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LLEWELLYN HILLETH THOMAS
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A Biographical Memoir by
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Biographical Memoir

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L. H. Thomas

LLEWELLYN HILLETH THOMAS

October 21, 1903–April 20, 1992

BY JOHN DAVID JACKSON

LLEWELLYN HILLETH THOMAS WAS BORN in London, England, on October 21, 1903. He died at age 88 in Raleigh, North Carolina, on April 20, 1992. An accomplished theoretical physicist and applied mathematician with wide-ranging interests and a practical bent, Thomas began his scientific career in the mid-1920s, just before the dawn of quantum mechanics. In a burst of creative energy in 1925-1926 while still a Ph.D. candidate, he made two major discoveries. The first was the famous “Thomas factor of $\frac{1}{2}$ ” that reconciled the magnitude of the atomic fine structure with the anomalous Zeeman effect, the $\frac{1}{2}$ being an unexpectedly large consequence of the “Thomas precession” of a spin axis, a general result of relativistic kinematics. His equations for the motion of spin in electromagnetic fields today provide the foundation for spin polarization studies in high-energy electron-positron colliders. The second discovery was the creation of a statistical model of the atom, now known as the Thomas-Fermi model, useful in describing average properties of heavy atoms and their response to external stimuli. All before he was 23.

In the 1930s while at Ohio State University, Thomas made a major contribution to accelerator physics with his invention of the isochronous, or radial-sector, cyclotron,

a precursor of the high-field isochronous cyclotrons currently used in proton beam therapy and nuclear physics. During the Second World War and subsequently, he used his applied mathematics skills on ballistics and shock-wave dynamics at the U.S. Army's Ballistics Research Laboratory and on magnetohydrodynamics in consultation at Los Alamos. A major shift in his career came in 1946 when he moved to the newly created Watson Scientific Computing Laboratory at Columbia University and soon had a joint appointment with the Columbia Physics Department. At the Watson Lab he had by all accounts a tremendous impact on both the development of numerical methods and on the design principles and hardware for electronic computers. He is credited with the early invention of a magnetic core memory and of electromagnetic delay lines for data storage. In 1968 he retired from the Watson Lab and Columbia to move to North Carolina State University, where he was University Professor until 1976. He was elected to the National Academy of Sciences in 1958, shortly after becoming a U.S. citizen.

EARLY YEARS

Thomas, the oldest of five children, was born in London, of Welsh stock; his father, Charles James Thomas, was a medical officer for health for London. Among family and friends Thomas was known as Hilleth and later in life on occasion signed his name as L. Hilleth Thomas. He was home schooled until the age of seven, being taught reading, writing, arithmetic, elementary geometry, and history by his mother, Winifred May (Lewis) Thomas. He then attended a private elementary school until age 11. A voracious reader, he recalls reading a popular account of Bohr's atom at that time.¹ From age 11 to 18 he attended Merchant Taylor's School in London on a scholarship. The curriculum reflected British private schools of the time (1914-1921): Latin and Greek

with an hour of mathematics every day until he switched to the science curriculum after three and a half years. Then he studied physics, chemistry, and mathematics, including J. J. Thomson's *Elements of Electricity and Magnetism*, Jeans's *Mechanics*, and Osgood's *Calculus*. He considered himself fortunate to have been taught how to write "a short English essay on almost any subject under the sun in a time of about an hour."¹ But in French and German he was taught just to read.

Thomas matriculated from Merchant Taylor with London University's examination in 1919 and its intermediate examination in 1921. He obtained a scholarship to Trinity College, Cambridge, by examination and began his studies there in 1921, focusing on pure and applied mathematics. He attended lectures by Charles G. Darwin, John E. Littlewood, Geoffrey I. Taylor, Ralph H. Fowler, Arthur S. Eddington, E. Arthur Milne, and Joseph Larmor, among others.² He seemed to have moved effortlessly through Trinity and the university, receiving a senior college scholarship in 1923 and an Isaac Newton Studentship based on his performance in the Mathematical Tripos in 1924, graduating with a first-class honors B.A. in mathematics with distinction in applied mathematics. He then began research with Fowler just as Fowler went away for a year to Niels Bohr's institute in Copenhagen.

