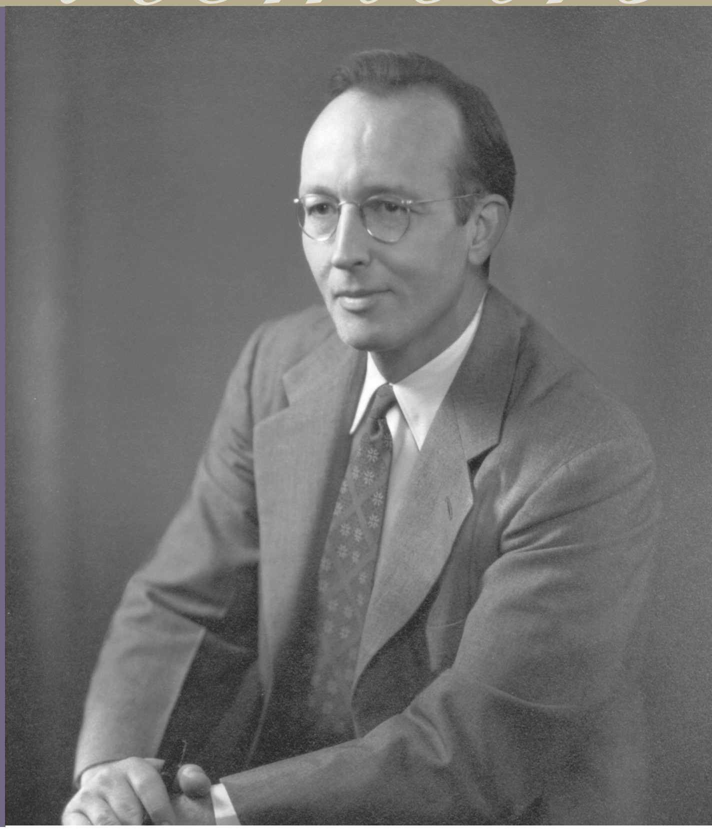
Saunders Mac Lane

1909–2005

BIOGRAPHICAL *Memoirs*



A Biographical Memoir by
Robion Kirby

NATIONAL ACADEMY OF SCIENCES

SAUNDERS **MAC LANE**

August 4, 1909–April 14, 2005

Elected to the NAS, 1949

Photography by Fabian Bachrach.

By Robion Kirby

The McLean clan came from the Highlands of Scotland, near Castle Duart overlooking the Straits of Mull. The clan was defeated by the British in 1746 in the Battle of Culloden (the last pitched battle fought on British soil), and eventually—in the early 1800s—Saunders Mac Lane's ancestors came to western Pennsylvania and Ohio. His grandfather, William Ward McLane, born in 1846, became a Presbyterian minister and then was charged with heresy due to preaching about Charles Darwin. He found refuge in New Haven, Connecticut, and became a Presbyterian pastor there.

Saunders' father, Donald McLane, born in 1882, studied at Yale University and the Union Theological Seminary in New York and became a Congregationalist minister. He married Winifred Saunders in 1908, and the following year Leslie Saunders MacLane was born; when the boy was only one month old, the first name Leslie was dropped. McLean had morphed into MacLane a generation earlier in order not to sound Irish, and MacLane acquired a space—becoming Mac Lane—when Saunders' wife Dorothy, who typed his Ph.D. thesis and later papers, found that variation easier to type.

Except for a few years in Boston around age seven, Saunders grew up in small towns, which it turned out he preferred. He started high school in Utica, New York, where his father fell ill (possibly due to the influenza epidemic of 1919) and died when Saunders was 14. The family then lived with Saunders' widowed grandfather, where Saunders got to know his uncles, one of whom decided to send him to Yale and fund him at the princely sum of $1,200 per year.

Entering Yale in 1926, Saunders chose chemistry for a major, given its practical applications in industry. But then he inquired about a career in math and was told that one could become an actuary, so he switched his major to math. A career in mathematical research had not yet occurred to him.

