Gene H. Golub

1932–2007

BIOGRAPHICAL *Memoirs*

A Biographical Memoir by
Dianne P. O'Leary

NATIONAL ACADEMY OF SCIENCES

GENE HOWARD **GOLUB**

February 29, 1932–November 16, 2007

Elected to the NAS, 1993

Gene Howard Golub was the most influential person of his generation in the field of numerical analysis, in particular, in the area of matrix computation. This was a consequence not just of his extraordinary technical contributions but was also due to his clear writing, his influential treatise on matrix computation, his mentorship of a host of students and collaborators, his service as founder and editor of two journals, his leadership in the Society for Industrial and Applied Mathematics (SIAM), his efforts to bring colleagues together in meetings and electronic forums, and his role as a research catalyst.

By Dianne P. O'Leary

Biography

Gene was born in Chicago, Illinois, on Leap Day, February 29, 1932, to Bernice Gelman Golub (an immigrant to the U.S. from Latvia) and Nathan Golub (from Ukraine). Gene's father worked as a "bread man" and his mother was a seamstress, and Gene was proud of his Jewish heritage. From the age of 12 Gene worked, first at his cousin's pharmacy and later at other jobs. After junior college and a year at the University of Chicago, he transferred to the University of Illinois in Urbana-Champaign, where he received his BS in Mathematics (1953), MA in Mathematical Statistics (1954), and PhD in Mathematics (1959). It was at Illinois that he first learned matrix theory, in a course taught by Franz Hohn. He also learned to program the ILLIAC I computer and implemented numerical methods as an assistant in the computer lab under John Nash.

Gene's employment history after his Ph.D can be stated quite concisely. A National Science Foundation fellowship took him to Cambridge University for a year. After brief stints at the Lawrence Radiation Laboratory and Space Technology Laboratories, Gene held temporary positions at Stanford University and at the Courant Institute. He was hired by George Forsythe in 1966 as associate professor in the newly-established

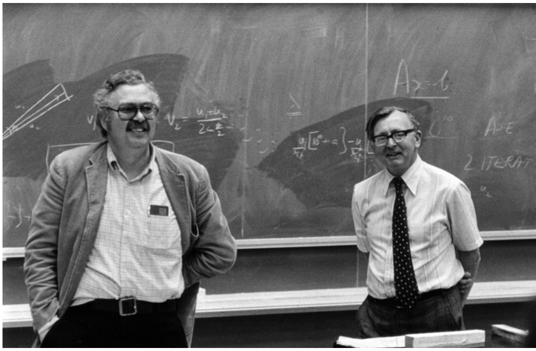
Computer Science Department at Stanford. He never left, although an astounding number of major universities and research institutes in the US and Europe hosted him for short- or long-term visits. He served as chair of his department from 1981 to 1984 and was named Fletcher Jones Professor of Computer Science in 1991.

Gene received ten honorary degrees from institutions including Linköping University, University of Grenoble, University of Waterloo, University of Dundee, University of Illinois, Universitat Louvain, University of Umea, Australian National University, Rostov State University in Russia, and Hong Kong Baptist University. He was a Guggenheim Fellow and had the rare distinction of being elected a member of both the National Academy of Engineering (1990) and the National Academy of Sciences (1993). In addition, he was a fellow of the American Academy of Arts and Sciences (1994), as well as a foreign member of the Royal Swedish Academy of Engineering Sciences.

Additional biographical information about Gene can be found in [Gre04].

Gene's research contributions

Gene was an exceptionally hard worker, comfortable collaborating everywhere from a train [CGO07, p.203] to an amusement park [CGO07, p.435]. His work in computational linear algebra was broad and deep, with foundational contributions to matrix decompositions, iterative methods for solving linear systems of equations, solving least squares problems, orthogonal polynomials and quadrature, eigenvalue problems, and applications. Some of his most influential papers are collected in a 2007 volume of selected works [CGO07] and a full bibliography of his work has been compiled by Nelson H. F. Beebe and Stefano Foresti.[1]



Gene Golub and James Wilkinson.
(Photo credit Gene Golub's collection.)

