NATIONAL ACADEMY OF SCIENCES

WILLIAM FOGG OSGOOD
1864–1943

A Biographical Memoir by

JOSEPH L. WALSH

Any opinions expressed in this memoir are those of the author and do not necessarily reflect the views of the National Academy of Sciences.

Biographical Memoirs, VOLUME 81

PUBLISHED 2002 BY
THE NATIONAL ACADEMY PRESS
WASHINGTON, D.C.

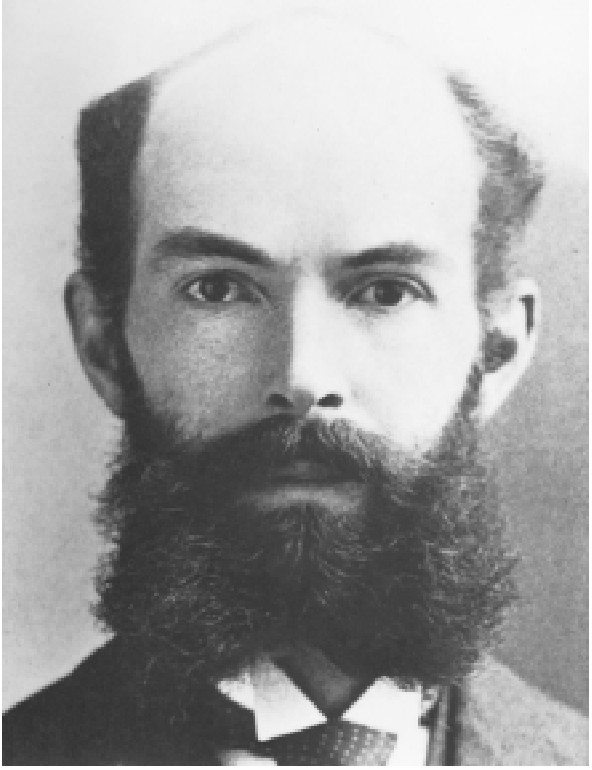

Photo courtesy of the American Mathematical Society.

William F. Osgood

WILLIAM FOGG OSGOOD

March 10, 1864–July 22, 1943

BY JOSEPH L. WALSH

WILLIAM FOGG OSGOOD WAS born in Boston, Massachusetts, the son of William and Mary Rogers (Gannett) Osgood. He prepared for college at the Boston Latin School, entered Harvard in 1882, and was graduated with the A.B. degree in 1886, second in his class of 286 members. He remained at Harvard for one year of graduate work in mathematics, received the degree of A.M. in 1887, and then went to Germany to continue his mathematical studies. During Osgood's study at Harvard, the great Benjamin Peirce (1809-1880), who had towered like a giant over the entire United States, was no longer there. James Mills Peirce (1834-1906), son of Benjamin, was in the Mathematics Department, and served also later (1890-1895) as Dean of the Graduate School and (1895-1898) as Dean of the Faculty of Arts and Sciences. William Elwood Byerly was also a member of the Department (1876-1913) and is remembered for his excellent teaching and his texts on the Calculus and on Fourier's Series and Spherical Harmonics. Benjamin Osgood Peirce (1854-1914) was a mathematical physicist, noted for

Reprinted with permission. Courtesy of the Harvard University Archives. Selected bibliography appended.

his table of integrals and his book on Newtonian Potential Theory. Osgood was influenced by all three of those named—they were later his colleagues in the department—and also by Frank Nelson Cole. Cole graduated from Harvard with the Class of 1882, studied in Leipzig from 1882 to 1885, where he attended lectures on the theory of functions by Felix Klein, and then returned to Harvard for two years, where he too lectured on the theory of functions, following Klein's exposition.

Felix Klein left Leipzig for Göttingen in 1886, and Osgood went to Göttingen in 1887 to study with him. Klein (Ph.D., Göttingen, 1871) had become famous at an early age, especially because of his Erlanger Program, in which he proposed to study and classify geometries (Euclidean, hyperbolic, projective, descriptive, etc.) according to the groups of transformations under which they invariant; thus Euclidean geometry is invariant under the group of rigid motions. The group idea was a central unifying concept that dominated research in geometry for many decades. Klein was also interested in the theory of functions, following the great Göttingen tradition, especially in automorphic functions. Later he took a leading part in organizing the *Enzyklopädie der Mathematischen Wissenschaften*, the object of which was to summarize in one collection all mathematical research up to 1900. Klein also had an abiding interest in elementary mathematics, on the teaching of which he exerted great influence both in Germany and elsewhere.

