NATIONAL ACADEMY OF SCIENCES

JOSEPH FELS RITT

1893—1951

A Biographical Memoir by
PAUL A. SMITH

Biographical Memoir

J. F. Ritt.

JOSEPH FELS RITT

1893–1951

BY PAUL A. SMITH

JOSEPH FELS RITT died on January 5, 1951, at a period in his life when he was making full use of his superb creative powers in mathematics. The years just preceding Ritt's death were years of exceptional accomplishment. During this period Ritt laid the foundations for a difficult and wholly unexplored extension of group theory. Simultaneously he recast completely his most important and characteristic early work in a volume which will remain definitive for many years. Ritt did not live to see the reviews which proclaimed this work a masterpiece.

Joseph Ritt was born on August 23, 1893, in lower Manhattan, New York City. He retained a vivid memory of life in the teeming city at the turn of the century and in later years he would spend an occasional afternoon visiting the old landmarks which he remembered from boyhood. Little is known about Ritt's experiences at school except that his progress was rapid and effortless. Many after-school hours were devoted to part-time jobs and the earnings were turned into the family treasury. Indeed, Ritt's feeling of responsibility toward his parents was unusually strong. As he grew older, he faced the problem of increasing his contributions to his parents by means consistent with his talents. Young Ritt considered the Civil Service as a possible solution and resolved to look for an opening at the earliest possible moment. An opportunity finally presented itself and led to his appointment to the staff of the Naval

Observatory at Washington at an annual salary of twelve hundred dollars. A substantial portion of this income was sent home regularly. At the time of his departure from New York, Ritt had attended City College for two years and had twice been awarded that college's Belden Mathematical Prize. In Washington Ritt managed to attend classes at George Washington University, where he soon completed his undergraduate work. This same university honored Ritt in 1932 with the degree of Doctor of Science.

During his years at Washington, Ritt became increasingly conscious of his mathematical talents, and it was inevitable that he should be drawn to one of the great scientific centers where he could perfect his training and test his creative talent. By "saving" a number of vacations, he was eventually able to attend a summer session in the graduate school at Columbia University. Ritt's gifts were quickly recognized at Columbia, especially by Edward Kasner, who was one of Ritt's instructors and was to become a lifelong friend and colleague. It was largely through Kasner's advice and encouragement that Ritt resigned from his post in Washington to remain at Columbia as a University Fellow. He received the doctorate from Columbia in 1917 for a dissertation on differential equations of infinite order which at once established him as a mathematician of power and originality. Ritt always regarded this work with special pride and, in later years, guarded carefully the few reprints which remained in his possession. The bestowal of one of these reprints was a special token of friendship.

After a brief period of war work (1917-1918) Ritt became a member of the mathematics department at Columbia University, where he remained until his death. In 1928 he married Estelle Fine, who created for him an atmosphere perfectly suited to his temperament. Ritt and his wife shared the toils of labor and its rewards. They traveled extensively, and worked together in the preparation of manuscripts. Ritt relied on the expert clerical assistance of his wife to such an extent that he rarely entrusted this work to other hands.

Judged externally, Ritt's life followed an even course to the end.

Ritt regarded his academic career as the central fact of his life and derived profound contentment from its pursuit. Honors came early and easily. He held a succession of administrative and editorial posts in the American Mathematical Society. In his late thirties he was named Colloquium Lecturer by the Society, and in 1933 he was elected to the National Academy of Sciences. At Columbia, Ritt was executive officer of the mathematics department during 1942-45. In this position, he displayed a remarkable capacity for the efficient handling of administrative details. Indeed, he took a certain amount of pleasure in administrative work and seemed to find in it an almost therapeutic relief from the strain of the sustained effort which his researches demanded of him.

In the classroom, Ritt was a memorable figure. Students always came away from his lectures aware that they had been listening to a master craftsman. He never approached any teaching assignment in a casual manner. Even in the most elementary subjects, he planned his progress carefully and rarely resorted to improvisation. He always made it a point to examine the historical evolution of his subject, the false steps and the slow surmounting of difficulties. Believing in the importance of recapitulation, he never hesitated to retrace his path in order to go forward on the basis of new viewpoints or sharper distinctions. "We shall look twice at our material, first casually, just to see how matters stand, then fully and squarely, with no turning away from hard realities." (Integration in finite terms.) But in contrast to this habit of careful preparation, Ritt always managed to give the impression of spontaneity. He believed that every teacher should be something of an actor. His delivery was characterized by distinguished diction, a keen rhetorical sense, and an abundant wit. He obviously enjoyed holding the stage, but he saw to it that each student could have his say. To a remarkable degree, he was able, in the classroom, to know and instruct his students as individuals. A phenomenal memory enabled him, in later years, to recall the personal characteristics of several generations of mathematicians who had passed under his scrutiny.

