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RICHARD DAGOBERT BRAUER

1901—1977

A Biographical Memoir by

J. A. GREEN

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Richard Brauer

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February 10, 1901–April 17, 1977

BY J. A. GREEN

RICHARD DAGOBERT BRAUER, Emeritus Professor at Harvard University and one of the foremost algebraists of this century, died on April 17, 1977, in Boston, Massachusetts. He had been an Honorary Member of the Society since 1963.

Richard Brauer was born on February 10, 1901, in Berlin-Charlottenburg, Germany; he was the youngest of three children of Max Brauer and his wife Lilly Caroline. Max Brauer was an influential and wealthy businessman in the wholesale leather trade, and Richard was brought up in an affluent and cultured home with his brother Alfred and his sister Alice.

Richard Brauer's early years were happy and untroubled. He attended the Kaiser-Friedrich-Schule in Charlottenburg from 1907 until he graduated from there in 1918. He was already interested in science and mathematics as a young boy, an interest which owed much much to the influence of his gifted brother Alfred, who was seven years older than Richard.

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His youth saved him from service with the German army during the first World War. He graduated from high school in September 1918, and he and his classmates were drafted for civilian service in Berlin. Two months later the war ended, and in February 1919 he was able to enroll at the Technische Hochschule in Berlin-Charlottenburg (now the Technische Universität Berlin). The choice of a technical curriculum had been the result of Richard's boyhood ambition to become an inventor, but he soon realised that, in his own words, his interests were "more theoretical than practical," and he transferred to the University of Berlin after one term. He studied there for a year, then spent the summer semester of 1920 at the University of Freiburg—it was a tradition among German students to spend at least one term in a different university—and returned that autumn to the University of Berlin, where he remained until he took his Ph.D. degree in 1925.

The University of Berlin contained many brilliant mathematicians and physicists in the nineteen-twenties. During his years as a student Richard Brauer attended lectures and seminars by Bierberbach, Carathéodory, Einstein, Knopp, von Laue, von Mises, Planck, E. Schmidt, I. Schur and G. Szegö, among many others. In the customary postscript to his doctoral dissertation [1], Brauer mentions particularly Bieberbach, von Mises, E. Schmidt and I. Schur. There is no doubt that the profoundest influence among these was that of Issai Schur. Schur had been a pupil of G. Frobenius, and had graduated at Berlin in 1901; he had been "ordentlicher Professor" (full professor) there since 1919. His lectures on algebra and number theory were famous for their masterly structure and polished delivery. Richard Brauer's first published paper arose from a problem posed by Schur in a seminar on number theory in the winter semester of 1921. Alfred Brauer also participated in this seminar. He was less fortunate than Richard, in that his studies were seriously interrupted by the war; he had served for four years with the army and been very badly wounded. The Brauer brothers succeeded in solving Schur's problem in one week, and in the same week a completely different solution was found by Heinz Hopf. The Brauer proof was published in the book by Pólya and Szegö (1925, p. 137, pp. 347-350), and some time later the Brauers and Hopf combined and generalized their proofs in their joint paper [2].

Richard Brauer also participated in seminars conducted by E. Schmidt and L. Bieberbach on differential equations and integral equations—a proof which he gave in a talk at this seminar in 1922 appears, with suitable acknowledgment, in Bieberbach's book on differential equations (1923, p. 129). But Brauer became more and more involved in Schur's seminar. As a participant in this, he reported on the first part of Schur's paper "Neue Anwendungen der Integralrechnung auf Probleme der Invariantentheorie" (1924), which shows how Hurwitz's method of group integration can be used for the study of the linear representations of continuous linear groups. In the second part of this work, Schur applied his method to determine all the irreducible (continuous, finite-dimensional) representations of the real orthogonal and rotation groups. He suggested to Brauer that it might also be possible to do this in a more algebraic way. This became Brauer's doctoral thesis [1], for which he was awarded his Ph.D. *summa cum laude* on March 16, 1926.