The mathematical atmosphere in Europe in 1887 was one of great activity. It included a clash of ideals, the use of intuition and arguments borrowed from physical sciences, as represented by Bernhard Riemann (1826-1865) and his school, versus the ideal of strict rigorous proof as represented by Karl Weierstrass (1815-1897), then active in Berlin. Osgood throughout his mathematical career chose the best from the two schools, using intuition in its proper place to suggest results and their proofs, but relying ultimately on rigorous logical demonstrations. The influence of Klein on “the arithmetizing of mathematics” remained with Osgood during the whole of his later life.

Osgood did not receive his Ph.D. from Göttingen. He went to Erlangen for the year 1889-1890, where he wrote a thesis, “Zur Theorie der zum algebraischen Gebilde $y^m = R(x)$ gehörigen Ableschen Functionen.” He received the degree there in 1890 and shortly after married Theresa Ruprecht of Göttingen, and then returned to Harvard.

Osgood’s thesis was a study of Abelian integrals of the first, second, and third kinds, based on previous work by Klein and Max Noether. He expresses in the thesis his gratitude to Max Noether for aid. He seldom mentioned the thesis in later life; on the one occasion that he mentioned it to me he tossed it off with “Oh, they wrote it for me.” Nevertheless, it was part of the theory of functions, to which he devoted so much of his later life.

In 1890 Osgood returned to the Harvard Department of Mathematics, and remained for his long period of devotion to the science and to Harvard. At about this time a large number of Americans were returning from graduate work in Germany with the ambition to raise the scientific level of mathematics in this country. There was no spirit of research at Harvard then, except what Osgood himself brought, but a year later Maxime Bôcher (A.B., Harvard, 1888; Ph.D., Göttingen, 1891) joined him there, also a student greatly influenced by Felix Klein, and a man of mathematical background and ideals similar to those of Osgood. They were very close friends both personally and in scientific work until Bôcher’s death in 1918.

Osgood’s scientific articles are impressive as to their high quality. In 1897 he published a deep investigation into the subject of uniform convergence of sequences of real continuous functions, a topic then as always of considerable importance. He found it necessary to correct some erroneous results on the part of du Bois Reymond, and established the important theorem that a bounded sequence of continuous functions on a finite interval, convergent there to a continuous function, can be integrated term by term. Shortly thereafter, A. Schoenflies was commissioned by the Deutsche Mathematiker-Vereinigung to write a report on the subject of Point Set Theory. Schoenflies wrote to Osgood, a much younger and less illustrious man, that he did not consider Osgood's results correct. The letter replied in the spirit that he was surprised at Schoenflies' remarkable procedure, to judge a paper without reading it. When Schoenflies' report appears (1900), it devoted a number of pages to an exposition of Osgood's paper. Osgood's result, incidentally, as extended to non-continuous but measurable functions, became a model for Lebesgue in his new theory of integration (1907).

In 1898 Osgood published an important paper on the solutions of the differential equation $y' = f(x, y)$ satisfying the prescribed initial conditions $y(a) = b$. Until then it had been hypothesised that $f(x, y)$ should satisfy a Lipschitz condition in y: $|f(x, y_1) - f(x, y_2)| \leq M\,|y_1 - y_2|$, from which it follows that a unique solution exists. Osgood showed that if $f(x, y)$ is merely continuous there exists at least one solution, and indeed a maximal solution and a minimal solution, which bracket any other solution. He also gave a new sufficient condition for uniqueness.

In 1900 Osgood established, by methods due to H. Poincaré, the Riemann mapping theorem, namely that an arbitrary simply connected region of the plane with at least two boundary points, can be mapped uniformly and conformally onto the interior of a circle. This is a theorem of great importance, stated by Riemann and long conjectured to be true, but without a satisfactory proof. Some of the greatest European mathematicians (e.g., H. Poincaré, H. A. Schwarz) had previously attempted to find a proof but without success. This theorem remains as Osgood's outstanding single result.

Klein had invited Osgood to collaborate in the writing of the *Enzyklopädie*, and in 1901 appeared Osgood's article "Allgemeine Theorie der analytischen Funktionen a) einer und b) mehrerer komplexen Grössen." This was a deep, scholarly, historical report on the fundamental processes and results of mathematical analysis, giving not merely the facts but including numerous and detailed references to the mathematical literature. The writing of it gave Osgood an unparalleled familiarity with the literature of the field.

