



Jürgen K. Moser

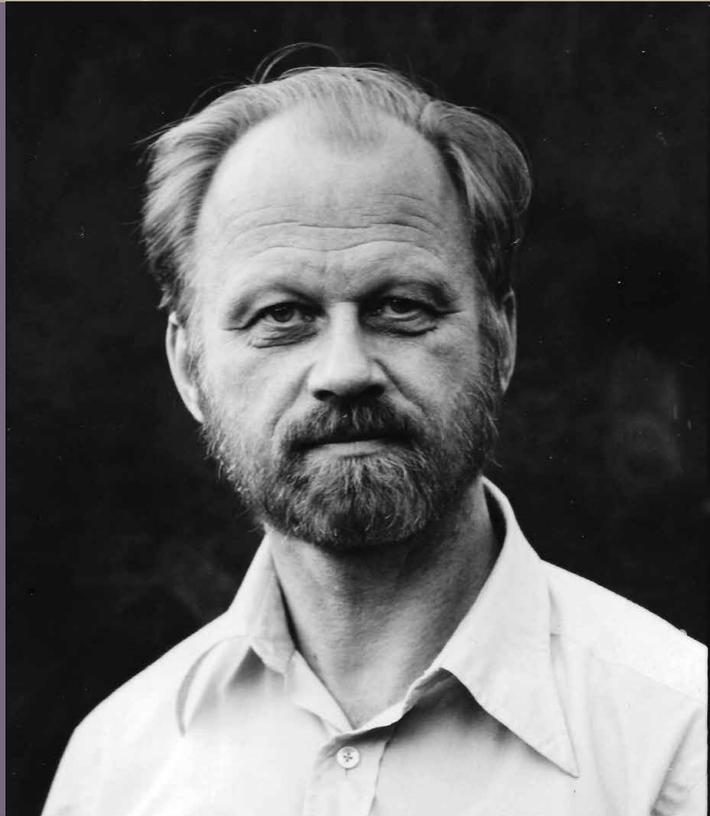
1928–1999

BIOGRAPHICAL

Memoirs

*A Biographical Memoir by
Paul H. Rabinowitz*

©2015 National Academy of Sciences.
Any opinions expressed in this memoir are
those of the author and do not
necessarily reflect the views of the
National Academy of Sciences.



NATIONAL ACADEMY OF SCIENCES

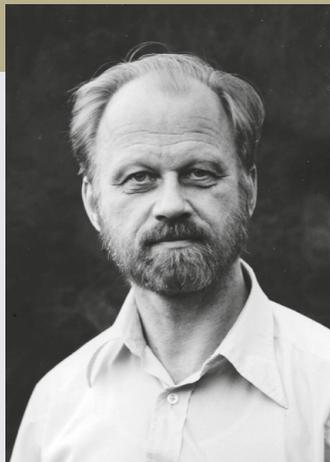
JÜRGEN KURT MOSER

July 4, 1928–December 17, 1999

Elected to the NAS, 1971

After the death of Jürgen Moser, one of the world's great mathematicians, the American Mathematical Society published a memorial article about his research. It is well worth beginning here with a lightly edited version of the brief introductory remarks I wrote then:

One of those rare people with a gift for seeing mathematics as a whole, Moser was very much aware of its connections to other branches of science. His research had a profound effect on mathematics as well as on astronomy and physics. He made deep and important contributions to an extremely broad range of questions in dynamical systems and celestial mechanics, partial differential equations, nonlinear functional analysis, differential and complex geometry, and the calculus of variations.



Jürgen Moser

By Paul H. Rabinowitz

To those who knew him, Moser exemplified both a creative scientist and a human being. His standards were high and his taste impeccable. His papers were elegantly written. Not merely focused on his own path-breaking research, he worked successfully for the well-being of mathematics in many ways. He stimulated several generations of younger people by his penetrating insights into their problems, scientific and otherwise, and his warm and wise counsel, concern, and encouragement. My own experience as his student was typical: then and afterwards I was made to feel like a member of his family. He had an important influence in his role as director of the Courant Institute of Mathematical Sciences (1967–70), as director of the Mathematics Research Institute of the ETH (Swiss Technical University) (1984–95), as president of the International Mathematical Union (1983–86), and through his active membership on numerous advisory bodies.

Moser was a lifelong music lover—he played the cello—and amateur astronomer. He enjoyed biking, swimming, and hiking. At the age of 60, he took up paragliding. With his untimely death at the age of 71, many of us in the mathematics community lost a hero, friend, and mentor (Mather et al. 2000).

Moser was born in Königsberg, Germany, at the time a beautiful old city located on the Baltic Sea not far from the border with the then Soviet Union. Königsberg is well known to mathematicians as the site of the seven bridge problem of Euler. Totally destroyed during the Second World War, the city was rebuilt, renamed Kaliningrad, and is now part of Russia.

Moser's father was a neurologist and the family—parents, older brother Friedrich, and Jürgen—lived comfortably in a large apartment that also contained his father's practice. A second brother, Klaus, was born in 1939.