Matrix decompositions: Gene fundamentally transformed the computational world through his ability to find precisely the right computational tool to solve a given matrix problem. If that tool did not exist, he invented it.

1. Nelson H. F. Beebe and Stefano Foresti, Bibliography of Gene H. Golub, http://ftp.math.utah.edu/pub/bibnet/authors/g/golub-gene-h.bib, accessed 28-03-2018.

A prime example is his work in developing an algorithm for computing the singular value decomposition (SVD) of a matrix, an orthogonal reduction of a matrix to diagonal form. For many matrices this reduction is computed through their eigenvalues and eigenvectors, but for "non-Hermitian matrices," the eigendecomposition has many undesirable qualities: it is non-orthogonal, possibly ill-conditioned, and it fails to exist for nonsquare matrices (number of rows different than number of columns) and for some square matrices, too. In contrast, the SVD exists for any matrix, even nonsquare ones, and since the reduction is orthogonal, it is numerically stable. Golub and W. Kahan developed a numerical method to reduce a matrix to bidiagonal form using orthogonal transformations [GK65], and Gene's later work with Christian Reinsch [GR70] and Peter Businger [BG69b] showed how to reduce the bidiagonal form to diagonal using the algorithm still in use today. The SVD is in some sense the Swiss army knife of matrix computations. It yields optimal low-rank approximations to a matrix and reveals orthogonal bases for its range space and null space. It is used in solving least squares problems, linear inverse problems, and signal processing problems. Through it we perform principal component analysis in statistics, compute canonical correlations, process natural language text through latent semantic indexing, and derive reduced-order models for the weather and other time-varying systems. It is hard to imagine numerical computation today without a stable and efficient means of computing an SVD. Gene was sometimes called Professor SVD for his contributions to the computation and application of the decomposition, and he chose "Prof SVD" for the license plate on his car.

Gene was an ardent advocate for the use of orthogonal transformations in matrix computations, because they preserve numerical stability. This is particularly important when the problem or the model changes and factorizations must be updated. For example, new observations might be added to a data set, or old ones might become obsolete. In optimization problems, as we explore the feasible space, one constraint might become important, and another might become irrelevant. It is important to be able to account for these changes in a matrix without needing to discard a factorization that has already been computed at considerable expense. Gene considered how triangular decompositions (LU) could be updated in the course of the simplex algorithm for linear programming problems [BG69a] and definitively attacked stable updates in a now classic and widely-cited paper with Philip Gill, Walter Murray, and Michael Saunders [GGMS74], showing how to efficiently and stably update triangular or QR factorizations after a rank-1 change in a matrix.

Gene's insight into matrix structure led him to do fundamental work on "fast Poisson solvers," efficient algorithms for determining steady-state distributions of temperature or electric potential in objects such as squares, boxes, circles, and spheres, as well as unions of these objects. The need to compute these distributions ran far ahead of the capabilities of early computers, and Gene's insight was critical in developing fast, low-storage decomposition algorithms and proving their stability [BGN70], as well as in finding applications to other partial differential equations and other domains [GHST98, BDGG71, CG73].

Iterative methods for solving linear systems: Gene's interest in iterative methods for solving large, sparse systems of linear equations (methods that compute approximate solutions to problems that are too complex for matrix decomposition methods such as Gaussian elimination to be effective) began with his work in his PhD dissertation [Gol59]. He studied a family of methods and concluded that Chebyshev semi-iteration was more effective than successive over-relaxation or the second-order Richardson method. As Gene was finishing up his work at the University of Illinois, a rather famous mathematician named Richard Varga visited and told Gene's advisor, Abraham Taub, about similar results he had obtained. Varga remembered:

> *Taub told Gene that 'if Varga publishes first, you will have to write a new thesis'. During that visit, I later met Gene, who was visibly shaken about all of this, and I suggested that we discuss this further over coffee. There was indeed overlap in our results, but Gene did things that I hadn't done, and conversely. [CGO07, p.45]*

Gene was always grateful to Varga for offering to publish the results jointly [GV61a, GV61b], in what became one of Varga's most highly cited works.

