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KURT OTTO FRIEDRICHS

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A Biographical Memoir by
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THIS MEMORIAL OF Kurt Otto Friedrichs is given in two parts. The first is about his life and its relation to his mathematics. The second part is about his work, which spanned a very great variety of innovative topics where the innovator was Friedrichs.

PART I: LIFE

Kurt Otto Friedrichs was born in Kiel, Germany, on September 28, 1901, but moved before his school days to Düsseldorf. He came from a comfortable background, his father being a well-known lawyer. Between the views of his father, logical and, on large things, wise, and the thoughtful and warm affection of his mother, Friedrichs grew up in an intellectual atmosphere conducive to the study of mathematics and philosophy. Despite being plagued with asthma, he completed the classical training at the local gymnasium and went on to his university studies in Düsseldorf. Following the common German pattern of those times, he spent several years at different universities. Most strikingly for a while he studied the philosophies of Husserl and Heidegger in Freiburg. He retained a lifelong interest in the subject of philosophy but eventually decided his real bent was in mathematics. He chose to complete his studies in Göttingen, the mecca of mathematics in the 1920s. There he met Richard Courant, director of the Institute of Mathematics, who admired enormously his talent but found him somewhat unworldly. In fact, Friedrichs's childhood asthma had prevented him from participating in the activities where children naturally socialize and he was painfully shy. Courant and Friedrichs enjoyed a lifelong friendship that included a great deal of mathematical stimulation, cooperation and interaction, a lot of practical advice from Courant, and a fair dose from Friedrichs of his logical approach to life and his special values. On the basis of my own observations for over twenty-five years, they dealt with each other's idiosyncrasies in a remarkably comfortable way.

Friedrichs stayed in Göttingen for five years. During this time he completed his first paper clarifying the logical significance of Einstein's general covariance postulate (1927,1) and then wrote his dissertation on boundary and eigenvalue problems for elastic plates (1927,2). This was followed by a paper with Hans Lewy (1927,3) on initial value problems for linear hyperbolic partial differential equations.

Those first three papers demonstrate most of Friedrichs's lifetime in mathematics—the first on the fundamental laws of the nature of matter, the second on applied mathematics viewed through analysis, and the third on basic theorems of wave propagation.

The paper on hyperbolic partial differential equations led naturally to what turned out to be one of the best-known and most used results of mathematics of the time (1928,2). In investigating whether difference schemes for a time-varying partial differential equation like the wave or heat equation yield a good approximate solution, Courant, Friedrichs, and Lewy were led to consider the stability of the difference scheme. In a simple difference scheme every partial derivative like $\partial f/\partial x$ is replaced by a difference of the approximating function at two values of x divided by the difference of "step" in the values of x. The three authors made the remarkable discovery that the steps in time could not be chosen arbitrarily but had to be smaller than some constant times the steps in the space variable. For the wave equation that constant was the reciprocal of the speed of propagation. For other equations there may be many such speeds or one may have a different kind of propagation, but there is always a limitation of space step (Δx) and time step (Δt). But the basic idea comes from this paper, and the constants are all known as CFL numbers. There is scarcely a talk or a paper on modeling phenomena governed by so-called explicit difference schemes where this number does not come up.

Friedrichs was interested, however, in proving existence theorems for partial differential equations by letting the mesh size and time step become vanishing small. The modeling aspect was a side product that only became important after World War II when one could compute something useful on a large computer. In later life, when Friedrichs was pressed to say something about the important role of computer modeling in applied mathematics to which he had made such a fundamental contribution, he simply would not bite.

After Friedrichs completed his dissertation and two years of an assistantship, according to Constance Reid's fascinating obituary[1] for the *Intelligencer*, Courant advised his shy young friend that in the severe competition for positions at German universities Friedrichs would need some special advantage and therefore he should become an applied mathematician. Thus, Friedrichs followed Theodore v. Karman to Aachen where v. Karman had become the first professor of aeronautical engineering. That was an exciting time in aerodynamics in Germany. First, it was a new and eminently practical subject, not like the exotic theories of quantum mechanics and relativity. Second, the interdiction of the building of airplanes in Germany in accordance with the Versailles treaty made the understanding of aerodynamics from a theoretical point of view a matter of vital concern to the nation. Friedrichs returned to Göttingen two years later with a deep knowledge of aerodynamics, which was to serve him later in America.