The family loved music. There was a Bechstein grand piano in the living room that mother and grandmother played, and father played the violin. Jürgen and Friedrich took piano lessons. Many evenings were devoted to chamber music. Later, at age 12, being somewhat bored by piano lessons, Jürgen began to study the cello, an instrument whose sound he was taken by. His affinity for the cello continued for the rest of his life. An aunt owned a large estate near town and Jürgen had happy boyhood memories of biking there during summer holidays with Friedrich, of fun-filled days of horseback riding, of picking mushrooms, and of working with the farm animals.

In 1933, the Nazis came to power in Germany. This did not affect Moser's life much until 1938, when he was 10 years old and pushed to go to a distant elite training school. His parents recognized the possible consequences of such training, and they managed to avoid it. Instead, he entered the *Wilhelmsgymnasium*, where the famous mathematician David Hilbert had also been a student. World War II began in 1939 and four years later, at the age of 15, Moser's entire class was drafted into the army.

The boys were trained to use anti-aircraft guns to defend Königsberg from air attacks. These guns were very large, and being one of the smallest boys in his class, Moser could not lift the heavy shells. So he was assigned the task of computing the paths of the attacking aircraft. These feeble defenses proved to be for naught; in two nights of concentrated bombing, the city was destroyed, and a period of trials and hardships

Planning to enter West Germany illegally during the night, train delays led to his reaching the border area at dawn. Another hazardous crossing resulted in which shots were fired, but he put his running ability to good use.

followed for most of its residents. “For more than two years Moser lived at the edge, always hungry and often in mortal danger, surviving by sheer will and wit” (Nasar 1999). Over 90 percent of his classmates were killed in the war, as was his brother Friedrich.

When the war ended, Moser was in the British occupied zone, and even though 16 years old he looked like an undernourished child. Having managed to obtain some civilian clothing and in that way avoiding the prisoner-of-war camps, he registered as a refugee and found a job repairing roads. After several months of searching for his parents, Moser finally found that they were living in a small town, Greifswald, in the Russian-occupied zone. He decided to visit them by illegally crossing the border but was captured by Russian soldiers and held without food in a water-flooded basement for six days. He barely escaped starvation, with only a stolen turnip to sustain himself. On a dozen later occasions he crossed the border again—still illegally but at better chosen times and places, and was never caught again. Upon his release by the Russians, he was able to reunite with his parents.

Returning to school, Moser completed his high-school education in April 1947 and sought admission to a university in the Russian zone. But both of his applications were turned down, given the requirement in those days that his father had to be a farmer or a worker. Furious, he decided to try to study at the University of Göttingen, located in West Germany and the first German university allowed to reopen after the war.

Moser’s high-school mathematics teacher supplied him with a letter of recommendation to Franz Rellich, director of the Mathematics Institute at Göttingen. Planning to enter West Germany illegally during the night, train delays led to his reaching the border area at dawn. Another hazardous crossing resulted in which shots were fired, but he put his running ability to good use. After arriving in Göttingen, he reached Rellich.

In an interview with mathematics writer Constance Reid many years later, Moser recalled his amazement at Rellich’s friendliness and interest. Göttingen was extremely crowded; and lodging, though crucial, was elusive. “One could not register as a student,” said Moser, “without having a place to live; and one couldn’t get a place to live unless he was

a student.” But Rellich helped him find a room. Indeed, Rellich was quite generous toward all of the students, many of whom were older [and needier] than usual because their lives had been disrupted by the war. “And then there was the matter of food,” Moser said. “We students were literally starving, and Rellich managed somehow to get CARE packages and distribute food to us” (Reid 1976).

So began Moser’s six years of mathematics study at Göttingen. Although historic and famous, the university had suffered severe losses of personnel, first because of the Nazis’ policies and then from the effects of the war. In particular Richard Courant, who had been director of the Mathematics Institute at Göttingen, had been forced out of his position and compelled to emigrate from Germany because he was Jewish. Rellich had been his student. In a letter to Courant 10 days after the American occupation of Göttingen, Rellich wrote: “The Mathematics Institute sleeps like Sleeping Beauty and obviously awaits the kiss of the prince. It would be wonderful if you felt like playing the role of the prince” (Reid 1976).

Moser chose mathematics for his studies because he had always liked the subject, though he did not have much formal training. However, during his military service and its aftermath, he had developed an effective way to distance himself from his surroundings. He would make up a mathematical problem and focus on solving it in his head. In an email to his grandson, Moser later observed: “[This process] was very soothing. The officers could yell as much as they wanted; they could not reach me. That is how I learned to think and concentrate.” While living in Greifswald—in a small attic room with his parents and younger brother, Klaus, often with no light—Jürgen would put a tea cozy over his head as a signal that he wanted to study and be focused on his mathematics. This too led to his development of great powers of concentration.

Moser studied at Göttingen from 1947 to 1953, together with some classmates—Erhard Heinz, Konrad Jorgens, Johannes Nitsche—who later became well-known mathematicians. Moser eagerly absorbed the material presented to him. It was all new but he understood it quite readily—as a result, he believed, of his ability to concentrate. He took courses from teachers such as Rellich, Wilhelm Magnus, and Helen Braun.