In the early 1970s, Gene became interested in the conjugate gradient and Lanczos family of iterative methods, which were, at the time, being advocated by a small group including John Reid and Chris Paige. These methods had been proposed in the 1950s, but had fallen into obscurity because of a mistaken perception that they should be considered direct methods. Gene was convinced that difficult problems in science and engineering could benefit from "preconditioned" versions of the methods, exploiting matrix splitting concepts that formed the basis of his dissertation. For my dissertation, Gene first asked me to study preconditioning and demonstrate its usefulness. This resulted in a paper by Paul Concus, Gene, and me [CGO76], the first in a long series of papers by Gene advocating for the method [GO89], developing variants that expanded its applicability [CG75, CGO78, GN82, CGM85, GY99, YGPC04, GRT07, Gv91], and explaining and exploiting its deep connection with Gaussian quadrature [Gol74, DEG72, GK89, BG91, GG90].

Orthogonal polynomials and quadrature: From his earliest work with iterative methods for solving linear systems of equations, Gene was fascinated with the ties between linear algebra and classical analysis [Gau02]. Many iterative methods can be expressed as a three-term recurrence, and this recurrence defines a related set of orthogonal polynomials. A set of orthogonal polynomials induces a "Gaussian quadrature rule" for numerical computation of integrals. This rich connection was always in the back of Gene's mind when he thought about iterative methods [GW69, GK83, Gv91, FG91, FG92, CGR94], and joint work with John Welsch [GW69, GK83] explained how the nodes and weights could be computed from the eigendecomposition of a tridiagonal matrix formed from the iteration coefficients. Gene later extended this study to Gauss-Kronrod [CGGR00], Gauss-Radau and Gauss-Lobatto quadrature [Gol73]. Other authors have generalized further.

The relation between matrices, moments, and quadrature was further exploited by Gene in a paper with Gerard Meurant about computing matrix functionals $u^T f(A) v$ where u and v are column vectors and f is a smooth function [GM94]. The main example they treat is computing a single element of the matrix A^{-1}. The fundamental insight is to express $u^T f(A) v$ as an integral and then estimate it using Gaussian quadrature formulas, generated through applying the Lanzcos algorithm to the matrix A using $u / \|u\|$ as the starting vector. The ideas were further developed in [GM97, GM10].

Statistical modeling: Work at the interface between mathematics and statistics was especially satisfying to Gene, building on his graduate study of both subjects.

A fundamental problem in science is fitting models to observed data. Doing this by minimizing the sum of squared differences between predictions and observations, the least squares method, has been a standard since the 1800s.

If the model is linear, $Ax = b+e$, then the problem takes the form $\min_x \|Ax - b\|_2^2$, where b is the vector of observed data, e is the unknown vector of errors in the observations, x is the vector of model parameters, and the matrix A derives from the model. The least squares problem can be solved by setting the gradient to zero, which yields the normal equations, a system of linear equations involving the matrix $A^T A$. In some sense this is

Gene Golub and some of his students, 60th Birthday celebration, Minneapolis, MN, 1992. Oliver Ernst, Alan George, Franklin Luk, James Varah, Eric Grosse, Petter Bjørstad, Margaret Wright, Daniel Boley, Dianne O'Leary, Steven Vavasis, Tony Chan, William Coughran, Michael Heath, Michael Overton, Nick Trefethen. (Photo credit Gene Golub's collection.)

a very attractive formulation, since the matrix is symmetric and positive semidefinite. Unfortunately, the matrix can be unnecessarily ill-conditioned, and, numerically, it is wiser to work with the original data matrix *A* rather than the symmetrized matrix. It was Gene who showed how to use a QR factorization of *A* to solve least squares problems in a provably stable way [BG65, Gol65].

If the model is linear in some variables but nonlinear in others, Gene, in joint work with Victor Pereyra, suggested separating the variables [GP73, GP76, GP03, GL79]. They observed that, given values for the nonlinear variables, optimal values for the linear variables can be obtained by solving a linear least squares problem, and they derived expressions for the partial derivatives of the minimization function in terms of the nonlinear variables. Their resulting algorithm for separable nonlinear least squares problems has proved valuable in applications, and it has been implemented in many computer languages.

