only a slight accent.

The sturdy, hardy immigrant boy quickly found employment as a farm laborer or worker in sawmills in Nebraska, Wyoming, Oregon and Washington. This phase lasted till 1892, when Blichfeldt was nineteen. He then got jobs with engineering firms for two years. All in all he crisscrossed the country "from East to West, from North to South, and back again" several times. One of his jobs was with a surveyors' party in the roughest sections of San Bernardino County, California. Apparently it was this engineering experience from 1892 to 1894 that was the turning point in his life. Years ago there was a legend at Stanford that Blichfeldt got his chance when he astonished the surveyors by doing in his head the calculations for which they required pencil and paper and mathematical tables. Like all legends, this one may have grown beyond fact with the passage of time, but there was nevertheless a good deal in it. Blichfeldt knew from his Danish school career that he had more than ordinary mathematical ability. His feats of mental calculation convinced the engineers that here was a real mathematician going to waste on tasks far below his capacity. They urged him to apply for admission to Stanford University, then only about six years old.

Blichfeldt was admitted in September, 1894, and by June 1896 had his A.B. degree; by June 1897, his A.M. At the time, there were no tuition fees at Stanford, and the "free elective system," adapted from Harvard, gave a student who knew where he wanted to go a chance to get there quickly. As Blichfeldt with the frugality and thrift characteristic of his family had saved a substantial part of his wages, he had no trouble with living expenses during his three years as a student at Stanford. He did not have to take odd jobs, as so many did, to pay his way. He gained some experience as a teacher in his year (1896-97) as a teaching assistant in mathematics. With the exceptions noted presently, the rest of his academic career was passed at Stanford: Instructor (in mathematics), 1898-1901; Assistant Professor, 1901-06; Associate Professor, 1906-1913; Professor, 1913-1938; Head of Department, 1927-1938; Emeritus, 1938 till his death in 1945. The Stanford experience was supplemented by teaching in the summer session of 1913 at the University of Chicago, and the like at Columbia University in 1925 and 1926. More important than these interludes, when Blichfeldt was an established mathematician, was an earlier one.

In the 1890's it was still the custom, inherited from the 1880's, for aspiring young mathematicians to go to Germany for advanced instruction in what were then the subjects of living interest. Germany led the mathematical world, and to the competence in modern mathematics which ambitious young American mathematicians acquired from the German masters, the United States owes its sudden emergence as a mathematical state able to hold its own. Blichfeldt decided to go to Germany for a year's study.

The outstanding mathematicians working in the Germany of the late 1890's were Sophus Lie (Norwegian, 1842-1899), Felix Klein (German, 1849-1925), and David Hilbert (German, 1862-1943). Lie was at Leipzig, Klein at Göttingen. Possibly because Lie was a fellow Scandinavian, Blichfeldt chose him. Hilbert was still on his way to the top and absorbed in his own researches. Moreover, he seems to have been somewhat unapproachable, especially to Americans. Klein, as the consequence of a nervous breakdown from overwork, had passed his peak. Nevertheless, as Blichfeldt said many years after his sojourn in Germany, "Klein was the daddy of them all"—a judgment, however, in which few today would concur. Blichfeldt spent his year with Lie, mastering the "Lie theory" of continuous groups as it then was. At the end of the year (1898) he was awarded the Ph.D. degree from the University of Leipzig, *Summa cum laude*. With the death of Lie (1899) interest in the Lie theory waned, and Blichfeldt gave it up as a main interest not long after his return to Stanford. One detail of his year in Germany should be recorded. In spite of all his thrift, Blichfeldt lacked sufficient funds to go abroad. Professor Rufus Lot Green of Stanford loaned him the necessary money. For this generosity, Blichfeldt was always grateful, and he repaid it in more ways than one. When he had earned an international reputation as a mathematician, Blichfeldt made many trips to Europe, renewing old friendships with some of the prominent European mathematicians.

Blichfeldt never married. The family ties were very close, and what he could spare went to help his relatives. His mother died in 1912 at the age of 81, his father in 1922 at 84.

In describing his father, Blichfeldt said he was "energetic, frugal, generous; he lived very simply and worked very hard all his life." He also said that his father had failed to accumulate a competence for his old age because "he trusted people"—the obverse side of generosity. In his last years the father lived with his son in a large, old-fashioned but comfortable house with a fine garden crammed with plants, many of them more suitable for lush Denmark than semi-arid California.

One of Blichfeldt's personal characteristics in middle life seems to have been the direct outcome of his early years of hard physical work and enforced frugality. The tall, strong man was physically too robust for the cramping routine of academic life and the sedentary business of mathematical research. He indulged in whatever exercise was popular—hiking, bicycling, swimming, motoring—but it was not enough, and he became a confirmed hypochondriac. Neither his friends nor his physicians could see anything the matter with him, except possibly a tendency to take things too easy. Check after check by specialists here and abroad resulted only in unnecessary expense. His complaints invariably proved to be imaginary, and his less sympathetic friends lost patience. But they soon got over their spells of exasperation, for Blichfeldt was a kind, generous and likable man. When he achieved a moderate prosperity, he shared with his friends all the good things he had missed as a young man and could now afford. He died at the age of 72, not of the heart disease he had imagined for forty years, but as the result of an operation.

Among his distinctions were the vice-presidency of the American Mathematical Society, 1912; member of the National Academy of Sciences since 1920; membership in the National Research Council, 1924-27; and Knight of the Order of Dannebrog (Denmark) 1939.