CAMBRIDGE AND COPENHAGEN

The dawn of quantum mechanics had not yet broken. In his first graduate year Thomas began a theoretical study of the passage of charged particles through matter, using work with the old quantum theory by Hartree on Bohr orbits of electrons in heavy atoms. During the year he attended only a few lectures, notably Ernest W. Hobson's valuable course on integral equations. He also completed a paper on adiabatic

invariants for an ensemble and won a Smith's prize for an essay based on that research.

At age 22 he went to Bohr's institute on a traveling fellowship for the year 1925-1926. In Copenhagen during that year, the air was electric with Heisenberg, Schrödinger, Dirac, and others establishing the foundations of quantum mechanics. By his own admission Thomas was "always slow in accepting new ideas and understood nothing of this for four or five years."^{2,3}

Nevertheless, pursuing his own course, he made important discoveries justifiably associated with his name. In continuing his work on charged particles passing through matter, he sought a simpler way to describe the averaged screened electric potential of a heavy atom of atomic number Z . He created a statistical model of the atom, now known as the Thomas-Fermi-Dirac model, that gave an electronic charge distribution and screened potential that scaled in size as $Z^{-1/3}$. The model was discovered independently a year later by Fermi, who focused on what it could say about the electronic shell structure of atoms, and later improved by Dirac and others. It continues to have application in diverse fields.^{4,5}

Thomas's other major accomplishment while in Copenhagen concerned atomic spectra. In 1925 Pauli introduced the concept of a two-valued additional degree of freedom for the electron. Uhlenbeck and Goudsmit identified Pauli's degree of freedom as an intrinsic spin of the electron and analyzed both the anomalous Zeeman effect and the atom's fine structure. They sent a copy of their paper to Bohr just before Christmas 1925. Bohr and Hendrik A. Kramers doubted the Uhlenbeck-Goudsmit hypothesis because the intrinsic magnetic moment that was necessary to understand the anomalous Zeeman effect made the doublet splitting of the fine structure a factor of two larger than experiment. Thomas suggested that the discrepancy might be caused

by the neglect of relativity. In his recollections about his discovery that resolved the dilemma, Thomas recalls:

Kramers who had known of the earlier work on the motion of the moon by De Sitter said to me, "It would be a very small relativistic correction. You can work it out, I won't." Over that weekend I looked at it. I had the advantage of having attended Eddington's lectures on relativity theory and I knew how to work the mathematics. I found that if you look at the change in the direction of the axis of the rotating electron, there should be a very considerable relativistic effect, in fact, a factor of two. I brought this idea back with a formula to Kramers and Bohr just after that one Christmas weekend. Bohr insisted that a letter be written to *Nature* [appearing in April 1926], which had this result in it."⁶

The famous factor of two in the fine structure of atoms, often described as the Thomas factor of $\frac{1}{2}$, is a result of the Thomas precession of the electron's spin, but one consequence of the non-commutativity of non-parallel Lorentz transformations.⁷ Later in 1926 Thomas elaborated on the motion of a spinning electron in general electromagnetic fields in a masterful paper published in the *Philosophical Magazine* in January 1927. Apart from its application to atomic physics, the paper lay largely forgotten for more than 30 years.⁸ Thomas's equations have seen their most frequent application in the spin polarization effects in high-energy electron-positron storage rings and in (g-2) experiments on the muon.

Thomas's time in Copenhagen was arguably the most productive of his career. His statistical model of the atom and his work on the relativistic precession and its effect on spin are the discoveries most strongly associated with his name. If we include the work of his first graduate year and work on the passage of charged particles through matter begun then but completed in Copenhagen, he had seven research papers to his credit by the time he returned to Cambridge in the fall of 1926 at age 23. He was elected a fellow of Trinity

College in 1926 before receiving his Ph.D. degree in 1927 and the customary M.A. a year later.⁹ In 1965 Cambridge awarded him an honorary doctorate of science.