In 1901 and 1902 Osgood published on sufficient conditions in the Calculus of Variations, conditions which are still important and known by his name. He published in 1903 an example of a Jordan curve with the positive area, then a new phenomenon. In 1913 he published with E. H. Taylor a proof of the one-to-oneness and continuity on the boundary of the function mapping a Jordan region onto the interior of a circle; this fact had been conjectured from physical considerations by Osgood in his *Enzyklopädie* article, but without demonstration. The proof was by use of potential theory, and a simultaneous proof by functional-theoretic methods was given by C. Carathéodory.

In 1922 Osgood published a paper on the motion of the gyroscope, in which he showed that intrinsic equations for the motion introduce simplifications and made the entire theory more intelligible.

From time to time Osgood devoted himself to the study of several complex variables; this topic is included in his *Enzyklopädie* article. He published a number of papers, gave a colloquium to the American Mathematical Society (1914) on the subject, and presented the first systematic treatment in his *Funktionentheorie*. He handled there such topics as implicit function theorems, factorization, singular points of analytic transformations, algebraic functions and their integrals, uniformization in the small and in the large.

It will be noted that Osgood always did his research on problems that were both intrinsically important and classical in origin—"problems with a pedigree," as he used to say. He once quoted to me with approval a German professor's reply to a student who had presented to him an original question together with the solution, which was by no means trivial: "Ich bestreite Ihnen das Recht, ein beliebiges Problem zu stellen und aufzulösen."

Osgood loved to teach, at all levels. His exposition was not always thoroughly transparent, but was accurate, rigorous, and stimulating, invariably with emphasis on classical problems and results. This may have been due in some measure to his great familiarity with the literature through writing the *Enzyklopädie* article. He also told me on one occasion that his own preference as a field of research was real variables rather than complex, but that circumstances had constrained him to deal with the latter; this may also have been a reference to the *Enzyklopädie*.

Osgood's great work of exposition and pedagogy was his *Funktionentheorie*, first published in 1907 and of which four later editions were published. Its purpose was to present systematically and thoroughly the fundamental methods and results of analysis, with applications to the theory of functions of a real and of a complex variable. It was more systematic and more rigorous that the French traités d'analyse, also far more rigorous than, say, Forsyth's theory of functions. It was a moment to the care, orderliness, rigor, and didactic skill of its author. When G. Pólya visited Harvard for the first time, I asked him whom he wanted most to meet. He replied "Osgood, the man from whom I learned function theory"—even though he knew Osgood only from his book. Osgood generously gives Bôcher part of the credit for the *Funktionentheorie*, for the two men discussed with each other many of the topics contained in it. The book became an absolutely standard work wherever higher mathematics was studied.

Osgood had previously (1897) written a pamphlet on Infinite Series, in which he set forth much of the theory of series needed in the Calculus, and his text on the Calculus dates from 1907. This too was written in a careful exact style, that showed on every page that the author knew profoundly the material he was presenting and its background both historically and logically. It showed too that Osgood knew the higher developments of mathematics and how to prepare the student for them. The depth of Osgood's interest in the teaching of the calculus is indicated also by his choice of that topic for his address as retiring president of the American Mathematical Society in 1907.

Osgood wrote other texts for undergraduates, in 1921 an Analytic Geometry with W. C. Graustein, which again was scholarly and rigorous, and in 1921 a revision of his *Calculus*, now called *Introduction to the Calculus*. In 1925 he published his *Advanced Calculus*, a masterly treatment of a subject that he had long taught and that had long fascinated him. He published a text on Mechanics in 1937, the outgrowth of a course he had frequently given, and containing a number of novel problems from his own experience.

After Osgood's retirement from Harvard in 1933 he spent two years (1934-1936) teaching at the National University of Peking. Two books in English of his lectures there were prepared by his students and published there in 1936: *Func-*

tions of Real Variables and *Functions of a Complex Variable.* Both books borrowed largely from the *Funktionentheorie*.

Osgood did not direct the Ph.D. theses of many students; the theses he did direct were those of C. W. Mcg. Blake, L. D. Ames, E. H. Taylor, and (with C. L. Bouton) G. R. Clements. I asked him in 1917 to direct my own thesis, hopefully on some subject connected with the expansion of analytic functions, such as Borel's method of summation. He threw up his hands, "I know nothing about it."

Osgood's influence throughout the world was very great, through the soundness and depth of his *Funktionentheorie*, through the results of his own research, and through his stimulating yet painstaking teaching of both undergraduates and graduate students. He was intentionally raising the scientific level of mathematics in America and elsewhere, and had a great part in this process by his productive work, scholarly textbooks, and excellent classroom teaching.