On September 17, 1925 Richard Brauer married Ilse Karger, a fellow-student whom he had first met in November 1920 at Schur's lecture course on number theory. Ilse Karger was the daughter of a Berlin physician. She studied experimental physics and took her Ph.D. in 1924, but she realized during the course of her studies that she was more interested in mathematics than in physics, and she took mathematics courses with the idea of becoming a school-teacher. In fact she subsequently held instructorships in mathematics at the Universities of Toronto and Michigan and at Brandeis University, and she eventually became assistant professor at Boston University. The marriage of Ilse and Richard Brauer was a long and very happy one. Their two sons George Ulrich (born 1927) and Fred Günther (born 1932) both became active research mathematicians, and presently hold chairs at, respectively, the University of Minnesota, Minneapolis, and the University of Wisconsin, Madison.

Brauer's first academic post was at the University of Königsberg (now Kaliningrad), where he was offered an assistantship by K. Knopp. He started there in the autumn of 1925, became Privatdozent (this is the grade which confers the right to give lectures) in 1927, and remained in Königsberg until 1933. The mathematics department at that time had two chairs, occupied by G. Szegö and K. Reidemeister (Knopp left soon after Brauer arrived), with W. Rogosinski, Brauer and T. Kaluza in more junior positions. The Brauers enjoyed the friendly social life of this small department, and Richard Brauer enjoyed the varied teaching which he was required to give. During this time he also met mathematicians from other universities with whom he had common interests, particularly Emmy Noether and H. Hasse.

This was the time when Brauer made his fundamental contribution to the algebraic theory of simple algebras. In [4], he and Emmy Noether characterized Schur's "splitting fields" of a given irreducible representation Γ of a given finite dimensional algebra, in terms of the division algebra associated to Γ. Brauer developed in [3],[5] and [7] a theory of central division algebras over a given perfect field, and showed in [13] that the isomorphism classes of these algebras can be used to form a commutative group whose properties give great insight into the structure of simple algebras. This group became known (to its author's embarrassment!) as the "Brauer group," and played an essential part in the proof by Brauer, Noether and Hasse [14] of the long-standing conjecture that every rational division algebra is cyclic over its centre.

Early in 1933 Hitler became Chancellor of the German Reich, and by the end of March had established himself as dictator. In April the new Nazi régime began to implement its notorious antisemitic policies with a series of laws designed to remove Jews from the "intellectual professions" such as the civil service, the law and teaching. All Jewish university teachers were dismissed from their posts. Later some exemptions were made—it is said at the request of Hindenburg, the aged and by now virtually powerless President of the Reich—to allow those who had held posts before the first World War, and those who had fought in that war, to retain their jobs. Richard Brauer came into neither of these categories, and was not reinstated. It is tragically well known that the "clemency" extended to those who were allowed to remain at their posts was short-lived. Alfred Brauer, whose war service exempted him from dismissal in 1933, eventually came to the United States in 1939. Their sister Alice stayed in Germany and died in an extermination camp during the second World War.

The abrupt dismissal of Jewish intellectuals in Germany in 1933 evoked shock and bewilderment abroad. Committees were set up and funds raised, particularly in Great Britain and the United States, to find places for these first refugees from Nazism. Through the agency of the Emergency Committee for the Aid of Displaced German Scholars, which had its headquarters in New York, and with the help of the Jewish community in Lexington, Kentucky, enough money was raised to offer Richard Brauer a visiting professorship for one year at the University of Kentucky. He arrived in Lexington in November 1933, speaking very little English, but already with a reputation as one of the most promising young mathematicians of his day. His arrival was greeted with sympathetic curiosity; the local paper reported an interview with the newcomer, conducted through an interpreter, and recorded Brauer's first impressions of American football. Ilse Brauer and the two children, who had stayed behind in Berlin, followed three months later. The friendly welcome which the Brauers found in Lexington, and their own adaptability, made the transition to life in the United States an easy one.