But his mathematical interest had shifted to the modern theory of operators in Hilbert spaces. He even rewrote a paper (1934) couched in classical language so that it was in von Neumann's "new" abstract language. He solved several problems in spectral theory and used the new method to solve the initial value problem for hyperbolic equations with only energy integrals. This led some years later to the idea of weak solutions, a concept that is the backbone of the modern theory of linear and some nonlinear equations.

In 1931 Friedrichs was called to Braunschweig to the Technische Hochschule as a full professor, a rare recognition in prewar Germany for a man of thirty. But the Hitler era was about to begin. After five years of increasing political difficulties at Braunschweig, a visit to Courant who had emigrated to New York, and most important, after meeting Nellie Bruell, his future wife, Friedrichs saw clearly that he too would have to emigrate, which he did in 1937.

Once again under Courant's influence, Friedrichs started dancing on what he liked to call his "other foot"—namely, doing applied mathematics. On the whole, until the end of the Second World War his main contributions were in fluid dynamics with Courant and in elasticity with J. J. Stoker.

When I first met him in 1946, Friedrichs was celebrating the end of the years of war effort in applied mathematics by teaching an innovative course in topology. But he and

Courant were also finishing their basic and still used book on compressible fluid dynamics (1948,1). I was selected to edit it mainly for English and thus I came to know Friedrichs's meticulous, very careful, and correct, but at times slightly pedantic, approach to the subject. Since English was after all my native language, I had some trouble with Friedrichs's desire to find a rule (mostly from Fowler's) to follow. His English was remarkable, but he never quite became idiomatic. Courant enlivened the text considerably, sometimes to Friedrichs's disappointment at the expense of correctness.

By 1951 his spectral theory work, which he had renewed after 1945, led him into his fundamental work in the quantum theory of fields. He published five monographs, which inspired a number of today's mathematical physicists, especially in the circle of Olga Ladyzhenskaya and Ludwig Fadeev in Russia. All the time as the spirit moved him he would do a basic piece for applied mathematics or a big chunk of writing for the famous Courant-Hilbert[2] volume 2, completed in German in 1937 but essentially rewritten in English by many of the faculty of the burgeoning institute that was to become the Courant Institute in the early sixties.

By the 1940s, Friedrichs was recognized as one of the new leaders in American mathematics. He was elected to the National Academy of Sciences in 1959. He received many awards and honorary degrees culminating in the National Medal of Science in 1976, which he received "for bringing the powers of modern mathematics to bear on problems in physics, fluid dynamics and elasticity."

Despite all the recognition, Friedrichs's modesty could be overwhelming. When we discussed what should be republished in his *Selecta,* he kept reiterating about each of his discoveries that someone else had done it better later and why should someone want to read his less effective original presentation. I succeeded in most cases in overruling him.

He was never shy mathematically and was, by the 1950s, much less shy socially. He had a great impact on what Courant's fledgling institute became and thereby he greatly influenced applied mathematics in America both directly and indirectly.

I must mention one other important influence. While afternoon teas as a fundamental ingredient of the intellectual life of a mathematician had been conceived at the Institute for Advanced Study in Princeton, Friedrichs carried the idea to the surroundings of NYU with his imposing principles. When the secretaries balked at washing the dishes after colloquia and Anneli Lax and I refused to take over, Friedrichs persuaded Courant to hire someone to make tea and coffee and wash up every day of the week. Today the notion that this custom is conducive to producing mathematics has been taken over almost everywhere.

He was a prodigious worker. Nellie shielded him from unnecessary dealings with the outside world, even to providing breakfast on a tray for his most productive early morning labors. But his five children, Walter, Liska, David, Christopher, and Martin, were a source of enormous interest and pleasure, and he meticulously guarded the time he spent with them from being interrupted by some mathematical spasm.

When he died at the age of eighty-one, one of Friedrichs's wishes was that his last works that dealt with the true way to regard the uncertainty principle and other quantum mechanical concepts should be properly understood and seen correctly from a philosophical point of view.

PART II: WORK

If Friedrichs's contributions to analysis and applied mathematics had a central theme, it was partial differential equations (p.d.e.), and it is appropriate to start there. We shall concentrate on three fundamental subjects covered in three long papers dealing with regularity of solutions for elliptic systems (1953), existence and uniqueness for symmetric hyperbolic systems (1954), and symmetric positive systems (1958). All of these papers might be considered natural applications of the watershed result (1944), where the weak extension of a system of first-order differential operators with C^1 coefficients is shown to be the same as the strong extension. Mollifiers, of which hints were given earlier, emerge as the main tool, as they are in the theory of distributions.

