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EDWARD JAMES MCSHANE
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A Biographical Memoir by

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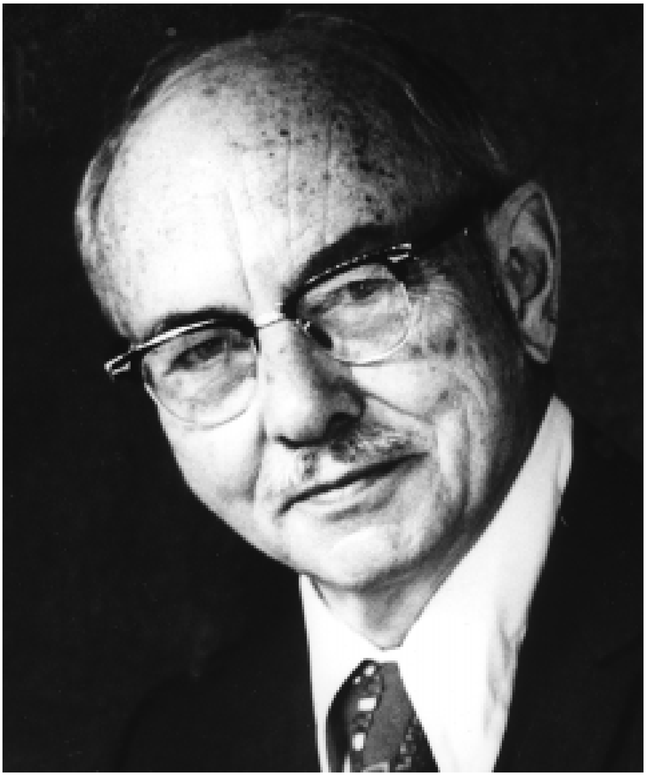

E. J. McShane

EDWARD JAMES MCSHANE

May 10, 1904–June 1, 1989

BY LEONARD D. BERKOVITZ AND
WENDELL H. FLEMING

DURING HIS LONG CAREER Edward James McShane made significant contributions to the calculus of variations, integration theory, stochastic calculus, and exterior ballistics. In addition, he served as a national leader in mathematical and science policy matters and in efforts to improve the undergraduate mathematical curriculum.

McShane was born in New Orleans on May 10, 1904, and grew up there. His father, Augustus, was a medical doctor and his mother, Harriet, a former schoolteacher. He graduated from Tulane University in 1925, receiving simultaneously bachelor of engineering and bachelor of science degrees, as well as election to Phi Beta Kappa. He turned down an offer from General Electric and instead continued as a student instructor of mathematics at Tulane, receiving a master's degree in 1927.

In the summer of 1927 McShane entered graduate school at the University of Chicago, from which he received his Ph.D. in 1930 under the supervision of Gilbert Ames Bliss. He interrupted his studies during 1928-29 for financial reasons to teach at the University of Wichita. It was at Chicago that McShane's long-standing interest in the calculus of variations began. From 1930 to 1932 he held a National Research Council Fellowship, spent at Princeton, Ohio State, Harvard, and Chicago. In 1931 he married Virginia Haun. The McShanes had three children, Neill (deceased), Jennifer, and Virginia.

Because of the Great Depression, openings in mathematics departments were virtually nonexistent in 1932. The McShanes spent 1932-33 at Gottingen, where he translated into English the two volumes of Courant’s *Differential and Integral Calculus*. They also saw firsthand some frightening aspects of the onset of Nazi power in Germany.

After two years (1933-35) on the Princeton faculty McShane joined the Department of Mathematics at the University of Virginia as a full professor in the fall of 1935. He remained there for the rest of his career, except for leaves of absence spent at other institutions. With the onset of World War II McShane agreed to head a mathematics group at the Ballistics Research Laboratory in Aberdeen, Maryland. During this time he wrote a book with John L. Kelley and Frank V. Reno entitled *Exterior Ballistics*, which is regarded as the definitive work on the subject. In 1947 Tulane University awarded him an honorary D.Sc. degree.

McShane served as president of the Mathematical Association of America during 1953-54. He took an active interest in efforts just then getting underway to revitalize undergraduate mathematics in the United States. As president, McShane appointed a committee to prepare texts and other material to improve the quality of undergraduate mathematical instruction. For several years after his term as president he chaired and served on this committee, which evolved into the Committee on the Undergraduate Program in Mathematics, a leader in these endeavors. McShane was elected to the National Academy of Sciences in 1948 and served on the National Science Board from 1956 to 1968. During 1958 and 1959 he was president of the American Mathematical Society. In 1964 he received the Mathematical Association of America's Annual Award for Distinguished Service to Mathematics.