A young Yale professor, Filmer Northrop, who had just studied with Alfred North Whitehead at Harvard, brought Bertrand Russell and Whitehead's three-volume *Principia Mathematica* to Saunders' attention. He read and annotated the first volume, and a lifelong interest in logic, foundations, and philosophy was born.

Saunders spent the academic years 1931-32 and 1932-33 at Göttingen. He experienced the institution in its full glory and then in its denouement in 1933.

Through most of his time as an undergraduate, Saunders was still of a mind to acquire knowledge, not create it. But in his senior year, Oystein Ore came to Yale, bringing notions from Emmy Noether's school of abstract algebra in Göttingen, Germany. Ore's lectures, and the reading of Otto Haupt's very abstract text in algebra, led Saunders to shift to the world of discovering knowledge.

Saunders was an excellent student at Yale; quantitatively, he had one of the highest GPAs in the college's history. Particularly fateful was an award he received at a 1929 event at which Robert Maynard Hutchins, just appointed president of the University of Chicago, also received an award. When Hutchins met Saunders he promptly recruited him to start graduate school at Chicago with a $1,000 fellowship. Ore was not pleased to hear that Saunders chose Chicago, pointing out that Harvard or Princeton (or presumably Yale) were better choices for mathematics study than was Chicago.

Nonetheless, Saunders went to Chicago, discovering upon arrival that he had not been admitted because Hutchins had neglected to inform the math department of the student's decision. But admission was quickly arranged.

Saunders was largely disappointed by his first year at Chicago. He perceived too much emphasis on pushing students to complete a thesis as a one-time piece of research before going out into the real world—rather than as a preparation for future research. However, there were positive aspects to that year: study with E. H. Moore, leading to a seminar talk (by Saunders) on the Zermelo-Frankel axioms for set theory; reading the most accessible source for topology, Veblen's *Analysis Situs*, from which he learned about Betti numbers but not that there were concepts like homology groups; a master's thesis in abstract algebra, in which he tried to axiomatize exponentials; and, most important, meeting Dorothy Jones, whom he later married (in 1933). Saunders saw little opportunity to write a Ph.D. thesis in logic at Chicago, so he applied for, and received, a fellowship to study at the University of Göttingen.

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David Hilbert, although retired, lectured there once a week during term; Edmund Landau gave careful, precise, but unmotivated lectures on Dirichlet series; Emmy Noether influenced Saunders greatly with her axiomatic view of algebra, which fit his interests in logic and foundations; Paul Bernays became Saunders' thesis adviser; Richard Courant was the administrative head of the Mathematics Institute; and Gustav Herglotz held the chair in applied mathematics.

In addition to these tenured mathematicians there were many outstanding assistants and graduate students, including Hans Levy, Franz Rellich, Ernst Witt, Oswald Teichmüller, and Fritz John.

In February 1933, the Nazis came to power and Hitler became chancellor. On April 7, 1933, the Law for the Restoration of the Professional Civil Service was passed. It required that all Jewish faculty members at the state-supported universities be immediately dismissed, except for a few academics with either 25 years of service or who had fought in the German army during World War I.

Courant was immediately fired. Landau, having served in WWI, was spared, but he left within months after his classes were boycotted by Nazi students. Noether went to Bryn Mawr College. Bernays went to the Eidgenössische Technische Hochschule in Zürich. Levy and many other younger mathematicians departed. The only full professor remaining was Herglotz.

This is Saunders' description of the situation:

> *Prior to 1933, the atmosphere at Göttingen crackled with enthusiasm for mathematical ideas and discoveries. It was a remarkable exhibit of the university as an amalgam of research, teaching, and inspiration. But the glory of mathematics at Göttingen came to an abrupt end when Hitler came to power. The Nazi government wrecked what had been the most prominent and influential center of mathematics in the world.*

Saunders had another year left on his Humboldt Fellowship, but he now rushed to finish his thesis, which he successfully accomplished. Two days after Saunders had passed his major and two minor doctoral exams, he and Dorothy were wed in a quiet ceremony at City Hall (July 21, 1933). Coincidentally, at City Hall they met Fritz John and his non-Jewish girlfriend, who were hastily getting married before such unions were outlawed; Saunders and Dorothy were witnesses sworn to secrecy, and the Johns then left and traveled separately to New York.