In that academic year 1933-34 the Institute for Advanced Study at Princeton came into full operation. Among its first permanent professors was Hermann Weyl. Brauer did not know Weyl personally, but had always hoped to do so from the time when he had been writing his thesis on the rotation group; Weyl's classic papers, in which he combined the infinitesimal methods of Lie and E. Cartan with Schur's group integration method to determine the characters of all compact semisimple Lie groups, appeared in 1925-26. It was therefore the fulfillment of a dream for Brauer to be invited to spend the year 1934-35 at the Institute as Weyl's assistant. Brauer's great admiration and respect for Weyl were returned. Many years later Weyl wrote that working with Brauer had been the happiest experience of scientific collaboration which he had ever had in his life. The famous joint paper on spinors [19] was written during this year, and also Brauer's paper [21] on the Betti numbers of the classical Lie groups. Pontrjagin had recently determined these numbers by topological means (1935), and Brauer, in response to a question by Weyl, was able to give in a few weeks an alternative purely algebraic treatment based on invariant theory. The references to Brauer in Weyl's book *The Classical Groups* (1939) make evident the esteem in which he held his younger colleague. Brauer collaborated with N. Jacobson, who had been Weyl's assistant during the second half of 1933-34, in writing up notes of Weyl's lectures on Lie groups, and of some of the seminar talks which followed. These appeared under the title *The Structure and Representation of Continuous Groups* (Princeton, 1934-1935).

The year at Princeton was very productive of new mathematical contacts for Brauer. The Institute was already a brilliant centre for mathematics. Besides its permanent professors (J. W. Alexander, A. Einstein, J. von Neumann, O. Veblen and Weyl) there were in the School of Mathematics that year four assistants and thirty-four "workers" (i.e. visiting members). Among the latter were W. Magnus, C. L. Siegel and O. Zariski, all of whom were to become lifelong friends of the Brauers. Brauer's mathematical contact with Siegel was particularly close, and bore fruit later in [52]. In addition to the mathematicians at the Institute, the mathematics faculty at Princeton University (who were then housed in the same building) included Bochner, Lefschetz and Wedderburn. The Brauers were also able to see Emmy Noether regularly, because she was giving a weekly seminar at Princeton that year. Emmy Noether was another refugee from Nazism, and held a post as visiting professor at Bryn Mawr College, Pennsylvania, from 1933 until her death in the spring of 1935.

It was a result of the account of him given by Emmy Noether when she visited the University of Toronto that Brauer was offered an assistant professorship there. He took up this post in the autumn of 1935, and was to remain in Toronto, holding in due course positions as associate and then full professor, until 1948. At Toronto, Brauer developed his famous modular representation theory of finite groups, which will probably always be regarded as his most original and characteristic contribution to mathematics. Some of the preliminaries to this theory appeared in 1935 in [18], but the first full treatment of modular characters, decomposition numbers, Cartan invariants and blocks was published jointly with C. J. Nesbitt in 1937 ([27]). Nesbitt was Brauer's doctoral student at Toronto from 1935-37, and he has given this interesting account of their collaboration. "Curiously, as thesis advisor, he did not suggest much preparatory reading or literature search. Instead, we spent many hours exploring examples of the representation theory ideas that were evolving in his mind. Eventually, I pursued a few of the ideas for thesis purposes, they received some elegant polishing by him, and later were abstracted and expanded by another great friend, Tadasi Nakayama. Professor Brauer generously ascribed joint authorship to several papers that came out of these discussions but my part was more that of interested auditor."

One of these joint papers with Nesbitt "On the modular characters of groups" [34] appeared in 1941 and remained for many years the only readily available reference for modular theory. An essential part of this theory was a new general representation theory of algebras, initiated by Brauer and developed by him, Nesbitt and Nakayama during this period.