In a letter to Courant 10 days after the American occupation of Göttingen, Rellich wrote: “The Mathematics Institute sleeps like Sleeping Beauty and obviously awaits the kiss of the prince. It would be wonderful if you felt like playing the role of the prince.”

Moser wrote his dissertation under the direction of Rellich. It was on the topic of perturbation theory for the continuous spectrum of differential operators. Another of Moser's legacies from Rellich was regular contact with physicists. Indeed when Rellich held a joint seminar with Werner Heisenberg, Moser developed an interest in quantum field theory.

Another, and more important, influence on Moser began in 1950 with the return of the famous mathematician C. L. Siegel to Göttingen after a 10-year stay in Princeton. Siegel, who taught courses on number theory and celestial mechanics, was an imposing figure, and Moser was greatly impressed by his style and impeccable lectures. It was because of Siegel that Moser developed a lifelong interest in astronomy. At Rellich's suggestion, Moser wrote the official notes for Siegel's celestial mechanics course, which became the first draft of Siegel's 1953 book on the subject. A second edition, revised by Moser, appeared under their joint authorship in 1971.

Siegel was a stern taskmaster, but also sometimes fickle. At one point in his writings, he asked Moser to calculate the characteristic polynomial of a 12 by 12 matrix, an onerous job that Moser couldn't complete in the allotted time. Siegel reproached him: "What! You haven't done your homework. It is very important for me that you calculate this polynomial." Consequently, with the aid of some friends, Moser spent several days on the computation and went triumphantly to Siegel to announce his achievement. Thereupon a bored Siegel responded with, "Oh yes, I quickly calculated it last night."

After completing the work for his Ph.D. in 1951 and receiving the degree in 1952, Moser was awarded a Fulbright Fellowship and an invitation from New York University to spend a year there. He had long wanted to leave Germany, in part because of his wartime experiences.

Courant, who had been forced out of the University of Göttingen's Mathematics Institute by the Nazis, moved to the United States in 1934 and began creating at NYU a new Institute for Mathematics and Mechanics. By 1953, the institute had become the primary center for research in partial differential equations and applied mathematics in the United States. Staffed mainly by a remarkable collection of European émigrés and talented young Americans, the institute provided Moser with a stimulating year, enabling him to meet senior mathematicians such as Kurt Friedrichs, Fritz John, Jim Stoker, and Lipman Bers as well as gifted younger people such as Peter Lax, Cathleen Morawetz,



Figure 1. Moser, 1957.

Louis Nirenberg, Joe Keller, and Jack Schwartz. At first it was a difficult and lonely period. The war was fresh in Americans' memories and Germans were viewed with suspicion. As Lax wrote, "After Jürgen had been in our circle for a short time, we realized he was very special, a prince among men, a knight in shining armor. He had all the German virtues: a devotion to hard work, a love of the outdoors, a love of beauty, of music" (Lax 2002)

Courant greatly enjoyed playing music and had regular gatherings for that purpose at his home in New Rochelle, a New York City suburb. Playing his cello in string quartets there, Moser met Courant's daughter Gertrude, who played the viola. She had recently been divorced and had a young son, Richie. When Jürgen and Gertrude soon fell in love, it gave him one more reason for remaining in the United States. Complicating

matters, however, was Moser's promise to Siegel that he would return to Göttingen as Siegel's assistant for the 1954–55 academic year. For Moser, this position represented not only an honor but also a duty that had to be carried out. Therefore he returned to Göttingen, as agreed. On informing Siegel of his plans to go back to New York and marry after the year was up, Siegel's reply was "Oh what a pity; I thought you wanted to become a mathematician."

Moser returned to New York in September 1955, married Gertrude, and began his second stay at NYU. Appointed assistant professor there, he worked on various problems in ordinary differential equations. An anecdote from that period illustrates both his fearlessness and presence of mind. The Mosers lived in a small second-floor apartment in Manhattan's Greenwich Village within easy walking distance of NYU. One afternoon

there were loud noises from the apartment below, and then came a knock at their door. It was the young daughter of the building manager, in tears, who said that her father was trying to kill her mother. Moser rushed downstairs to the manager's apartment, where the drunken man was chasing his wife around the kitchen table with a butcher knife in his hand. Moser's immediate reaction was to call out loudly, "Where is the package that was delivered for me earlier today?" The distracted manager stopped to look for the nonexistent package and the crisis passed.



Figure 2. The Moser family, 1959.

Moser stayed at NYU until 1957. Then, as a result of the strong recommendation of Norman Levinson, one of the leading members of the Massachusetts Institute of Technology's Mathematics Department, MIT offered him a position as an associate professor. Somewhat to the vexation of Courant, he accepted and moved there in September, shortly after the birth of his first child—a daughter, Nina. A second daughter, Lucy, was born a year later. With their young family, the Mosers spent three happy years living in a small rented house in Needham, a suburb of Boston. For his part, Moser's time at MIT was productive, and he met and became friends with several of the young mathematicians there, including John Nash, Lennart Carleson, and Paul Cohen. In 1959, Moser became a U.S. citizen.