Osgood's favorite recreations were touring in his motor car, and smoking cigars. For the latter, he smoked until little of the cigar was left, then inserted the small blade of a penknife in the stub so as to have a convenient way to continue.

Osgood was a kindly man, somewhat reserved and formal to outsiders, but warm and tender to those who knew him. He had three children by Mrs. Teresa Ruprecht Osgood: William Ruprecht, Freida Bertha (Mrs. Walter Sitz, now deceased), Rudolph Ruprecht. His years of retirement were happy ones. He married Mrs. Celeste Phelpes Morse in 1932, and died in 1943. He was buried in Forest Hills Cemetery, Boston.

SELECTED BIBLIOGRAPHY

1896

Some points in the elements of the theory of functions. *AMS Bull.* 2:296-302.

1897

Introduction to Infinite Series. Cambridge. (2nd ed. reprinted with corrections, 1902; 3rd ed. reprint, 1910; other reprints in 1920, 1928, and 1936. French trans. by A. Sallin: *Sèries Infinies. Exposé théorique et pratique illustré par de nombreux Exemples développés et des Exercices à resoudre empruntés la Géométrie, la Mécanique, la Physique et l'Astronomie,* Paris, 1935.

Non-uniform convergence and the integration of series term by term. *AJM* 19:155-90.

1898

Example of a single-valued function with a natural boundary, whose inverse is also single-valued. *AMS Bull.* 4:417-24.

Selected topics in the general theory of functions. Six lectures delivered before the Cambridge Colloquium, August 22-27, 1898. *AMS Bull.* 5:59-87.

Beweis der Existenz einer Lösung der Differentialgleichung $dy / dx = f(x, y)$ ohne Hinzunahme der Cauchy-Lipschitz'schen Bedingung. *MMP* 9:331-45.

1899

Note über analytische Functionen mehrerer Veränderlichen. *MA* 52:462-64; Zweite Note, *MA* 53(1900):461-64.

1900

Üeber einen Satz des Herrn Schönflies aus der Theorie der Functionen zweier reeller Veränderlichen. *GN* 94-97.

On the existence of the Green's function for the most general simply connected plane region. *AMS Trans.* 1:310-14, 2(1901):484-85.

1901

Sufficient conditions in the calculus of variations. *AM,* s. 2, 2:105-29 (Polish trans., *Wiadomoœci Matematyczne* 5:179-210).

On the existence of a minimum of the integral $\int_{x_0}^{x_1} F(x, y, y')dx$ when x_0 and x_1 are conjugate points, and the geodesics on an ellipsoid of revolution: A revision of a theorem of Kneser's. *AMS Trans.* 2:166-82, 486.

On a fundamental property of a minimum in the calculus of variations and the proof of a theorem of Weierstrass's. *AMS Trans.* 2:273-95; 3(1902):500.

Allgemeine Theorie der analytischen Funktionen: a) einer und b) mehrerer komplexen Grössen. *Encyk. d. Math. Wiss.* II-2:1-114. French trans.: Fonctions analytiques, exposé d'après l'article allemand de W. F. Osgood . . . par P. Boutroux . . . ét Jean Chazy . . . *Encycl. d. Sci. Mathém.* tome 2, v. 2, fasc. 1, 1911, pp. 94-96. Only these pages of the introduction were ever published.

1902

Problems in infinite series and definite integrals; with a statement of certain sufficient conditions which are fundamental in the theory of definite integrals. *AM,* s. 2, 3:129-46.

1903

A Jordan curve of positive area. *AMS Trans.* 4:107-12.

On the transformation of the boundary in the case of conformal mapping. *AMS Bull.* 9:233-35.

1907

Lehrbuch der Funktionentheorie, vol. 1. Leipzig. (Vol. 2, 1924).

1909

A First Course in the Differential and Integral Calculus. New York.

1913

With E. H. Taylor. Conformal transformations on the boundaries of their regions of definition. *AMS Trans.* 14:277-98.

1914

Topics in the theory of functions of several complex variables. *AMS Colloq. Pub.* 7:111-230.

1916

On functions of several complex variables. *AMS Trans.* 17:1-8.

1917

Factorization of analytic functions of several variables. *AM,* s. 2, 19:77-95.

1918

Singular points of analytic transformations. *AMS Trans.* 19:251-74.

1921

With W. C. Graustein. *Plane and Solid Analytic Geometry.* New York.

1922

On the gyroscope. *AMS Trans.* 23:240-64.