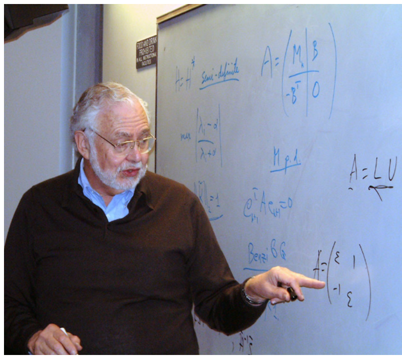
Gene also studied errors-in-variable modeling, which he termed the total least squares problem [GV79, GV80, SVG04, CGGS99, CGGS97]. Where least squares accounts for errors in the observations, total least squares also allows unknown errors E in the matrix A, resulting in the model $(A + E)x = b + e$ and the objective to minimize $\|[E, e]\|_F^2$, the sum of squares of entries in E and e. Gene recognized it as an eigenvalue problem embedded in a least squares problem. He used the SVD to understand the character of the problem and compute solutions. His contributions are fundamental to the field, and he extended the use of the SVD to regularization of ill-posed problems [FGHO97, GHO99, GHW79].

A significant contribution to the statistical literature, Gene's joint work with Michael Heath and Grace Wahba on "Generalized cross-validation as a method for choosing a good ridge parameter" [GHW79] is his most cited article.

Eigenvalue problems: Gene made extensive contributions to the computation of eigenvalues of matrices, especially large sparse ones, and to non-standard eigenvalue problems.

He gave particular attention to inverse eigenvalue problems [CG05]. For example, given an increasing sequence of n numbers and a sequence of $n-1$ numbers that interlace them, find a symmetric tridiagonal matrix for which the n numbers are its eigenvalues and the $n-1$ numbers are the eigenvalues of its leading principal submatrix of order $n-1$. Gene returned to variants on this problem throughout his career, tying them to problems involving orthogonal polynomials and Gaussian quadrature [BG78a, BG78b, BG87, CG02, EMG04, dG78].

My personal favorite among Gene's papers is his single-authored work on modified matrix eigenvalue problems [Gol73]. He considers in turn a variety of interesting problems and asks whether being able to easily compute a solution to a related problem can be of use. The problem areas are constrained optimization, inverse eigenvalue problems, finding the intersection of subspaces, computing eigenvalues of a matrix with a rank-one update, up/downdating least squares problems, and computing Gaussian quadrature rules with preassigned nodes. For each, he explains a surprisingly simple and simply beautiful solution. The eighteen clearly-written pages showcase his voice, his aesthetic, his nimble mind, his talent for exposition, and his exuberance when explaining his results.



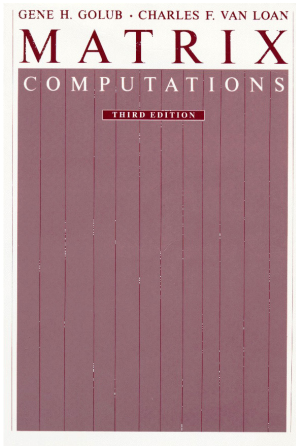
Gene Golub, lecturing.
(Photo credit Jack Dongarra.)

Applications: Gene was driven by curiosity and always wanted to know what people were working on. Gene's impact on research was magnified by collaborations with scientists and engineers. He took the time to learn what the important problems were and how they might be expressed in matrix terms. Then he provided practical insights and solutions to computational problems. He made direct contributions to areas as diverse as waves and fluids [GJY75, CGP76, CGS02], photogrammetry [GLP79], geodetics [GP80], control theory [BG84], economics [GG84], signal and image processing [CG90, TPG93, XZGK94, TZG96, NMG01a, NMG01b, TEMG05], medical imaging [OGG+ 03], and genomic analysis [AG04, AG05, KGP05, AG06]. His unpublished contributions to Google's PageRank algorithm earned him a small stake in the company.

In addition to his own collaborations, there were countless collaborations born of his matchmaking. His fertile mind saw connections that no one else did, and he was a catalyst for many research projects. He loved to organize conferences that brought together groups of people who ordinarily would not interact, and he was forever introducing people by saying, "Come talk to X, who has some great problems in field Y that could benefit from your expertise in Z."