McShane had a lifelong interest in music. His early interest in opera led him to learn to read Italian libretti. In addition to Italian he was fluent in Dutch, French, German, and Spanish. His knowledge of Italian, in turn, led Gilbert Ames Bliss to suggest to McShane that he read the then new book *Fondamenti di Calcolo delle Variazione* by Leonida Tonelli, which started McShane on his study of multiple integral problems in the calculus of variations. Later, in the 1950s, McShane learned to play the cello and became an amateur chamber music performer.

The injustices suffered by some of his colleagues during the post-World War II anti-Communist hysteria deeply offended McShane. He himself, in response to the question on the Aberdeen Proving Ground security form that asked whether he had ever been involved with organizations that at any time advocated the overthrow of the U.S. government by force and violence, replied that, yes, he was an employee of the Commonwealth of Virginia. During the McCarthy era, the House Un-American Activities Committee (HUAC) "invited'' him to express his views, but he was not subpoenaed. He did not cooperate with HUAC but wrote a letter in which he stated his views and backed them up with quotations from various sources.

Victor Klee, recalling his experience as a graduate student at Virginia from 1945 to 1949, wrote: "He [McShane] was very popular with the graduate students because of his clear lectures, his amusing anecdotes, and unusual kindness." Klee went on to tell how McShane turned his office over to the graduate students, who had no offices of their own: "His generosity contributed a lot to the quality of the graduate program by providing a place for the graduate students to meet with each other and talk about mathematics. . . . It is simply impossible, in a few words, to convey the extent of the graciousness, kindness, and hospitality that have been [and are] exhibited by Virginia and Jimmy McShane in their relations with those lucky enough to know them. These go far beyond professional matters.''

E. J. McShane died of congestive heart failure on June 1, 1989, in Charlottesville, Virginia.

Most of McShane's work in the 1930s was in the calculus of variations. The late 1920s and 1930s saw many changes in the calculus of variations. Leonida Tonelli's book had introduced the "direct method," which was advantageous for proving semicontinuity and the existence of absolute minima. The solution to Plateau's problem by Jesse Douglas and Tibor Rado stimulated the rapid development of the calculus of variations for multiple integral problems and the theory of Lebesgue area of surfaces. (The Plateau problem is to find a surface of minimum area with given boundary.) McShane was at the forefront of these developments. While still a graduate student McShane obtained the necessary condition of Weierstrass for quasiconvex variational problems with an arbitrary number of functions of several variables. Soon afterward he turned to questions of semicontinuity and existence of a minimum for multiple integral geometric calculus of variations, of which the Plateau problem was a prototype. Hidden in these problems were notorious analytical and topological difficulties, which were later overcome by other mathematicians as part of Lebesgue surface area theory. McShane provided an elegant solution for geometric variational integrands that do not vary spatially. The key idea was that it suffices to find the minimum in the smaller class of "saddle surfaces," which are representable parametrically by a vector function monotone in Lebesgue's sense.

In 1939 McShane published a paper in the *American Journal of Mathematics* entitled "On Multipliers for Lagrange

Problems," which was important in itself and some 20 years later had a profound, but not generally recognized, influence on optimal control theory and nonlinear programming. A solution of the problem of Lagrange satisfies two first-order necessary conditions, the Euler equations and the Weierstrass condition. Prior to the appearance of this paper the Weierstrass condition could only be established under the assumption that the Euler equations satisfied a condition called normality. This condition is not verifiable a priori. In this paper McShane established the Weierstrass condition without assuming normality. The proof was novel and consisted of first constructing a convex cone generated by first-order approximations to the end points of perturbations of the optimal trajectory and, second, showing that optimality implies that this cone and a certain ray can be separated by a hyperplane. Twenty years later, this idea was used by Lev S. Pontryagin and his coworkers in their proof of the necessary condition for optimal control now known as the Pontryagin maximum principle. The classic book by Pontryagin, Boltyanskii, Gamkrelidze, and Mischchenko entitled *The Mathematical Theory of Optimal Processes*, which collected their previous work and for which they received the Lenin Prize, popularized the convex cone and separation constructions. These constructions were subsequently used by most authors in deriving necessary conditions not only for control problems but also for nonlinear programming problems and abstract optimization problems.