Brauer's teaching contribution to mathematics at Toronto was considerable; his lectures and seminars were well-attended, and he had several Ph.D. students apart from Nesbitt, including R. H. Bruck, S. A. Jennings, N. S. Mendelsohn, R. G. Stanton and R. Steinberg. Brauer was elected to the Royal Society of Canada in 1945. With his Toronto colleagues H. S. M. Coxeter and G. de B. Robinson he was involved in the Canadian Mathematical Congress and the founding of the Canadian Journal of Mathematics. During his years in Canada he kept up many contacts with the United States; he was visiting professor at the University of Wisconsin in 1941, and a visiting member of the Institute of Advanced Study in 1942. In 1942 he also spent some time with Emil Artin at Bloomington, Indiana. Brauer had met Artin briefly in Hamburg, but this was their first real mathematical and personal contact. Their discussions and correspondence over the ensuing years resulted in Brauer's famous proof [51] of Artin's *L*-function conjecture, and a series of subsequent papers relating to class-field theory, for which he received the American Mathematical Society's Cole Prize in 1949. Artin and Brauer were to remain close friends until Artin's death in 1962.

By 1948 Brauer was becoming one of the leading figures on the international mathematical scene, and it can have surprised no one when he moved back to the United States in that year, to a chair at the University of Michigan, Ann Arbor. Nesbitt was on the faculty there, but by then had moved into another area of mathematics, and the few graduate courses in algebra were being taught by R. M. Thrall, who already had considerable contact with the work of Artin, Brauer and Nakayama. Brauer at once set about enlarging the graduate programme in algebra and number theory, and he took on a big personal load of advanced lectures, seminars and Ph.D. supervision. There was no National Science Foundation to support research in those days, but many of the best international researchers were prepared to lecture at summer schools in the United States. Michigan had always had a particularly good and well-attended summer programme in mathematics, which was now enhanced by the attraction of Brauer. When Brauer was not involved in such an Ann Arbor summer, he and Ilse would take vacations at Estes Park, Colorado, where there were usually other algebraists present—for example Reinhold Baer, a former school-fellow of Brauer's in Berlin, and now at the University of Illinois at Urbana. Michigan became one of the liveliest centres of algebra, with a remarkable young generation—Ph.D. students of Brauer's included K. A. Fowler, W. Jenner and D. J. Lewis; and W. Feit, J. P. Jans and J. Walter were students while Brauer was at Michigan, although they did not take their doctorates with him. A. Rosenberg was a post-doctoral fellow at Michigan during this time, and the junior faculty included M. Auslander and J. McLaughlin.

About 1951 Brauer, together with his pupil K. A. Fowler, found the first group-theoretical characterization of the simple groups $LF(2, q)(q \geq 4)$. At nearly the same time, M. Suzuki in Japan had proved a similar theorem for the case $q = p$ (prime), and later introduced important simplifications in the proof of the general case with his method of "exceptional characters". G. E. Wall, who was then at Manchester, had also arrived at Brauer's theorem independently by about 1953. The final version, a joint paper by Brauer, Suzuki and Wall [70], did not appear until 1959. This work, together with Brauer and Fowler's paper "On groups of even order" [64], marked the beginnning of a new advance in the theory of finite groups. A few years later W. Feit and J. Thompson made another breakthrough with their long proof (Feit, Thompson 1963) of the old conjecture of Burnside that every non-Abelian finite simple group has even order. Most of the great progress in the understanding and classification of finite simple groups, which has dominated algebra in the past 25 years, can be traced to these pioneering achievements. Brauer was to remain a leading contributor to this progress.

The Brauers were very happy at Ann Arbor, and expected to stay there for the rest of their lives. However in 1951

Brauer was offered a chair at Harvard University, which he accepted. He took up this post in 1952, and stayed at Harvard until he retired in 1971; he and Ilse lived at Belmont, Massachusetts until his death in 1977.