OHIO STATE UNIVERSITY AND ABERDEEN PROVING GROUND

In 1929, with R. H. Fowler's recommendation, Ohio State University offered Thomas an assistant professorship of physics, which he accepted. He remained there, apart from wartime service, for 17 years, advancing through the ranks to full professor in 1936. The archival holdings in the library at North Carolina State University show that at Ohio State he taught a wide range of advanced courses in physics, astrophysics, and astronomy. His research ranged widely over atomic and nuclear physics, astrophysical applications of fluid dynamics, quantum theory of solids and complex molecules, design of cyclotrons, self-energies in quantum field theory, and mobilities of electrons and ions in gases, as well as applied mathematical topics such as numerical methods. The M.S. and Ph.D. thesis topics of his students ranged similarly.¹⁰ Special mention should be made of his pioneering work (1930) on radiation hydrodynamics, especially radiative viscosity, relevant for stellar interiors and nuclear explosions. His paper on the three-nucleon problem and the structure of ^3H is notable in that it is still cited today in a number of different fields. Some of Thomas's 24 papers published between 1929 and 1942 were collaborations with students or colleagues such as Willard H. Bennett, Alfred Landé, and Harald H. Nielsen. Notable among his graduate students was Leonard I. Schiff, who received his M.S. degree in 1934.

Perhaps his most noteworthy research while at Ohio State was the work on the isochronous, or variable-sector, cyclotron in which the customary azimuthally symmetric magnetic field $B_z(r)$ is replaced by $B_z(r, \varphi)$ with dependence in the azimuthal angle φ periodic with period $\pi/2$. He showed

that with the main magnetic field increasing with radius and the relative magnitude of the azimuthal variation of the field of order v/c , the orbits are stable and isochronous up to order $(v/c)^3$, thereby circumventing (to that order) the effect of the relativistic increase in momentum beyond the nonrelativistic $p = mv$. This discovery was an important breakthrough; it stimulated subsequent developments that led today to numerous isochronous radial-sector and spiral-sector cyclotrons for nuclear physics and medical purposes with kinetic energies of the order of 50-500 MeV.

From 1943 to 1945 Thomas worked on ordnance science (ballistics and explosions) at the Ballistics Research Laboratory of the Aberdeen Proving Ground, Aberdeen, Maryland. Colleagues included Gregory Breit, Subrahmanyan Chandrasekhar, Edwin Hubble, and Joseph Mayer. John L. von Neumann and Isador I. Rabi served on the laboratory's advisory board. While there Thomas wrote a number of internal reports and one published paper on shock fronts. After the war he served as a member of the Ballistics Research Laboratory's Scientific Advisory Committee. He returned to Ohio State University only for the 1945-1946 academic year.

WATSON SCIENTIFIC COMPUTING LABORATORY
AND COLUMBIA UNIVERSITY

The Watson Scientific Computing Laboratory at Columbia University was founded in 1945 with funding from International Business Machines Corporation and Wallace J. Eckert, professor of astronomy, as director. Upon the recommendation of Rabi and von Neumann in 1946 Rabi and Eckert lured Thomas from Ohio State as the third scientist at the Watson Lab, with the job title of "technician" since none of the other IBM titles fit.¹¹ Shortly after his arrival, Rabi arranged a faculty appointment for Thomas, although he was paid by IBM. He abandoned his slide rule and mental

arithmetic to work on the use of large computing machines for scientific research. In 1946 Eckert, Grosch, and Thomas taught the first for-credit courses in computing; Thomas lectured on numerical solutions of differential equations. Grosch remembers Thomas as a brilliant but unworldly scientist whose dress and mannerisms were orthogonal to the buttoned-down IBM orthodoxy.¹¹ Peter Price, an IBM colleague, described him as reserved but when engaged, of firmly held opinions and a warm sense of humor.¹² He had a huge influence in physics, mathematics, and machine design at the Watson Lab. So much so that Director Eckert was prompted to write in a report on the laboratory's activities, "Perhaps there should be a box on the organizational chart labeled L. H. Thomas."¹³

Thomas conceived forms of the magnetic core memory in 1946, two years before others, and made important contributions to design principles and hardware, including an electromagnetic delay line for data storage.¹³ His personal research included relativistic particle dynamics, hydrodynamic flow, and computation of statistical charge densities and X-ray scattering factors for the whole periodic table and several degrees of ionization. In the area of computers he published both internal reports and papers in the open literature on numerical methods, software, and hardware. He taught regularly in the Physics Department, offering courses in general relativity, group theory and quantum mechanics, relativistic quantum theory, and magnetohydrodynamics. His vast knowledge of mathematical physics made him known in the Physics Department as "the sage of 116th Street" (the first home of the Watson Lab). He also supervised a number of graduate students to Ph.D. theses.¹⁴

In 1954 he achieved some notoriety by proposing ingenious and inexpensive countermeasures against large potentially hostile satellites in a talk to the American Rocket Society at

a time when Wernher von Braun had a pet project for such a large but friendly satellite.¹⁵ According to his family this brought some unwelcome McCarthy-era inquiries about just what this alien scientist was doing at the Watson Lab. In any event he felt that his non-U.S. citizenship was an impediment to doing classified work for the government; he became a naturalized citizen in 1957. During his Watson Lab-Columbia years Thomas consulted for the Ballistics Research Laboratory, the Naval Ordnance Laboratory, and the Sherwood Project of the Atomic Energy Commission. He retired from the Watson Lab and Columbia in 1968.