Another body of work, which was definitive for problems in the calculus of variations in one independent variable, was the series of three papers that appeared in 1940 in volumes six and seven of the *Duke Mathematics Journal*. These papers concerned a broad class of problems (called of Bolza type) without convexity assumptions needed to ensure that there exists an ordinary curve that is minimizing. McShane showed that if the problem of Bolza is phrased in terms of generalized curves (which were introduced in 1937 for simple problems in the plane by L. C. Young) then the problem of Bolza has a solution. He then derived the generalizations of the standard necessary conditions that must hold along a minimizing generalized curve. Finally, he gave conditions under which the minimizing generalized curve is an ordinary curve. Definitive as this work was, it did not seem to attract attention outside the circle of cognoscenti in the calculus of variations until 20 years later, in the 1960s, when generalized curves were rediscovered by control theorists as relaxed controls, or sliding states. In a 1967 *SIAM Journal on Control* paper McShane adapted his 1940 work to the control theory setting. This paper is more elementary and self-contained than most treatments of relaxed controls and reflects McShane's dedication to teaching as well as research.

After the war McShane developed a serious interest in providing completely rigorous mathematical foundations for quantum field theory. Although the ambitious program that he undertook in this direction did not reach fruition, the attempt profoundly influenced his subsequent work on integration processes and stochastic calculus. This is seen, for example, in his excellent *Bulletin of the American Mathematical Society* survey article "Integrals Designed for Special Purposes" (1963) and his book *Stochastic Calculus and Stochastic Models* (1974), which is the definitive treatment of his approach to that subject.

In *Proceedings of the American Mathematical Society* (1967) and corrigenda and addenda (1969) McShane and R. B. Warfield proved a general version of Filippov's implicit function theorem. This lemma gives conditions that guarantee the existence of a measurable solution to an equation whenever a point-wise solution exists and is one of the basic tools in optimal control theory.

His 1973 paper in the *American Mathematical Monthly* entitled "The Lagrange Multiplier Rule" is another example of McShane's interest in instruction. Here he gave a penalty function proof of the Fritz John and Kuhn-Tucker necessary conditions for nonlinear programming problems that is short and accessible to anyone who knows standard undergraduate real analysis. Later other authors applied the arguments used here to obtain necessary conditions for a variety of control and optimization problems.

Over the years McShane achieved an extraordinarily deep understanding of integration processes as they arise in various guises. He wrote three books on integration, in addition to a number of research articles and the 1963 *Bulletin of the American Mathematical Society* survey already mentioned. His 1944 volume *Integration* gave a readable introduction to the Lebesgue theory at a time when few such books existed in English. The 1953 monograph, *Order Preserving Maps and Integration Processes*, was an outgrowth of his search for a mathematically correct setting in which to treat divergent integrals in quantum physics. In 1957 J. Kurzweil defined a modification of the Riemann integral, which turned out to be more general than the Lebesgue integral. McShane's 1983 volume *Unified Integration* develops in a similar vein a complete theory of integrals, together with a wealth of applications to physics, differential equations, and probability. An appealing feature of this approach from a pedagogical standpoint is that point-set topology and measurability issues can be deferred.

During the 1960s and 1970s McShane's interests turned toward developing a stochastic differential and integral calculus. The Ito stochastic calculus was by then already in existence. It provided a convenient way to represent an important class of stochastic processes, called Markov diffusions, as the solutions to stochastic differential equations.

The random inputs to an Ito-sense stochastic differential equation are Brownian motion processes whose formal time derivatives are “white noises.” At that time, however, there was considerable confusion in the engineering literature about the correct interpretation if an idealized white noise is replaced either by a physical “wide band’’ noise or by some discrete process introduced for numerical approximation to the solution of the stochastic differential equation. This issue was clarified by the work of McShane, Stratonovich, and Wong-Zakai.

McShane’s approach provided a particularly satisfying resolution of this issue. His stochastic integral has an important consistency property that ensures that solutions of differential equations representing physical systems driven by wide-band noises tend in the white noise limit to the solution of the corresponding McShane-sense stochastic differential equation. McShane’s *Stochastic Calculus and Stochastic Models* (1974) gave a definitive account of this work. Even today the consistency question is often not addressed in the applied literature in such areas as chemical physics, financial economics, and biology. Consistency becomes a more delicate matter for systems on a time interval of length T that is large (or infinite), as happens in questions of large deviations or ergodicity. It is perhaps ironic that it has been left to probabilists to sort out these practical consistency questions.

Edward James McShane will be remembered for his many important and often definitive contributions to mathematics, for his service to and leadership in the mathematical community and for his warmth and kindness to his students and colleagues.

THIS BIOGRAPHICAL MEMOIR is a revision of a tribute to E. J. McShane, written by us for the September 1989 issue of the *SIAM Journal on Control and Optimization*, which was a collection of research papers contributed in his honor. Our preparation of that tribute was greatly facilitated by unpublished biographical material that McShane provided a few months before his death the same year. We also wish to thank McShane's family and Victor Klee for many helpful suggestions and for their warm encouragement.

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