The first major outcome was the regularity for elliptic systems using mollifiers as a tool. These regularity results are now standard theory. To mollify a not-so-smooth function you convolute it with a very smooth one concentrated in a neighborhood of a point. The convolution is close to the identity operator. Then one presses out the desired estimates by letting the neighborhood shrink. By this device Friedrichs avoided using the Lebesgue theory, where, as he liked to put it, you have to write almost everywhere, almost everywhere.

The second major outcome was for symmetric hyperbolic systems. Friedrichs's interest in this problem goes back much further, to his work with H. Lewy and to the paper described in Part I with Courant and Lewy. The basic idea of the work with Lewy (1932) was to base not only uniqueness but also existence of solutions to the wave equation on energy estimates for the solution and its higher derivations. This idea was taken up subsequently by Schauder, Petrovsky, and Sobolev. The two authors would have gone further, via difference equations, but were stumped by the need to prove that the expansion around the origin in three-space of

$$\frac{1}{(1-\beta)(1-\gamma)+(1-\alpha)(1-\gamma)+(1-\alpha)(1-\beta)}$$

has positive coefficients. An elegant proof was supplied by G. Szego, but the new paper had become so involved and technical that the effort was set aside. By the time Friedrichs returned to the linear problem in his paper "Symmetric Hyperbolic Differential Equations" (1954), he was able to devise a more direct difference scheme with positive coefficients. The problem has a long history at various levels of generality going back to Hadamard. It was of deep physical importance since almost all the key hyperbolic systems, those of electromagnetic theory, compressible flow, and magneto fluid dynamics can be reduced, as Friedrichs showed for the last two, to the symmetric case.

The key ingredients Friedrichs used are energy estimates, the projection theorem in Hilbert space, and mollifiers. Finite difference methods are used to establish differentiability. The result is the existence and uniqueness for all time of a solution to mixed initial boundary value problems for very wide classes of initial data and boundary conditions.

The third major result is on "Symmetric Positive Linear Differential Equations" (1958). The type of equation given in the title is special and is defined as follows. A first-order linear operator

$$K=\sum \alpha^{\rho}\frac{\partial}{\partial x^{\rho}}+\gamma,$$

α^{ρ} and γ square matrix valued functions, is called *symmetric positive* if $K + K^{*}$ is formally positive, where K^{*} is the formal adjoint of L:

$$K^* = -\sum \frac{\partial}{\partial x^\rho} \alpha^{\rho *} + \gamma^* \,.$$

Along with the operator K go special boundary conditions of positive type.

A remarkably wide variety of classical and nonclassical ones, such as various boundary value problems for a certain class of elliptic systems, the Cauchy problem for a certain class of hyperbolic equations, mixed initial boundary value problems for hyperbolic equations, and, last but not least, certain boundary value problems for equations of mixed type, such as Tricomi's equation, are symmetric positive. Friedrichs's main motivation was to treat systematically equations of mixed type and, as he often said, to establish a method of proof that was "deaf," as he put it, to changes of type and would at the same time yield the number and kinds of boundary conditions necessary for well-posed problems. He succeeded in "deaf" proofs only in some isolated cases but instead introduced a fundamental new approach to weak existence.

Pseudo-differential operators came into being near the end of Friedrichs's career. He grasped their importance immediately, especially for symmetrizing recalcitrant differential operators. He made many useful technical innovations, and he invented the name for the subject.

Two of the big applications of partial differential equations are in fluid dynamics and elasticity. Aside from the "bible" of shock wave theory written with Courant, Friedrichs made many contributions to fluid dynamics, several of them unpublished results from wartime work of the forties. These include work on flow through nozzles, over surfaces of revolution, in detonations, and deflagrations. From a mathematical point of view, his most important contribution to elasticity was in simplifying and clarifying the very long proof of Korn's inequality (1947). This has been recently reduced to two pages[3] by Olga Oleinik. Friedrichs showed how to solve the natural boundary value problems of elasticity. His work with J. J. Stoker (1941, 1942) on buckling problems broke new ground in a nonlinear problem that was of great importance to engineers. The methods were basically asymptotic boundary layer methods adapted from Prandtl's fluid boundary layer theory. An important key was matching two asymptotic expansions, a so-called inner and an outer expansion. This technique, which Friedrichs used rigorously, also entered the folklore of applied mathematics, often less rigorously. Friedrichs loved this work, particularly because it was the basis of a long-lasting cooperation with his friend Stoker.