Other contributions to the research community

Gene was an exceptionally fine mathematical writer and emphasized clear, concise exposition when educating his students. As a lecturer, his bubbly enthusiasm and excitement over beautiful results was infectious, and his voice can be clearly heard in the book *Matrix Computations*, the first edition of which was derived from a short course he gave at Johns Hopkins University [GV83]. The 1983 book was updated in 1989 and 1996, and the fourth edition was published in 2013. Written with Charles Van Loan, the book is incredibly influential as a textbook and reference volume in the mathematical and applications communities, with tens of thousands of citations.



Third edition of *Matrix Computations*, the last completed before Gene died. (Photo credit Charles Van Loan.)

Although late to begin using email, Gene quickly recognized its potential for bringing the numerical community together. He founded NA-Digest [DGG+ 08], emailed weekly to keep the community abreast of news and announcements, and a related service that provided contact information for members of the community.

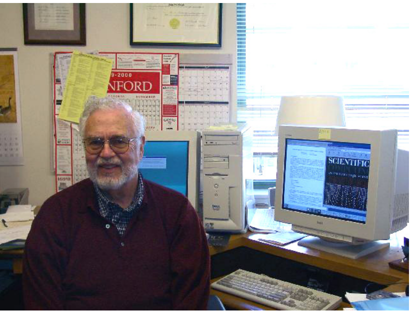
He was quick to identify research trends as well. He founded two journals for SIAM to showcase important emerging themes, that were incompletely served by other scholarly publications, *SIAM Journal on Scientific and Statistical Computing* (1980) and *SIAM Journal on Matrix Analysis and Applications* (1988).[2] Gene ably served as editor-in-chief until each was well established.

Gene made other important contributions to *SIAM*. He served as its president in the mid-1980s and always pushed the organization towards inclusiveness and innovation. He left a substantial bequest to SIAM, and in appreciation of Gene's outreach, a committee of his close colleagues proposed using it to establish the Gene Golub *SIAM* Summer

2 This journal replaced the *SIAM Journal on Algebraic and Discrete Methods*, founded in 1980.

School, held each year to give graduate students a hands-on introduction to a central topic in applied mathematics while fostering technical interaction and creating a network among the students and the instructors.

One can't summarize Gene's contributions to research without mentioning Serra House. It was an old, rather disheveled building that was formerly the home of the president of Stanford. It had been rather clumsily converted to office space, and for many years the numerical analysis group occupied the ground floor. Gene directed as many as 14 students at a time (after George Forsythe's untimely death), and the building was always bursting with activity from students, postdocs, and a steady stream of visitors—some renowned, others at the beginning of their careers, all receiving a helping hand from Gene. The roster of visitors shows a special emphasis on people Gene met while visiting countries not known for scientific excellence. For decades, numerical analysts could be certain that if someone was speaking about "Gene," it surely meant Gene Golub.


Gene Golub in his office at Stanford University, circa 1999. (Photo credit Michael Heath.)

Gene advised 31 Ph.D. students, whom he counted as his family, and he currently has almost 300 academic descendants.

Gene's hospitality was renowned. His beautiful home in the hills above Stanford, a short bicycle ride from his office, was usually filled with house guests whether Gene was in town or away, and it was the setting for many gracious gatherings where social and technical connections were made across multiple generations. When Gene hosted a seminar or chaired a session at a meeting, he had the endearing custom of introducing each attendee to the speaker. His memory for people was prodigious, and his attention to each individual was sincere.

Family, friends, and passions

Family was very important to Gene, and he enjoyed hosting his widowed mother at his home. His brother Alvin and sister-in-law Shirlee were also important in his life, as were their two children, Neal and Ellen. Sadly, Ellen was killed by a drunk driver in the 1980s as Neal was driving her back to Gene's house after a concert in San Francisco. Then in 2004, Neal was murdered by his ex-wife in a custody dispute over their 6-year-old daughter. These twin tragedies affected Gene deeply.