NORTH CAROLINA STATE UNIVERSITY

Upon his retirement from Columbia, Thomas took up a Visiting University Professorship of Physics and Mathematics at North Carolina State University, where he remained active in teaching and research. He continued his role as encyclopedic scholar, directing inquiring graduate students to references in the library on almost any subject. He supervised one M.A. student.¹⁶ He retired a second time in 1976. He is remembered in the Physics Department as a profound stimulus to its intellectual life, a man of great intellect and erudition, and a quick wit. He and his wife added greatly to the department's social life. Since 1980 the Physics Department has honored his memory with an annual L. H. Thomas Lecture, presented by a succession of eminent scientists. He died in Raleigh, North Carolina, on April 20, 1992, at the age of 88.

HONORS, MEMBERSHIPS, FAMILY

While a student and postgraduate researcher at Cambridge, Llewellyn Hilleth Thomas held numerous awards, including the Isaac Newton Studentship, the Rouse Ball Travelling Fellowship at Copenhagen, an 1851 Exhibition Scholarship,

and a Fellowship of Trinity College. He was elected to the National Academy of Sciences in 1958. He was made an IBM Fellow in 1963. He was awarded an honorary doctor of science degree by Cambridge University in 1965. In 1982 he was awarded the Davisson-Germer Prize by the American Physical Society, "for his early pioneering contributions to the theory of the spin-orbit interaction in atoms and to the statistical model of atoms."

He was a fellow of the American Physical Society and American Association for the Advancement of Science and a member of Sigma Xi, British Association for the Advancement of Science, Cambridge Philosophical Society, and Royal Astronomical Society.

The family has a poignant story of what led to Thomas's marriage. Naomi Estelle Frech was a graduate student in physics at Ohio State in the fall of 1929 when Thomas arrived as a young assistant professor. Some time after that Ms. Frech suffered a terrible accident in the laboratory that injured one eye with flying glass. Thomas then visited and read poetry to her in the hospital as she recovered. Ms. Frech received her M.A. in spectroscopy in 1932. She and Thomas were married in 1933. In the next nine years they had four children: Charles Frederick, who died at age eight; James Rhys Thomas, now (2009) living in Palm Coast, Florida; Ann Rhonwen May (Thomas) Viele, now in Westwood, New Jersey; and Margaret Olwen (Thomas) DeAngelis, now residing in Mendham, New Jersey. There are 13 grandchildren.

During Thomas's Watson Lab-Columbia years, Mrs. Thomas taught physics and mathematics at the Packer Collegiate Institute in Brooklyn, New York. After her husband's death, Mrs. Thomas donated his voluminous papers to the library of North Carolina State University.¹⁷ She outlived her husband by almost 16 years.

I am grateful to the faculties and staffs of the physics departments as well as the libraries (through their online library catalogs) at Ohio State, Columbia, and North Carolina State for providing valuable information on Thomas and his students. I thank Dimitri Mihalas for stressing the historical importance of Thomas's work on radiation hydrodynamics. Special thanks go to the Thomas's children for providing me with family facts, and to Professors Jasper Memory, Michael Paesler, and Richard Patty of North Carolina State for providing the frontispiece photograph, impressions of the Thomases, and information crucial to finding the family.