In 1945, they introduced the notion of category in a paper titled *General Theory of Natural Equivalences*. Saunders referred to it later as "offbeat" and "far out," and when I was a student at Chicago the subject was sometimes referred to as "generalized abstract nonsense" both by its afficionados and detractors.

After Göttingen, Saunders bounced around, first at Yale as a postdoc (1933–34), then Harvard (1934–36), Cornell (1936–37), and finally Chicago (1937–38). During those years, Saunders' research seems, retrospectively, to have been somewhat unfocused. There was some logic; some algebra, partly under the influence of Oystein Ore at Yale; and some topology. The latter had begun with Saunders' study (as a Yale undergraduate) of Felix Hausdorff's book *Grundzüge der Mengenlehre* [Basic Set Theory], then with the study of *Analysis Situs* as a graduate student at Chicago, next with Hassler Whitney (who was working on the four-color problem and graphs) at Harvard, and then with V. W. Adkisson at the University of Arkansas (where Saunders and Dorothy were visiting her relatives). Adkisson was a student of J. R. Kline, and with Saunders he studied whether a homeomorphism of a graph G in S^2 could be extended to a homeomorphism of S^2. These bits of topology were a far cry from Saunders' later work in this field with Samuel Eilenberg, but they served as a warm-up.

Saunders' daughter Gretchen was born during his year at Chicago. Then the lure of an assistant professorship at Harvard brought the family to Cambridge, MA, in 1938. Right away, Saunders and Garrett Birkhoff began offering an undergraduate course in algebra; Birkhoff emphasized algebras, lattices, and groups while Saunders addressed group theory, Galois theory, and an axiomatic treatment of vector spaces. Soon they combined their notes and in 1941 *A Survey of Modern Algebra* was published. It was the first American text to use the ideas of Emmy Noether and eventually it was adopted beyond the elite departments.

During his Harvard years, Saunders spent some time studying group extensions and crossed-product algebras. In 1941, he gave the Ziwet lectures at the University of Michigan, speaking about group extensions. Eilenberg was in the audience, and in conversation they realized that group extensions were related to the homology of the p-adic solenoid; thus did their famous collaboration begin, resulting eventually in 15 joint papers.

In 1945, they introduced the notion of category in a paper titled *General Theory of Natural Equivalences*.[6] Saunders referred to it later as "offbeat" and "far out," and when I was a student at Chicago the subject was sometimes referred to as "generalized abstract nonsense" both by its afficionados and detractors. The original idea of categories with objects and morphisms, and natural transformations between them, has since blossomed almost beyond recognition.

But it was the two mathematicians' papers on cohomology of groups and Eilenberg-Mac Lane spaces that made their reputations, at least among topologists.

Here is a quote from Alex Heller and coauthors, writing about Eilenberg in the *Memoirs of the National Academy of Sciences*:

> *With Mac Lane he developed the theory of cohomology of groups, thus providing a proper setting for [Heinz] Hopf's remarkable theorem on the homology of highly connected spaces. This led them to the study of the Eilenberg-Mac Lane spaces and thus to a deeper understanding of the relations between homotopy and homology. Their most fateful invention perhaps was that of category theory, responding, no doubt, to the exigencies of algebraic topology but destined to radiate across most of mathematics.*[2]

And here is Saunders himself writing about the genesis of his collaboration with Eilenberg.[4] The setting was the University of Michigan, where Saunders gave the 1941 Ziwet Lectures:

> *At that time, I had been fascinated with the description of group extensions and the corresponding crossed-product algebras, which had entered into my research with O. F. G. Schilling on class field theory. So group extensions became the topic of my Ziwet lectures. I set out the description of a group extension by means of factor sets and computed the group of such extensions for the case of an interesting abelian factor group defined for any prime p and given generators* a_n *with* $pa_{n+1} = a_n$ *for all n. When I presented this result in my lecture, Sammy [Eilenberg] immediately pointed out that I had found Steenrod's calculation of the homology group of the p–adic solenoid. This solenoid, already studied by*

Sammy in Poland, can be described thus: Inside a torus T_1, wind another torus T_2 p–times, then another torus T_3 p–times inside T_2, and so on. What is the homology of the final intersection? Sammy observed that the Ext group I had calculated gave exactly Steenrod's calculation of the homology of the solenoid! The coincidence was highly mysterious. Why in the world did a group of abelian group extensions come up in homology? We stayed up all night trying to find out 'why.' Sammy wanted to get to the bottom of this coincidence.