At MIT, Moser continued to do research on dynamical systems, and on celestial mechanics problems in particular. When he had been at Göttingen, Siegel introduced him to a class of problems in that field involving "small divisors"—more precisely, regarding linearized operators that could be inverted formally. But in calculating this formal inverse as an infinite series, one had to divide by terms that approached zero—i.e., the small divisors. Therefore convergence of the series was dubious. Siegel had urged Moser to pursue such questions, especially the stability problem for fixed points of area-preserving maps.

By chance, while at MIT, Moser received a request from the journal *Mathematical Reviews* to review the published version of a talk given by the famous Russian mathematician Andrey Kolmogorov at the 1954 International Congress of Mathematicians held in Amsterdam. Kolmogorov had discussed some remarkable work on the existence and stability of quasiperiodic solutions of near-integrable Hamiltonian systems, a result that greatly excited Moser because of its implications for the stability questions to which Siegel had steered him. But despite carefully reading the corresponding paper, Moser was unable to follow a crucial part of the argument that dealt with convergence; nor did the actual published sketch of the proof in the Russian journal *Doklady* shed any further light on this point. Therefore he wrote to Kolmogorov for clarification, but received no reply. As Moser wrote in a later article, “I never believed in ‘proof by authority,’” and he simply pointed out that he found the convergence discussion unconvincing (Moser 1999). He did not doubt that Kolmogorov could prove the result, but he continued to try to find his own proof for the special case of area-preserving twist maps.

One person at MIT whose research especially intrigued Moser, and with whom he conferred many times, was John Nash. A few years earlier, Nash had proved a celebrated embedding theorem. This difficult problem, which had features such as a so-called “loss of derivatives” for an associated linearized operator, seemed to be related to the puzzling small-divisor problems that were of special interest to Moser. Nash at that time was obsessed with questions pertaining to the regularity of weak solutions of quasilinear elliptic and parabolic partial differential equations (PDEs). In papers appearing in 1957 and 1958, Ennio De Giorgi and Nash independently made major advances in this area, De Giorgi for the elliptic case and Nash for the parabolic. Moser was drawn to such regularity matters, and in 1959 he found an important new approach to the elliptic case that involved an ingenious and delicate iteration scheme using the weak form of the equation to get estimates for weak solutions. This research attracted a lot of attention, including an offer from NYU to return as a full professor, which he accepted.

The Mosers came back to New York in fall 1960 and found a sizable house they liked, replete with a large garden and tall trees, in the suburb of New Rochelle, where several of the senior members of the Mathematics Department—Courant, Lipman Bers, Kurt Friedrichs, Fritz John, and Wilhelm Magnus—also resided. Gertrude took a half-time job at the Albert Einstein Medical Center, where she did biology research in the area of her PhD thesis. Given that both parents worked, they regularly had an au pair girl to help with the kids and the house. And because the au pairs usually did not know how to drive a car, Moser routinely gave them driving lessons; at least one of his Ph.D.

students—the author of this memoir—benefited as well from this sideline of his. But from Moser’s suburban locale, he himself had a lengthy commute to the city, combining bike, train, subway, and foot to get to NYU, as he had opted to leave the car at home.

Now began Moser’s most creative period, with several ideas flowering in succession. In 1961, he was trying to extend his iteration method to obtain a Harnack inequality for positive weak solutions of elliptic equations. One day, when he happened to be on the train headed home, fellow commuter Fritz John told him about some work that he was doing with Louis Nirenberg on functions of bounded mean oscillation (BMO). That evening, Moser realized that he could use their BMO result to complete the proof of his Harnack inequality. A few years later, in 1964, he produced a version of his Harnack inequality for parabolic PDEs. After the publication of some important work by Enrico Bombieri, which bypassed the use of the John-Nirenberg result to prove the elliptic Harnack inequality, Moser was able to do the same for the parabolic Harnack inequality. These and other tools and ideas that Moser introduced during this period had a major impact on subsequent research in regularity theory for elliptic and parabolic PDEs.

In another direction, the one that became his main occupation for many years, Moser discovered a common approach to the loss-of-derivatives and small-divisors questions that had been such a mystery to him before. As he later wrote (Moser 1999), these questions could be tackled by combining three ideas. One came from Kolmogorov’s work: an iteration scheme of Newton type—i.e., one that converged very rapidly. Another idea, which came from Nash’s proof of the embedding theorem, involved a suitable smoothing approximation process. Finally, one had to combine the gain in accuracy given by the rapid convergence with the loss in accuracy due to the smoothing operation; how to do this successfully was aided by insights he obtained from his recent work on the elliptic regularity question. In a paper introducing his deep and useful merging of these ideas (Moser 1961), Moser also supplied an application—a concise new proof of Nash’s Embedding Theorem.

Thereafter followed a lengthy series of papers, lasting well into the 1970s, in which he applied this method to a variety of other important problems, mainly those involving the existence of quasi-periodic solutions of differential equations. The first of these publications appeared in 1962 in what many consider to be Moser’s greatest paper. It presented a proof of what is generally called the Invariant Curve Theorem, a problem posed by G. D. Birkhoff many years earlier; Moser had been pursuing this question on and off since his days with Siegel.