Friedrichs's years in Göttingen coincided with the rising interest among mathematicians, notably Hilbert and von Neumann, to put the rapidly developing new physics on a logical basis. In his basic book on quantum mechanics von Neumann had identified the states of a quantum mechanical system with unit vectors in a Hilbert space and observables as self-adjoint operators. A self-adjoint operator, according to von Neumann and Marshall Stone, is an operator L defined on a domain $D(L)$, whose adjoint has the same domain as L and coincides with L there. To apply this definition, one needs an exact description of the domain of L. In practice this is a painfully pedantic process, straining the abilities of mathematicians and never accepted by physicists. Friedrichs was able to eliminate the need for such pedantry. He showed that if an operator is bounded from below—and almost all Schrödinger operators are so bounded—one can start with a crude skeleton of the operator, defined on a much smaller set, and then reconstruct the true operator from the skeleton by a process known ever since as the Friedrichs extension. Such important cases as the harmonic oscillator, the hydrogen atom, and the boson quantum field could be treated immediately. Other cases have followed.

In 1938 Friedrichs investigated in several crucial examples what happens under perturbation to the spectrum of an operator that has continuous or mixed continuous and discrete spectrum. This work was prompted by the work for discrete spectra of F. Rellich[4] but was applicable to quantum mechanical scattering. When Heisenberg and Møller introduced the scattering matrix and wave operators, respectively, in the forties, Friedrichs had already developed the tools for a time-independent approach in his early work. This work is described in English with some additions in "On the Perturbations of Continuous Space" (1948,1).

Friedrichs went on to write a series of books in quantum physics. In the first he gave precise definitions to many of the basic but somewhat confused notions of quantum field theories. Many mathematical physicists have used these books as starting points for their quantum field work. One of Friedrichs's last contributions to quantum mechanics was his paper "Unobserved Observables and Unobserved Causality" (1981). This paper is a contribution to one of the great controversies of twentieth-century physics, the debate between Niels Bohr and Albert Einstein over the completeness of quantum mechanics. Einstein felt that the laws of nature should be deterministic or causal, and for this reason he felt the quantum mechanical notion of the state of a particle was incomplete. Friedrichs introduced the "intrinsic state" of a particle and argued that for this intrinsic state causality is valid but, in accordance with quantum theory, not verifiable. It is yet to be seen whether this truly reconciles the two sides of the controversy.

There are still many other areas where Friedrichs made fundamental and deep contributions. The most notable of these was asymptotic theory, which grew out of his nonlinear elasticity work. In 1954 Friedrichs gave his Gibbs lecture, a semipopular expository talk in applied areas, at the American Mathematical Society on this subject. There he surveyed the role of discontinuities in many physical phenomena, pointing out how important they are to us as observers (see 1954 paper). Shock waves, boundary layers, shadows, edge effects, and Stokes phenomena in ordinary differential equations are all examples. They all come about because of a singular limiting process where some parameter that could be viscosity, frequency, the reciprocal of Hooke's constant, etc., goes to zero. Every one of these phenomena is, however, slightly different, and Friedrichs helped clarify and rigorize the process for many of them. Friedrichs ended the Gibbs lecture by showing how asymptotics enter the adiabatic theorem of quantum mechanics. It is another mark of his modesty that there is only one reference to himself in the bibliography, although he had made so many vital contributions to the subject.

Taken altogether, as one looks at Friedrichs's very extensive list of publications, one cannot help but be struck by the variety of ways and the number of times Friedrichs broke new ground and then, with the passage of only a few years, how many of his ideas were absorbed into modern analysis and applied mathematics and became standard, so that it is almost forgotten today that they were Friedrichs's.

As I write this, the spirit of Friedrichs hangs over me fussing over the details that are not quite right and reminding me that I have left out this and I should have put in that, but I remind him that it was he who said, "Open your own newly published work on any random page and you will find a mistake." For those who would like to read his works in detail, I refer to the *Selecta* published by Birkhauser

(1986) from which I have drawn heavily, in particular from David Isaccson's article about his role in physics.

NOTES

1. C. Reid. K. O. Friedrichs. *Mathematical Intelligencer.* 5:(1983):23-30.

2. D. Hilbert and R. Courant. *Methoden den Mathematische Physik.* Berlin: Springer (1931, 1937). (English translation and revision, Interscience Publishers, 1953).

3. V. Kondratiev and O. Oleinik. On Korn's inequalities. *C. R. Acad. Sci.* 308(1989):483-87.

4. F. Rellich. Storungstheorie der Spektralzerlegung. *Mathematische Annalen* 113(1936):600-19; 113(1937):677-85; 116(1939):555-70; 117(1939):355-82; 118(1942):462-84.

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