Gene was a life-long bachelor until a surprising twist late in life, in the mid 1990s:

> *Here's some exciting news. I have a girlfriend whom I knew originally 35 years ago! I first met Barbara at Cambridge and I have thought of her often. She's a widow now and has two grown children. Barbara teaches math in a fancy London high school but she may be moving to CA. As you can imagine, this has made life very sweet.*[3]

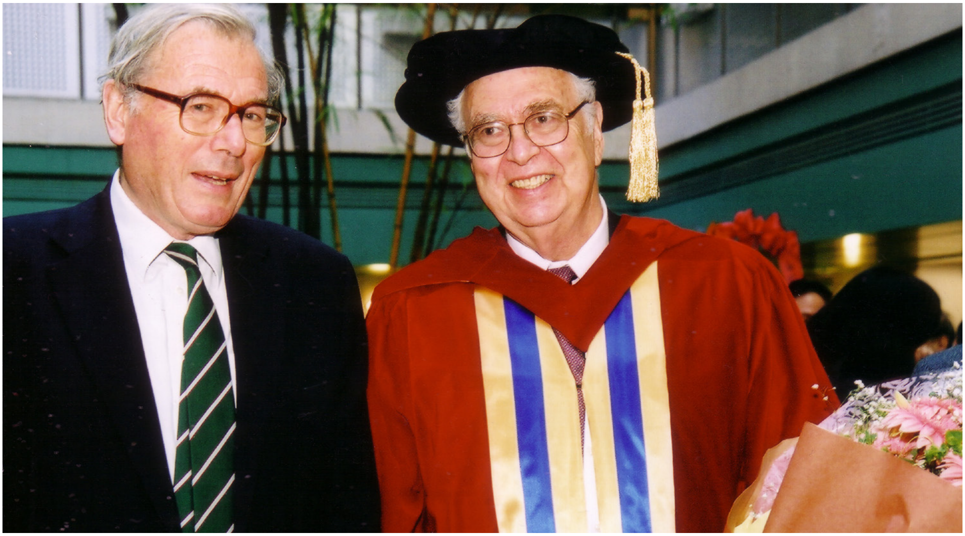
By March, 1997, Gene had taken on a new role, "delighted to be married; So now I'm settling down to my post- adolescent years."[4] Unfortunately, the marriage was short lived. There was a stronger love in Gene's life, Stanford University, and when Barbara decided to move back to England in 2000, the couple separated and eventually divorced.

Stanford was indeed the love of Gene's life. Despite his occasional displeasure with the university or his department, and despite numerous very attractive offers from other institutions, Gene could not bring himself to leave the institution. Retirement held no attraction for him; he drew his energy from his students and his professional friends. He continued his frequent travels up until the end of his life, accumulating many adventures, such as being stranded in Newfoundland after planes were diverted from US airspace after the 9/11 attack in 2001.

Gene visited Hong Kong in fall, 2007, and began feeling unwell. He had scheduled a short visit home before a trip to Switzerland to receive an honorary degree from ETH. During that respite he was hospitalized and diagnosed with leukemia. Many of us who were his friends visited or talked to him by phone during his brief stay in the hospital. Local friends sent frequent optimistic updates, until a final message that Gene had died unexpectedly on November 16, 2007. Cleve Moler, the founder of MathWorks, summarized Gene's impact: "Our community has lost its foremost member." [Sta07]

In response, Gene's friends organized a memorial multi-site meeting in February 2008, centered at Stanford, to celebrate his almost 76 years of life on what would have been his 19th February 29th birthday. Over 1000 people participated in 32 events in 23 countries on 6 continents. Since then, there have been many February memorial meetings, the most recent at Hong Kong Baptist University in 2017.

3. Gene H. Golub. Email to Dianne P. O'Leary, private communication, 22 April 1996.
4. Gene H. Golub. Email to Dianne P. O'Leary, private communication, 02 March 1997.



Michael Powell and Gene Golub, Hong Kong Baptist University honorary degree, 2002. (Photo credit Gene Golub's collection.)

Summary

It is hard to formulate a realistic mathematical model that does not have an underlying matrix. Because of this, it is rare to find an important computational algorithm that has not been influenced by some of Gene Golub's foundational work on linear systems, eigenvalue problems, and matrix decompositions. But beyond his technical achievements, presented in over 350 articles and 5 books, his legacy is defined by his 31 students, almost 300 academic descendants, approximately 250 coauthors, and thousands of researchers helped by him during his remarkable life.

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