A mapping of an annulus into itself—a mapping that preserves circles by rotating them by an angle that increases as the radius increases—is called a twist map. Suppose that a perturbation of the map has the property that each circle intersects its image. An important case for applications in which this latter condition is satisfied occurs when the perturbed map is area-preserving and transforms the inner circle into itself. Moser proved that if the perturbation is sufficiently small and smooth, corresponding to each invariant circle whose rotation angle (or rotation number) satisfies an appropriate Diophantine condition, the perturbed map possesses a closed invariant curve with the same rotation number that is near the original circle.

In applications, such maps arise as Poincaré maps of a two-dimensional section of a three-dimensional flow, with the closed invariant curves corresponding to almost-periodic solutions of the associated differential equation. Moreover, the existence of these invariant curves leads to important stability results. Indeed, Moser further showed that the theorem leads to stability of elliptic fixed points of general type for area-preserving mappings, thus resolving a question Siegel that had earlier posed to him (Moser 1962). Even Siegel was impressed with this work.

With these papers and his subsequent efforts treating higher-dimensional problems in which invariant curves are replaced by invariant tori, Moser extended Kolmogorov's result from the analytic category to the C^∞ setting. In 1963, a detailed proof of Kolmogorov's result was given by Kolmogorov's brilliant student, Vladimir Arnold, also in the analytic setting. Such were the origins of what is now known as KAM (for Kolmogorov-Arnold-Moser) theory, one of the great mathematical achievements of the 20th century, and Moser's remarkable contributions to KAM theory gave him an international reputation. For example, he was invited to present his results at the International Congress of Mathematicians held in Stockholm in 1962. But although people refer to this work as KAM theory and likewise speak of Nash-Moser theory, Moser himself always viewed these achievements as a collection of ideas and techniques rather than theories. He made this point, and also shared some of his memories of the early development of KAM, in the brief article "Recollections" (Moser 1999). In September 1963, Moser visited Moscow and met Kolmogorov, Arnold, and many of the outstanding young Soviet mathematicians working in dynamical systems and PDE.

Moser attracted many Ph.D. students, 24 in all over the years. He also had a steady stream of postdocs and visitors. He was a clear thinker and as such his lectures were always beautifully organized. Because he saw things so transparently himself, in the

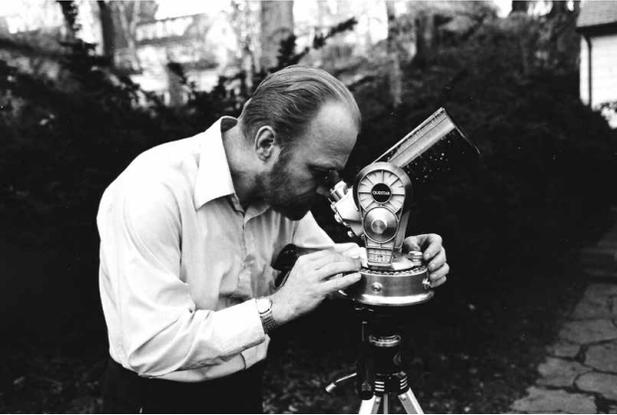


Figure 3. Moser with his telescope, 1979.

courses Moser taught he didn't fuss as much with the details as some do, but filling them in was usually a rewarding learning experience for the students. He was very generous with his time and his ideas. Speaking to Moser was likely to be a good investment; given his keen insight, he generally had something worthwhile to say about your problems, mathematical or otherwise. He helped many talented young people to find jobs and others to restart stalled careers.

Mark Adler, one of his students, wrote of his meetings with Moser:

I remember vividly how after I would get a new result and show it to Jürgen, he would first praise it and then say 'Now you need to understand the result.' After I came back with what I thought was understanding, he would [again] suggest that I need to really understand the result. In fact, in my thesis years I was exposed to a number of problems, but my real goal was ... to understand what the whole field (integrable systems) was really about—a very Jürgenish attitude—and [that] the problems I [encountered] were just a means to that end.

Another of Moser's students, Mark Levi, recalled:

After my thesis clicked, I described to Jürgen the geometrical picture, which was correct in principle but not in every detail. Jürgen praised it and also said that this is still half-baked stuff (actually, he stated this rather more profanely); somehow he made it sound like a compliment, maybe because he exuded such a sense of benevolence, and I knew what he meant. It felt from his reaction that he was pleased with the progress, and the remark was a relaxed way of saying that it's too early to celebrate.

As the above anecdote indicates, before he would accept the veracity of a result, it was essential for Jürgen that all of the critical details be tied down. Here is one such

experience of my own: Several years after my Ph.D., he was visiting the University of Wisconsin at Madison and attended a seminar talk that I gave about periodic solutions of a nonlinear wave equation. In fact it was a generalization I had long sought of some of my thesis work. Jürgen had just seen a preprint of work by Alan Weinstein on periodic solutions of prescribed energy for Hamiltonian systems and asked whether the variational techniques I was employing could treat such problems. I thought about it, and upon seeing him a couple of days later I said that I believed so. “Well,” he said, “if you can do it, I’ll give you a quarter.” Six months later, I sent him a preprint and duly received a quarter in the mail. I still have it!