In his younger days, Ritt acquired an accurate knowledge of geography from his frequent travels in Europe and Asia. Through his habit of reading constantly and widely, he became acquainted with the classics of world literature. He had an excellent working knowledge of several languages. Words seemed to fascinate him, and his interest in the origin of words became a minor hobby. He often asserted that to him each word suggested a definite color. A keen observer would indeed have seen that this sound-color association was very strong and remarkably self-consistent. In the sciences, Ritt seems to have confined his reading largely to mathematics, although he did have more than a passing interest in psychology, particularly in irregular behavior patterns and the psychology of mobs. He frequently occupied himself with analyzing the motives for specific acts. He judged men and events shrewdly and, although he had little respect for reputations which he believed were based on false values, he went to great lengths to put in a good word when he believed it was deserved. He showed a special kindliness in his relations with his students and younger colleagues.

We shall describe briefly the main features of Ritt's scientific work.* Broadly speaking, his contributions lay in the fields of foundation theory and analysis, and he derived his inspiration from the great masters of the nineteenth century. The problems which he attacked were of classical simplicity. This in itself indicates the order of the difficulties which Ritt was accustomed to face. For, the problems which are most intuitive, most easily stated, are often the most difficult to solve. An excellent example is the problem of determining all pairs of rational functions $f(z), g(z)$ such that $f(g(z)) = g(f(z))$. Partial results on this question had been obtained by a number of European mathematicians. By bringing a more powerful analysis to bear on the problem, Ritt was able to achieve a final solution. The paper in which this is done is notable for a style that is both poised and compelling. Ritt also considered the elementary

* For an excellent technical account, see Joseph Fels Ritt, by E. R. Lorch, Bull. Am. Math. Soc., 57: 307-318.

problem of determining those functions which cannot be integrated in elementary terms. It is known to every student of calculus that such functions exist, yet the theory which leads to their recognition is subtle and, if one relies only on classical literature, rather inaccessible. Ritt brought the theory into a state of perfection in his beautiful monograph Integration in Finite Terms. Ritt's methods are essentially those which Liouville published over a century ago in the *Journal de Mathématique,* which he founded. "Liouville's methods," says Ritt "are ingenious and beautiful. From the formal standpoint, they are entirely sound. There are, however, certain questions connected with the many-valued character of the elementary functions which could be pressed back behind the symbols in Liouville's time but which have since learned to assert their rights. . . . It might be great fun to talk just as if the elementary functions were one-valued. I might even sound convincing to some readers; I certainly could not fool the functions."

Ritt's most important achievement was the creation of an algebraic theory of ordinary and partial differential equations. The theory presents a deep analysis of the structure of systems (possibly infinite) of differential equations and of the manifolds of their solutions. It has been suggested that this work owes its origin to the fact that the concepts of "general solution" and "singular solution" had never been clearly formulated, although the question had been considered by such masters as Laplace and Lagrange. The correct formulations were discovered by Ritt and are part of his general theory. This is another instance of Ritt's preoccupation with fundamental classical problems. His work in this field culminated in his magnificent book *Differential Algebra;* this is the work referred to in the opening paragraph of this memoir.

In his final sequence of papers, Ritt introduced and developed the concept of differential group. This is a bold generalization of the classical concept of continuous group. The n independent variables which, in the classical theory, represent the parameters of the group are replaced by n indeterminates which may be thought of as arbitrary functions. Group composition is then given by a formal power series in 2 n indeterminates and their derivatives. Ritt made a complete analysis of differential groups for $n = 1$ and $n = 2$. There exist two types in the first case, thirteen in the second, whereas in the classical case the corresponding numbers of types are one and two respectively. Ritt regarded the determination of types in the case $n = 2$ as the most difficult piece of analysis he had ever attempted. The concepts which are involved here are in their very formulation dependent on the exacting art of manipulating formal power series and, for this reason, they lack the immediate appeal which they may ultimately possess. Thus it may be said that Ritt's last work is only a beginning: it will stand as a challenge until it is thoroughly understood and its significance evaluated.