It finally turned out that the answer involved the relation between the (integral) homology groups $H_n(X)$ of a space X with the cohomology groups $H^n(X,C)$ of the same space, with coefficients in an abelian group G. It was then known that there was an isomorphism Θ,

$$\theta : H^n(X,G) \rightarrow Hom(H_v(X),G)$$

where the right-hand group is that of all homomorphisms of $H_n(X)$ into G. But we found that this map Θ had a kernel that was exactly my group of abelian group extensions, $Ext(H_{n-1}(X),G)$. In other words, we found and described a short exact sequence

$$0 \rightarrow Ext(H_{n-1}(X),G) \rightarrow H^n(X,G) \rightarrow Hom(H_n(X),G) \rightarrow 0$$

In effect, this 'determines' the cohomology groups in terms of the integral homology groups, and this explains why the algebraically introduced groups have a topological use. This exact sequence is now known as the 'universal coefficient theorem.'

The definition and construction of what became known as Eilenberg-Mac Lane spaces, $K(G,n)$, occurred in the mid-1940s. The homotopy groups of a topological space $X, \pi_n(X)$, are defined by the homotopy classes of continuous maps of the n-sphere into *X*, a much simpler definition than that of homology groups $H_n(X;Z)$ (which nonetheless had been defined much earlier in the work of Poincaré and were easier to compute). The fundamental group $\pi_1(X)$ may be non-Abelian, but the higher homotopy groups were Abelian. Hopf showed that $\pi_3(S^2) = Z$, in contrast to $H_3(S^2,Z) = 0$. So homotopy groups were different from homology groups, and as it turned out, were much harder to compute. To this day, not all of the higher homotopy groups of even the 2-sphere are known.

Eilenberg and Mac Lane stepped in and constructed, $K(G,n)$, a space whose n^{th} homotopy group is any finitely presented Abelian group G and all other homotopy groups are zero. These form essential building blocks for more complicated spaces. There are only a few simple examples, namely $K(Z,1)=S^1$, the circle; $K(Z/2,1)=RP^\infty$, the infinite-dimensional real projective space; and $K(Z,2)=CP^\infty$, the infinite dimensional complex projective space. Other examples can be obtained from the Dold-Thom theorem, for if the only non-zero homology of X is G in dimension n, then the infinite symmetric product of X is a $K(G,n)$. E.g., take $X=S^n$.

Saunders spent 1938–47 at Harvard (Irving Kaplansky and Roger Lyndon were the best known of his Harvard students), but he then was lured back to the University of Chicago in 1949 by Hutchins (who remained chancellor until 1952) and the new department chair Marshall Harvey Stone. Saunders spent the intervening year (1948) touring Europe, and in particular talking with J. H. C. Whitehead in Oxford. They sorted out the first case of what later became Postnikov systems, defining $k^3 \in H^3(\pi_1,\pi_2)$. (Postnikov systems are roughly "twisted" Cartesian products of Eilenberg-Mac Lane spaces.)

Saunders stayed at Chicago for the rest of his mathematical career. He often recounted the years 1947–59—the "Stone Age"—when the university's math department was arguably the best in the world.[5] The senior faculty were Adrian Albert, S.-S. Chern, Stone, Andre Weil, Antoni Zygmund, and of course Mac Lane; the junior faculty included Paul Halmos, Kaplansky, Irving Segal, and Edwin Spanier, plus many eventual stars among the graduate students.

It all came apart in the late 1950s, when Stone retired, Weil went to the Institute for Advanced Study, Chern and Spanier opted for Berkeley in 1958, and Halmos and Segal departed as well. Still, it had been a great time, and many of the stars remained.