Jürgen’s students, postdocs, and visitors were regular dinner visitors at his home. He enjoyed the discussions there and didn’t pay much attention to the food. Gertrude often remarked, “You could feed him sawdust and he wouldn’t notice!” As Peter Lax wrote, “He was exceedingly good company to do things with, like hiking in the mountains. He was very good [at] describing in detail something interesting that happened to him. He loved adventure and to test his powers; he had great self-confidence” (Lax 2002).

Jürgen possessed exceptional foresight, and he not only anticipated more things than most of us do but also devised practical solutions and implemented them with a cool head. For example, consider the fact that he went by bicycle to the New Rochelle train station every weekday. Riding home one rainy evening, a car coming out of a gas station did not notice Jürgen and ran into him. Jürgen saw that the car was about to hit him, but given that he had considered and planned for such a possibility, just before the collision occurred he jumped up strongly, using the pedals and the handlebars for leverage exactly as anticipated. Thus when the car (moving horizontally) hit Jürgen he was moving vertically, and he went flying over the car instead of landing under it, as typically happens.

The driver stopped immediately and got out. When he saw the broken bicycle in front of his car, and started looking for the victim underneath. Jürgen, somewhat dazed, got up and went to the driver, who said, “I think I hit a bicyclist, but I can’t find him.” Not quite realizing what had happened, Jürgen started helping the driver look. Then it occurred to Jürgen that he was the victim, and told the driver, who was quite relieved but also surprised, considering the damage to the bike. It was completely bent out of shape, but Jürgen had only a minor bump on his leg.

Because the bike was rendered useless, the driver offered Jürgen a ride home. When they arrived, Gertrude was totally nonchalant. She considered Jürgen indestructible and just consoled the distraught driver, basically ignoring poor Jürgen. The fact that Gertrude

acted as if everything was fine just illustrates what many of us expected of Jürgen: he was going to find a way to deal with any problem, so he was the last person to worry about.

But Jürgen was not an iron man. He had his sensitive side, especially regarding his family. During a three-month visit to Stanford in 1966, his young daughter Nina broke her arm after falling off a horse. The Mosers rushed her to a hospital and Jürgen was with Nina in the emergency room when her arm was being set. They gave her a shot and Jürgen keeled over at the sight of the needle. The nurse ran to Gertrude in the waiting room and said, “Your husband just fainted.” To which his wife replied, “He does that all the time!”

Moser assumed the directorship of NYU’s Courant Institute of Mathematical Sciences (as it had been renamed) in 1967. That Vietnam War period was a trying time at American universities, but Moser’s foresight, good judgment, and patience made for what colleagues considered a very successful term of office. Nevertheless, he gladly stepped down when his term ended in 1970. When Courant expressed disappointment that Moser did not continue as director, Peter Lax pointed out that Moser disliked the position. Courant replied, “Jürgen can’t have disliked it that much, for then he would not have done such an excellent job” (Lax 2002).

In 1967, Moser received the James Craig Watson Medal of the National Academy of Sciences for his contributions to dynamic astronomy. In subsequent years, many other honors and awards followed. For example, he was awarded the first G. D. Birkhoff Prize in Applied Mathematics in 1968 and elected to the NAS in 1971.

Although he made forays back to earlier areas of research, Moser regularly went on to learn work in new ones. In particular, in the mid-1970s, Moser developed a new interest: complex geometry. A basic question in that subject, going back to Élie Cartan, involves a pair of real analytic hypersurfaces in \mathbf{C}^n that are locally holomorphically equivalent. In 1974, Moser and the well-known differential geometer S. S. Chern published an influential paper on this question. In 1983, Moser and S. M. Webster studied the higher-codimensional case.

A second new area of Moser’s during that period involved a series of questions about the old topic of complete integrability of Hamiltonian systems. For a long time, the subject matter consisted of some theory and a collection of nice examples. In 1890, Henri Poincaré showed that most Hamiltonian systems were not completely integrable. But the subject then essentially disappeared from view until 1965, when Martin Kruskal and Norman Zabusky made the intriguing discovery of solitons for the Korteweg-de Vries

equation (KdV), which models the motion of long waves in shallow water. A crucial piece of work by Lax in 1968 led to the discovery in 1974, by Clifford Gardner, J. M. Greene, Kruskal, and Robert Miura, that KdV was an infinite-dimensional completely integrable Hamiltonian system. An avalanche of papers soon followed that established the complete integrability of other important infinite-dimensional Hamiltonian systems.

Moser's first paper on the subject came out in 1974, when he analyzed Toda's lattice—a linear collection of particles connected by exponential restoring forces. His clear analysis served as a model many subsequent researchers followed. He himself made many other seminal contributions to the study of complete integrability. One reason he liked this area was that he could draw upon the broad knowledge of classical geometry that he acquired in Göttingen. Moser was a very modern mathematician, but also a lover of the classical beauty of the older mathematics. (Related information can be found in Henry McKean's contribution to Mather et al. 2000.)