KEY TO ABBREVIATIONS

Am. J. Math. = American Journal of Mathematics
Am. Math. Soc. Semi-centennial Pub. = American Mathematical Society Semi-centennial Publications
Ann. Math. = Annals of Mathematics
Astron. J. = Astronomical Journal
Bull. Am. Math. Soc. = Bulletin of the American Mathematical Society
C. R. Acad. Sci., Paris. = Comptes-rendus Académie des Sciences, Paris.
J. U. S. Artil. = Journal of United States Artillery.
Math. Ann. = Mathematische Annalen
Math. Zeit. = Mathematische Zeitschrift
Proc. Nat. Acad. Sci. = Proceedings of the National Academy of Sciences
Trans. Am. Math. Soc. = Transactions of the American Mathematical Society

BIBLIOGRAPHY

1913

A Control for Least Square Solutions. Astron. J., 28:73–74.

1914

Note sur la fonction sin $[(n+1)$ arc cos $x]$. Nouvelles Annales de Mathématiques, 14:184–186.

1916

On Certain Real Solutions of Babbage's Functional Equation. Ann. Math., 17:113–122.

On the Derivatives of a Function at a Point. Ann. Math., 18:18–23.

1917

On a General Class of Linear Homogeneous Differential Equations of Infinite Order with Constant Coefficients. Preceded by a note on The Resolution into Partial Fractions of the Reciprocal of an Entire Function of Genus Zero. Dissertation. Trans. Am. Math. Soc., 18:21–49.

1918

On the Differentiability of Asymptotic Series. Bull. Am. Math. Soc., 24:225–227.

Sur l'itération des fonctions rationelles. C. R. Acad. Sci. Paris, 166:380–381.

1919

On the Differentiability of the Solution of a Differential Equation with Respect to a Parameter. Ann. Math., 20:289–291.

1920

On the Iteration of Rational Functions. Trans. Am. Math. Soc., 21:313–320.

On Weighing Factor Curves in Flat Fire. J. U. S. Artil., 53:404–410.

1921

On the Conformal Mapping of a Region into a Part of Itself. Ann. Math., 22:157–160.

Note on Equal Continuity. Bull. Am. Math. Soc., 27:351–353.

1922

Periodic Functions with a Multiplication Theorem. Trans. Am. Math. Soc., 23:16–25.

Prime and Composite Polynomials. Trans. Am. Math. Soc., 23:51–66.

On Algebraic Functions Which Can Be Expressed in Terms of Radicals. Trans. Am. Math. Soc., 24:21–30.

1923

Sur les fonctions rationnelles permutables. C. R. Acad. Sci. Paris, 176:60–68.

On the Integrals of Elementary Functions. Trans. Am. Math. Soc., 25:211–222.

Permutable Rational Functions. Trans. Am. Math. Soc., 25:399–448.

1924

Equivalent Rational Substitutions. Trans. Am. Math. Soc., 26:221–229.

Analytic Functions and Periodicity. Bull. Am. Math. Soc., 30:406–409.

Note on the Dirichlet Series with Complex Exponents. Ann. Math., 26:144.

1925

Elementary Functions and Their Inverses. Trans. Am. Math. Soc., 27:68–90.

New Proofs of the Two Well-Known Theorems on Quadratic Forms. Ann. Math., 26:202–204.

1926

Sur les fonctions méromorphes qui admettent un therorème d'addition ou de multiplication. C. R. Acad. Sci. Paris, 182:201–202.

Simplification de la méthode de Liouville dans la théorie des fonctions elémentaires. C. R. Acad. Sci. Paris, 183:331–332.

Transcendental Transcendency of Certain Functions of Poincaré. Math. Ann., 95:671–682.

1927

On the Integration in Finite Terms of Linear Differential Equations of the Second Order. Bull. Am. Math. Soc., 33:51–57.

An Assemblage-Theoretic Proof of the Existence of Transcendentally Transcendental Functions. With Eli Gourin. Bull. Am. Math. Soc., 33:182–184.

Meromorphic Functions with Addition or Multiplication Theorems. Trans. Am. Math. Soc., 29:341–360.

Real Functions with Algebraic Addition Theorems. Trans. Am. Math. Soc., 29:361–368.