Blichfeldt's mathematical output was not voluminous, and although what he himself considered his main contributions (Nos. 7, 10, 19 in the bibliography) are impressive, it cannot be said that his publications give a just estimate of his potentialities. He attacked difficult problems, many of them important in the mathematics of his day, disposed of some, but left many more incomplete—"I never finish things." After he had seen through some problem to its end, too often the drudgery of reducing his notes to printable form was too repulsive for him to undertake, and—as a competent expert said—Blichfeldt left behind him "a barrel of good stuff." This barrel is unlikely to be tapped now; mathematical tastes have changed since the good stuff was laid down. In the late 1920's, for instance, Blichfeldt had a sudden outburst of activity in the structure theory of algebras, and for several weeks sent at least one special delivery letter a day to his algebraic friends announcing his latest finds. But, with the end in sight, he lost interest, and possibly all that he did is now more simply done in the manner of modern abstract algebra.

Brief descriptions of the material in some of the papers listed in the bibliography follow:

No. 1. This gives explicit determinations for the sides of all triangles described in the title. It was Blichfeldt's first paper.

No. 2. An investigation of the class of all finite continuous groups of n-space having not less than $m > 1$ invariant points, while all invariants of $s > m$ points are expressible in terms of the invariants of the system of m points which are a subset of the system of s points. For 3-space and $m = 2$, Lie discussed this problem in connection with his researches in the foundations of geometry. Blichfeldt extended this to $m = 3$, finding 8-, 7-, and 6-parameter groups according as the number of independent invariants for 3 points is 1, 2, or 3.

No. 3. This paper introduced a new method for the determination in question. The leading idea is that a group of point-transformations in the plane, in more than two parameters, leaves invariant at least one differential equation which is integral and algebraic in the derivatives.

No. 4. This belongs to an extensive department of approximate analysis originated by Tchebycheff. As an application, Blichfeldt finds as the greatest approximation to $\sin x$ in the form $a_1x + a_2$, in the interval $0<x<h<\pi/2$ the value

$$\frac{\sin h}{h}x+\tfrac{1}{2}\left[1-\left(\frac{\sin h}{h}\right)^2\right]^{\frac{1}{2}}-\frac{\sin h}{2\,h}\cos^{-1}\left(\frac{\sin h}{h}\right).$$

No. 5. A rigorous elementary proof, independent of the parallel axiom, of the famous theorem on the angle-bisectors.

No. 6. In n-space the distance between two arbitrary points is expressed as an algebraic function of their distances from a set of fixed points. It is postulated that a "distance-relation" exists between the mutual distances of $n + 2$ points.

No. 7. A specimen theorem must suffice. In every finite group of linear homogeneous substitutions on the n-variables, those substitutions whose orders are divisible only by primes greater than $(n\text{-}1)$ $(2n + 1)$, form a subgroup. The theory of group characters is used.

No. 8. A continuation of No. 6. An application determines all surfaces having a distance-relation between 5 arbitrary points with the property that between their rectilinear or geodesic distances two or more relations hold.

No. 9. Gives a characteristic invariant for central conics.

No. 10. Continuation of No. 7.

No. 11. One of the applications proves Frobenius' theorem that if the order of a group is divisible by n, then the number of those elements of the group whose orders divide n is a multiple of n.

No. 12. New proof and generalization of C. Jordan's fundamental theorem on finite groups of linear homogeneous substitutions in n-variables and their abelian subgroups.

No. 13. A complete determination.

No. 14. Continuation of Nos. 7, 10.

No. 15. This recognizes the importance of Galois fields in studies of representations of finite groups as groups of linear homogeneous substitutions.

No. 16. Application of the methods developed in Nos. 7, 10 to the complete determination of the groups in question.

No. 17. Extensions, with proofs of several theorems (1901) of Frobenius.

No. 18. Continuation of Nos. 7, 10, 14.

No. 19. This refers to what, after Minkowski is called the geometry of numbers, and it is here a question of trapping lattice points in n-dimensional continua by translations of the coordinate system. For an exact description, the paper itself must be consulted. One theorem is comprehensible without elaboration: Let F be a positive definite quadratic form of determinant D. Then integers $l_1, \ldots, l_n$, not all zero, may be substituted for the n variables such that the numerical value of F is not greater than

$$\frac{2}{\pi}\left[\Gamma\left(1+\frac{n+2}{2}\right)\right]^{2/n} D^{1/n},$$

where Γ indicates the usual gamma function. Asymptotically the value of this expression is $nD^{1/n}/2\pi e$, which is one half of that obtained by Minkowski.— This paper started Blichfeldt on a long series of researches on the minima of quadratic forms; the theorem cited is an example.

No. 20. Blichfeldt's section of this useful text was clearly and attractively written.

No. 21. A systematic treatise containing some of the author's original work.

No. 22. Too brief to be of much interest either as history or exposition. The promised amplied exposition evidently was abandoned, as it never appeared.

No. 23. This is the direction of approximations to linear functions, with rational coefficients, of linearly independent irrational numbers, of which the first example was given by Liouville. Blichfeldt approximates simultaneously to n linear forms of a special type.

No. 24. This is a sequel to No. 19. The minimum of a positive definite quadratic form in n variables with given determinant D is $\leqq \gamma_n D^{1/n}$, where γ_n depends only on n. Blichfeldt had obtained on upper limit for γ_n which he here sharpens.

No. 25. Explicit minima $\left(\frac{64}{3}\right)^{\frac{1}{6}}$, $(64)^{\frac{1}{7}}$, 2 respectively for the forms named in the title.

No. 26. A considerable generalization of certain work of Minkowski on a similar topic.

No. 27. Sufficiently described by the title.

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