In the late 1970s, Moser was soon to face an empty nest and tiring of his long commutes. Although he had had a happy and productive time at NYU, he started to think about returning to Europe. He had no interest in a position in his native Germany; extended earlier visits had shown him that he no longer felt at home there. But a door was opened for him elsewhere when in 1978 Moser received an unexpected offer of a professorship from the Swiss Technical University (ETH) in Zürich, where he had presented the Pauli Lectures in 1975. Although he found himself intrigued by this possibility, Moser replied that he was not willing to think about such matters until his daughters completed college two years later. When the offer was repeated in 1980, after gaining Gertrude's slightly reluctant consent he accepted.

The circumstances leading to the offer make for a good story. The Mathematics Department of the ETH was seeking a new appointment, and the leading candidate's case was presented to the university president. When he requested documentation of the candidate's quality, he was presented with a very strong letter of recommendation written by Moser. "And who is this Moser?" asked the president.

The reply: "The best researcher in the world working on dynamical systems."

"So why don't we try to get him?" the president responded.

And they did!

The Mosers purchased a house in Schwerzenbach, a Zürich suburb. It had plenty of room for their visitors and a swimming pool, of which they took full advantage. Moser still had a commute, but a much less onerous one than in New York. The transition to the more structured life of Switzerland from their earlier informal lifestyle was not easy, but, as on earlier occasions, the Mosers made many new friends there, often through their playing in music groups. Meanwhile, their children went on to begin noteworthy careers. Richie became a successful lawyer in New York. Nina, the older daughter, became a theater/film designer in Seattle. Lucy, the younger, earned a Ph.D. in mathematics at the University of Geneva in 1987 and in 1998 became a professor at the University of Bourgogne, France.

After arriving at the ETH, Moser initially continued to pursue some of his earlier mathematical interests, but he also started working on some questions involving spectral theory for Schrödinger equations with quasiperiodic potentials. And he maintained a very active program of visitors. In 1981, Moser and his former postdoc Edi Zehnder co-organized a weeklong meeting on dynamical systems, which was held at the Mathematical Research Institute in Oberwolfach, Germany. This meeting became the first of an influential series that attracted the leading figures in the field and that continues to this day. Moser and Zehnder served as co-organizers until Moser's retirement from the ETH in 1995.

In the mid-1980s, Moser's research moved in a new direction. He became interested in extending Aubry-Mather theory from its familiar setting in dynamical systems to nonlinear elliptic PDEs; and he soon came to enjoy this topic, which drew on ideas from many areas, including the calculus of variations, geometry, and PDEs.

One time during Moser's ETH years, when Zehnder was visiting, he brought his old dog, Max. Zehnder got into a lengthy discussion with a colleague, and because it didn't involve Moser he decided to take Max for a walk. They encountered a young girl with a large dog that took an immediate dislike to Max, and when she lost control of her dog it attacked Max. The viciousness of the scene was not very conducive to human intervention, but it was clear to Moser that without it Max would be severely injured. So



Figure 4. Playing the cello.



Figure 5. Hang gliding in Rio.

he jumped on top of the belligerent dog and held him down, saving Max. Age had not diminished Moser's courage and quick thinking.

In 1983, Moser was elected president of the International Mathematical Union—a significant new responsibility. In this capacity, among other achievements he helped bring the quadrennial International Congress of Mathematicians to Zürich in 1986. In 1984, Moser assumed the directorship of the Mathematics Research Institute at the ETH. Over the years, the institute had lost some of its earlier luster, which Moser sought to restore. Such attempts are never simple, but Moser could be quite forceful, and with his usual good judgment and high standards, he played a major role in making the institute a more prestigious venue for mathematicians. He continued as director until he reached the mandatory retirement age of 67 in 1995.

Earlier, when he had turned 60, Moser was appointed to an honorary professorship at the Institute for Pure and Applied Mathematics (IMPA) in Rio de Janeiro—one among many such honorary professorships he received during his career. But his visit to Rio became especially noteworthy when he observed people engaged in hang gliding. Moser

thought the activity looked so graceful and pleasurable that he decided to try it himself, then and there. On his return to Switzerland, he passionately took up the related sport of paragliding,¹ and it became a great joy for him until his health no longer permitted it.

Among the major honors Moser received in his final years were the Cantor Medal of the German Mathematical Society (1993) and Israel's Wolf Prize for Mathematics (1995). Although he retired in 1995, Moser continued to do research until his final illness. He died from complications of prostate cancer in 1999 at the age of 71, far too early.

FURTHER READING

Much more about Moser's research and its impacts can be found in the reference articles below by John Mather, Louis Nirenberg, Henry McKean, and Paul Rabinowitz (Mather et al. 2000); by Edi Zehnder (Zehnder 1993); by Peter Lax (Lax and Moser 2002); and by Boris Hasselblatt and Anatole Katok (Hasselblatt and Katok 2002).

ACKNOWLEDGMENTS

The author thanks Mark Adler, Mark Levi, Nina Moser, and Lucy Moser-Jauslin for their help with this article. In particular, I am grateful to Nina Moser for providing the photos.