A Factorization Theory for Functions $\sum_{i=1}^{n} a_i e^{\alpha_i z}$. Trans. Am. Math. Soc., 29:584–596.

1928

On Certain Points in the Theory of Dirichlet Series. Am. J. Math., 50:73–86.

1929

Calculus, Differential and Integral. Encyclopedia Britannica 14th ed., 4:555–558.

Algebraic Combinations of Exponentials. Trans. Am. Math. Soc., 31:654–679.

On the Zeros of Exponential Polynomials. Trans. Am. Math. Soc., 31:680–686.

1930

On a Certain Ring of Functions of Two Variables. Trans. Am. Math. Soc., 32:155–170.

Manifolds of Functions Defined by Systems of Algebraic Differential Equations. Trans. Am. Math. Soc., 32:569–598.

Representation of Analytic Functions as Infinite Products. Math. Zeit. 32:1–3.

1931

Systems of Algebraic Differential Equations. Proc. Nat. Acad. Sci., 17:366–368.

1932

Differential Equations from the Algebraic Standpoint. Am. Math. Soc. Colloquium Publications, Vol. 14. 173 pp.

1933

Integral Functions Obtained by Compounding Polynomials. Bull. Am. Math. Soc., 39:627–632.

Systems of Algebraic Difference Equations. With J. L. Doob. Am. J. Math., 55:505–514.

1934

Algebraic Difference Equations. Bull. Am. Math. Soc., 40:303–308.

1935

System of Algebraic Differential Equations. Ann. Math. (2), 36:293–302.

Jacobi's Problem on the Order of a System of Differential Equations. Ann Math. (2), 36:303–312.

1936

Indeterminate Expressions Involving a Function and Its Derivatives. Monatshefte für Mathematik and Physik., 43:97–104.

On the Singular Solutions of Algebraic Differential Equations. Ann. Math., 37:552–617.

1938

On Certain Points in the Theory of Algebraic Differential Equations. Am. J. Math., 60:1–43.

Systems of Differential Equations, I: Theory of Ideals. Am. J. Math., 60:535–548.

Algebraic Aspects of the Theory of Differential Equations. Am. Math. Soc. Semi-centennial Pub., 2:35–55.

1939

On Ideals of Differential Polynomials. Proc. Nat. Acad. Sci., 25:90–91.

On the Intersections of Algebraic Differential Manifolds. Proc. Nat. Acad. Sci., 25:214–215.

Ideal Theory and Algebraic Difference Equations. With H. W. Raudenbush, Jr. Trans. Am. Math. Soc., 46:445–452.

On Certain Ideals of Differential Polynomials. With. E. R. Kolchin. Bull. Am. Math. Soc., 45:895–898.

1940

On the Intersections of Irreducible Components in the Manifold of a Differential Polynomial. Proc. Nat. Acad. Sci., 26:354–356.

On a Type of Algebraic Differential Manifold. Trans. Am. Math. Soc., 48:542–552.

1941

Complete Difference Ideals. Am. J. Math., 63:681–690.

1943

Bezout's Theorem and Algebraic Differential Equations. Trans. Am. Math. Soc., 53:74–82.

A Family of Functions and Its Theory of Contact. Bull. Am. Math. Soc., 49:109–113.

1945

On the Manifolds of Partial Differential Polynomials. Ann. Math., 46:102–112.

Analytical Theory of Singular Solutions of Partial Differential Equations of the First Order. Ann. Math. 46:120–143.

1946

On the Singular Solutions of Certain Differential Equations of the Second Order. Proc. Nat. Acad. Sci., 32:255–258.

1947

Theory of Functions. King's Crown Press. 181 pp.

1948

Integration in Finite Terms. Columbia University Press. 100 pp.

1949

Abel's Theorem and a Generalization of One-Parameter Groups. Trans. Am. Math. Soc., 67:491–497.

1950

Associative Differential Operations. Ann. Math., 51:756–765.

Differential Groups and Formal Lie Theory for an Infinite Number of Parameters. Ann. Math., 52:708–726.

Differential Algebra. Am. Math. Soc. Colloquium Publications, Vol. 33. 186 pp.

Differential Groups. Proceedings of the International Congress, Harvard, 1950.

1951

Differential Groups of Order Two. Ann. Math., 53:491–519.

Subgroups of Differential Groups. Ann. Math., 54:110–146.