I was a student at Chicago during 1954–64, and I recall Saunders as an ebullient man and a lively teacher who often spoke and lectured with a not quite suppressed smile.

In 1949, at the relatively young age of 39, Saunders was elected to the National Academy of Sciences. Although his most influential work was with Eilenberg on group extensions, cohomology of groups, and Eilenberg-Mac Lane spaces (the appreciation of category theory came later), Eilenberg was not elected for another 10 years. It may be that Saunders' abilities outside research (plus being well accepted in the "old-boys" network that still existed in those days) helped account for his early election.

In 1959 Saunders became chair of the editorial board of the *Proceedings of the National Academy of Sciences*, and he served for eight years. Traditionally, the *PNAS* had no system of refereeing; it simply accepted almost all papers submitted by members or communicated by members. As might be expected, the quality was mixed. However, the right of a member to publish or communicate has gradually been whittled away, (this right is infrequently exercised now), and papers are now refereed and accepted/rejected as normally done in good math journals.

In the 1950s, mathematics papers formed the largest proportion of *PNAS* articles. But during Saunders' chairmanship, the treasurer of the NAS, being worried about the *PNAS*'s finances, instituted page charges. The result was that mathematicians, lacking big grants, largely stopped publishing in *PNAS*. At the time of this publication, the journal will not charge mathematicians, as it is trying to increase their representation in its pages.

Saunders was elected president of the American Mathematical Society (AMS) in 1972, a time of some tumult due to the Vietnam War and the emerging women's movement in mathematics. Chicago had always admitted women and produced quite a few female Ph.D.s over the years, a fact that Saunders seemed to be proud of. However, I recall that he showed up at a meeting of the Council of the AMS wearing a tie painted with little white piglets and MCPs (male chauvinist pigs). I'd guess he did it to tease the more ardent feminists, but I'm not sure it went over well. About the same time he nominated and helped elect Julia Robinson to the NAS, the first female member in mathematics (she shortly went on to be elected president of the AMS, an honor that at that time was traditionally reserved for members of the NAS). I believe that he also helped get Stephen Kleene elected concurrently, for Kleene had had a long and distinguished career in logic and was equally deserving of the honor.

While president, Saunders encouraged the AMS to be more proactive in government affairs—for example, he helped it establish (with the Society for Industrial and Applied Mathematics [SIAM] and the Mathematical Association of America [MAA]) the Joint Projects Board in Mathematics to influence public policy.

Saunders continued to be active in mathematics and mathematical governance nearly to the end of his life. I recall him lecturing with vigor on something categorical in the opening term of the Newton Institute in Cambridge, UK, in 1992.

Saunders' wife Dorothy had been infected with a variant of encephalitis, and as her condition was exacerbated by the progression of Parkinson's disease and arthritis, it became more and more difficult for them to travel together. They celebrated their 50th wedding anniversary in 1983, and on February 3, 1985, Dorothy died. Saunders subsequently wrote very warmly of Dorothy and their marriage.

In 1986, Saunders married Osa Skotting (ex-wife of Irving Segal), and they enjoyed his remaining years together.

Because Saunders had a fondness for clever verse, often about mathematics or mathematicians, I think the best way to end this essay is with two examples that he often quoted. They are followed by another poem, written by a colleague, about Saunders himself.

Here's to Marston, Mickey Morse,
A man experienced in divorce.
His opinion of himself, we charge,
Like nose and book is in the large.

Here's to Lefschetz, Solomon L.,
Irrepressible as hell.
When he's at last beneath the sod,
He'll then begin to heckle God.

Steve Awodey[1] wrote the following ditty to commemorate the verses that Saunders enjoyed:

To Saunders Mac Lane, of the plaid sport coat,
with his thingamajigs and the Homs he wrote,
in mathematics he stood alone,
in a category of his own!

AUTHOR'S NOTE

This memoir is based mostly on Saunders' own *Saunders Mac Lane: A Mathematical Autobiography.*[3] It is also written from the perspective of a topologist; for a different view, see the excellent articles by Colin McLarty on Saunders' philosophical work.[7,8,9]

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