Brauer was fifty-one years old when he went to Harvard. It is a striking fact of his career that he continued to produce original and deep research at a practically constant rate until the end of his life. About half of the 127 publications which he has left were written after he was fifty; the years 1964-77 produced 44 papers. The mathematical atmosphere at Harvard and at the neighboring Massachusetts Institute of Technology was very congenial to Brauer, who had many contacts at both places. He had an impressive catalogue of successful students at Harvard, including D. M. Bloom, P. Fong, M. E. Harris, I. M. Isaacs, H. S. Leonard, J. H. Lindsey, D. S. Passman, W. F. Reynolds, L. Solomon, D. B. Wales, H. N. Ward and W. Wong—and this list, like those which we have given of Brauer's students at Toronto and Michigan, is far from complete. Besides students, there were many visitors who came to Harvard because Brauer was there. The Brauers were a hospitable couple, and had always liked to entertain colleagues and students in their home. Everyone who had contact with Brauer in his years at Harvard, whether as student, colleague or visitor, has spoken of the great warmth and personal interest which he and Ilse brought to the mathematical community in the Boston area.

The Brauers travelled abroad regularly, usually to Europe where there were old friends. They visited the Baers in Frankfurt, after they had returned to Germany in 1956. They regularly spent summer vacations at Pontresina in Switzerland with C. L. Siegel, and also visited him in Göttingen—Brauer held the Gauss professorship at the Akademie der Wissenschaften there for a semester in 1964. In 1959-60 he was visiting professor at Nagoya University in Japan at the invitation of T. Nakayama, whom the Brauers had known for many years. They visited England frequently to stay with the Rogosinskis in Newcastle. Brauer was made honorary member of the London Mathematical Society in 1963, and was Hardy Lecturer in 1971. He and Ilse spent a term at Warwick in 1973, which is remembered with great pleasure; Brauer's paper [126] had its origin in the seminar on modular representations which he held on this occasion. In 1972 Brauer was visiting professor at Aarhus University in Denmark.

Early in 1969 Brauer began to suffer from myasthenia gravis, a neurological disease which causes a selective weakening of the muscles, in his case the muscles of the eye. Although he could still read, this partial paralysis impaired his side vision and made him see double from beyond a certain distance. He adjusted himself with great fortitude to this distressing condition, and managed to lead an almost normal life in spite of it.

Brauer received many honours in the course of his life, and a list of these is given at another place in this notice. We mention here his election to the National Academy of Sciences in 1955, the Cole Prize of the A.M.S. in 1949 for his work on class-field theory, and the National Medal for Scientific Merit awarded to him by the President of the United States in 1971.

In 1976 Brauer became sufficiently ill to require hospital treatment on two occasions—in his own words, "For the first time in my life I have seen hospital rooms at night." He made a good recovery and continued his busy working life. But in the middle of March 1977 he had to be rushed to the hospital again. He was suffering from aplastic anaemia, a condition in which the body no longer produces enough blood cells, and consequently loses its natural defenses against infection. He knew that he was very ill, but did not doubt that he would recover eventually. He continued to deal with his correspondence from his hospital bed, dictating letters to Ilse, who stayed with him throughout his illness. A general sepsis led to his death on 17 April.

Richard Brauer has been one of the most consistent and effective influences in algebra this century. His work provides an example of mathematical research and scholarship at its best. He solved important problems which had long been outstanding in group representation and number theory, and in the process he made major theoretical advances which have since become incorporated into the groundwork of algebra. We shall mention here only one example, the theory of linear associative algebras. This was enriched by Brauer in two ways: first by his theory of simple algebras, which led to the paper by him, Noether and Hasse on rational division algebras, and which was the result of Brauer's studies on the Schur index of a representation. His second contribution to the theory of algebras was his analysis of the regular representations of a non-semisimple algebra, which led to the idea of projective and injective modules, the local (p-adic) theory of orders in a semisimple algebra, and to Nakayama's researches on Frobenius algebras. This work was one of the byproducts of Brauer's theory of group representations over a field of finite characteristic.