¹ Whereas paragliding involves what is essentially a parachute, hang gliding allows the participant to fly with the aid of a large shaped sail supported by an aluminum frame.

REFERENCES

- Hasselblatt, B., and A. Katok. 2002. The development of dynamics in the 20th century and the contribution of Jürgen Moser. *Ergodic Theory and Dynamical Systems* 22:1343–1364.
- Lax, P. D. 2002. J. Moser. *Ergodic Theory and Dynamical Systems* 22:1337–1342.
- Mather, J. N., H. P. McKean, L. Nirenberg, and P. H. Rabinowitz. 2000. Jürgen K. Moser (1928–1999). *Notices of the American Mathematical Society* 47(11):1392–1405.
- Moser, J. 1999. Recollections. In *The Arnoldfest: Proceedings of a conference in honour of V. I. Arnold for his sixtieth birthday*, edited by E. Bierstone, B. Khesin, A. Khovanskii, and J. Marsden. pp. 19–21. Toronto: Fields Institute Communications.
- Nasar, S. 1999. Jürgen Moser, who proved celestial theory, dies at 71. *New York Times*, December 21.
- Reid, C. 1976. *Courant in Göttingen and New York*. New York: Springer.
- Zehnder, E. 1993. Cantor Medaille für Jürgen Moser. *Jber. d. Dt. Math. – Verein* 95:85–94.

SELECTED BIBLIOGRAPHY

- 1958 On a generalization of a theorem of A. Liapounoff. *Comm. Pure Appl. Math.* 11:257–271.
- 1960 A new proof of De Giorgi's Theorem concerning the regularity problem for elliptic differential equations. *Comm. Pure Appl. Math.* 13:457–468.
- 1961 A new technique for the construction of solutions for nonlinear differential equations. *Proc. Nat. Acad. Sci. U.S.A.* 47:1824–1831.
- On Harnack's Theorem for elliptic differential equations. *Comm. Pure Appl. Math.* 14:577–591.
- 1962 On invariant curves of area-preserving maps of an annulus. *Nachr. Akad. Wiss. Göttingen* 1:199–226.
- 1964 With R. K. Brayton. A theory of nonlinear networks, I and II. *Quarterly of Appl. Math.* 22:1–33 and 22:81–104.
- A Harnack inequality for parabolic differential equations. *Comm. Pure Appl. Math.* 17:101–134.
- 1965 On the volume elements on a manifold. *Transactions of the American Math. Soc.* 120:286–294.
- 1966 A rapidly convergent iteration method and nonlinear partial differential equations, I. *Ann. Scuola Normale Sup. Pisa* 20:265–315
- A rapidly convergent iteration method and nonlinear partial differential equations, II. *Ann. Scuola Normale Sup. Pisa* 20:499–535.
- 1970 Regularization of Kepler's problem and the averaging method on a manifold. *Comm. Pure Appl. Math.* 23:609–636.
- 1971 With C. L. Siegel. *Lectures on celestial mechanics (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen)*, volume 187. Berlin: Springer-Verlag.
- On a pointwise estimate for parabolic differential equations. *Comm. Pure Appl. Math.* 24:727–740.
- A sharp form of an inequality by N. Trudinger. *Indiana Univ. Math. J.* 20:1077–1092.

- 1973 *Stable and random motions in dynamical systems: With special emphasis on celestial mechanics.* Annals of Math. Studies, vol.77. Princeton, NJ: Princeton University Press.
- 1974 With S. S. Chern. Real hypersurfaces in complex manifolds. *Advances in Math.* 113:219–271.
- 1975 Three integrable Hamiltonian systems connected with isospectral deformations. *Advances in Math.* 16:197–220.
- 1976 Periodic orbits near an equilibrium and a theorem of Alan Weinstein. *Comm. Pure Appl. Math.* 29:724–747.
- 1977 With H. Airault and H. P. McKean. Rational and elliptic solutions of the Korteweg-deVries equation and a related many-body problem. *Comm. Pure Appl. Math.* 30:95–148.
- 1978 With M. Adler. On a class of polynomials associated with the Korteweg-de Vries equation. *Communications in Mathematical Physics* 61:1–30.
- 1982 With R. Johnson. The rotation number for almost-periodic potentials. *Communications in Mathematical Physics* 84:403–438.
- 1983 *Integrable Hamiltonian systems and spectral theory.* Fermi Lectures given in 1981 at the Scuola Normale Superiore. Pisa: Accademia Nazionale Dei Lincei.
- 1986 Minimal solutions of variational problems on a torus. *Ann. Inst. H. Poincaré Analyse Non Linéaire* 3:229–272.
- 1988 A stability theorem for minimal foliations on a torus. *Ergodic Theory and Dynamical Systems.* 8:251–281.

Published since 1877, *Biographical Memoirs* are brief biographies of deceased National Academy of Sciences members, written by those who knew them or their work. These biographies provide personal and scholarly views of America's most distinguished researchers and a biographical history of U.S. science. *Biographical Memoirs* are freely available online at www.nasonline.org/memoirs.