The progress of this "modular" theory of group representations shows all of Brauer's remarkable mathematical qualities at work. Frobenius and Burnside had revolutionized the theory of finite groups in the first decade of this century, and some of their deepest results were those obtained by the application of the new theory of group characters. The idea of a modular theory of group representations was not new; Dickson had already done some pioneering work in the early 1900s (Dickson 1902, 1907). Schur suggested, in lectures at Berlin, an "arithmetic" approach: a given rational prime p generates, in the integral group ring of ZG of a given finite group G, an ideal whose prime divisors, in a suitable order containing ZG, correspond to the types of irreducible representations of G over a field of characteristic p. But it was Brauer who solved, one by one, the enormous technical and conceptual problems which stood between Schur's idea and a theory which could contribute to the understanding of the structure of the group G. Brauer always considered that the aim of his theory was to give information about the structure of groups; more particularly, he used modular theory as a way of obtaining refined "local" information on the *ordinary* character table of G—his beautiful theory of blocks being the principal means to this end. His judgment was brilliantly vindicated in the event, and it is hard to imagine any other contemporary algebraist with the superb creative and technical resource necessary to carry through Brauer's programme.

Brauer's many students, and others who were influenced by his teaching at an early stage in their careers, are now to be found at universities throughout the United States and Canada. To them he transmitted a fine tradition of German algebra and number theory which can be traced back through Schur to Frobenius and Kronecker. Brauer's lectures were carefully prepared and undramatic; he was very concerned to give proofs in complete detail (in contrast to the prevailing fashion), and would sometimes go back and rephrase an argument two or three times in order to make things clearer. Some students found this tedious, but there were others who came to realize that Brauer had few equals as an expositor, both of mathematical ideas and of techniques. A former student at Michigan has said of his lectures, "You had the feeling you were seeing a magnificent structure being built before your eyes by a master craftsman, brick by brick, stone by stone." Many people have expressed the hope that some of Brauer's lecture courses might eventually be published.

It is not possible to separate Richard Brauer's mathematical qualities from his personal qualities. All who knew him best were impressed by his capacity for wise and independent judgment, his stable temperament and his patience and determination in overcoming obstacles. He was the most unpretentious and modest of men, and remarkably free of self-importance. He was embarrassed to find his name attached adjectivally to some of his discoveries, and rebuked a student, gently but seriously, for referring to "Brauer algebra classes" in the theory of simple algebras--this was at Harvard, and the offending terminology had been standard for at least twenty years!

Brauer's interest in people was natural and unforced, and he treated students and colleagues alike with the same warm friendliness. In mathematical conversations, which he enjoyed, he was usually the listener. If his advice was sought, he took this as a serious responsibility, and would work hard to reach a wise and objective decision.

Richard Brauer occupied an honored position in the mathematical community, in which the respect due to a great mathematician was only one part. He was honored as much by those who knew him for his deep humanity, understanding and humility; these were the attributes of a great man.

HONORS AND HONORARY POSTS HELD BY RICHARD BRAUER

Elected memberships of learned societies

Royal Society of Canada 1945
American Academy of Arts and Sciences 1954
National Academy of Sciences 1955
London Mathematical Society (Honorary Member) 1963

Akademie der Wissenschaften Göttingen (Corresponding Member) 1964

American Philosophical Society 1974

Prizes, etc.

Guggenheim Memorial Fellowship 1941-42

Cole Prize (American Mathematical Society) 1949

National Medal for Scientific Merit 1971

Honorary doctorates

University of Waterloo, Ontario 1968

University of Chicago 1969

University of Notre Dame, Indiana 1974

Brandeis University, Massachusetts 1975

Presidencies of Mathematical Societies

Canadian Mathematical Congress 1957-58

American Mathematical Society 1959-60

Editorships of learned journals

Transactions of the Canadian Mathematical Congress 1943-49

American Journal of Mathematics 1944-50

Canadian Journal of Mathematics 1949-59

Duke Mathematical Journal 1951-56, 1963-69

Annals of Mathematics 1953-60

Proceedings of the Canadian Mathematical Congress 1954-57

Journal of Algebra 1964-70

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