NOTES

1. Biography of Llewellyn Hilleth Thomas, unpublished personal biographical sketch, undated, National Academy of Sciences files.
2. Reminiscences LHT (of his student years), unpublished, undated, National Academy of Sciences files.
3. Evidence of a continuing reluctance to accept new ideas may be found in Charles H. Townes's memoir *How the Laser Happened, Adventures of a Scientist*, New York: Oxford University Press, 1999. On p. 69 Townes recounts a conversation with Thomas at Columbia in which Thomas told him that "due to basic physics principles," the yet-to-be-demonstrated maser would not work. Here Thomas was in good company. Townes reports that Niels Bohr said, "But that is not possible" when learning of the performance of the successful maser.
4. N. H. March. *Electron Density Theory of Atoms and Molecules*. London: Academic Press, 1992.
5. E. H. Lieb. *The Stability of Matter: From Atoms to Stars: Selecta of Elliott H. Lieb*, ed. W. Thirring. New York: Springer, 2001.
6. L. H. Thomas. Recollections of the discovery of the Thomas precessional frequency. In *High Energy Spin Physics—1982*, ed. G. M. Bunce. AIP Conference Proceedings No. 95, Particles and Fields Subseries No. 28, pp. 4-5. New York: American Institute of Physics, 1983.

7. The Thomas factor of $\frac{1}{2}$ is the result of a cancellation between two terms in the fine structure interaction. One term is the naive expression, proportional to $(g/2)$, where g is a measure of a particle's magnetic moment (for an electron, $g = 2$ to high accuracy, consistent with the anomalous Zeeman effect). The second term of opposite sign is the result of the relativistic Thomas precession of the electron's spin as the electron moves in its curved path under the influence of the nuclear Coulomb force. It gives a contribution of $(-1/2)$, leading to a total coefficient of $(g-1)/2$, whereas Uhlenbeck and Goudsmit had $(g/2)$.
8. Thomas's equations were rediscovered in 1959 by V. Bargmann, L. Michel, and V. L. Telegdi, *Phys. Rev. Lett.* 2(1959) 435-4366. These authors were unaware of Thomas's 1927 paper and made no reference to it. For many years the equations for spin motion in electromagnetic fields were known as the BMT equations.
9. A Cambridge M.A. is awarded upon application to holders of a B.A. after at least six years from first being in residence and two years after the B.A. It confers membership in the University Senate.
10. Among his M.S. and Ph.D. students at Ohio State were Herman M. Roth, "Two Problems Concerning the Internal Structure of the Stars" (Ph.D., 1932); Leonard I. Schiff, "The Variation of the Vector Potential of a Beam of Light with Depth below a Metallic Surface According to Quantum Theory" (M.S., 1934); Walter S. McAfee, "The Normal State of the Neutral Helium Atom" (M.S., 1937); Edward C. Campbell, "A Numerical Method for Obtaining the Lowest Characteristic Value of the Problem of Three Bodies in Quantum Mechanics for an Attractive Force of Small Range" (Ph.D., 1938); Clyde W. Harris, "The Fifth Order Aberrations of the Symmetrical Optical System" (M.S., 1939); Edward H. Gamble, "Numerical Solution of Torsional Stress Distribution of a Circular Shaft with Circumferential Cut" (M.S., 1942); Arthur B. Clymer, "Mechanical Integrators" (M.S., 1946); Lloyd Smith, "Identification of the Positive Particles Emitted by Negative Beta Ray Sources" (Ph.D., 1946); R. R. Newton, "Internal Molecular

- Motion of Large Amplitude Illustrated by the Symmetrical Vibration of Ammonia” (Ph.D., 1947).
11. Herbert R. J. Grosch, an astronomer/computational scientist, was the second employee. Numerous glimpses of Thomas can be found in Grosch’s memoir of life at the Watson Lab, *Computer: Bit Slices of a Life*, (Underwod-Miller, Novato, Calif., 1991; 3rd ed., 2003. Available on line at <http://www.columbia.edu/acis/history/computer.html>.
 12. P. J. Price. Llewellyn H. Thomas (obituary). *Phys. Today* 47(9):115-116.
 13. Jean Ford Brennan. *The IBM Watson Laboratory at Columbia. A History*. (Copyright 1971 International Business Machines Corporation, N.Y.), p.40-41. Available online at <http://www.columbia.edu/acis/history/brennan/index.html>, p.26-27.
 14. Ph.D. students supervised by Thomas, as found in the Columbia University Library catalog: C. C. Grosjean, “The Exact Mathematical Theory of Multiple Scattering of Particles in an Infinite Medium” (1951); B. Bakamjian, “Relativistic Particle Dynamics “ (1953); B. Zondek, “Stability of a Limiting Case of Plane Couette Flow” (1954); D. H. Tycko, “Numerical Calculation of the Wave Functions and Energies of the 1^1S and 2^3S States of Helium” (1957); K. M. King, “Analysis of the Statistical Density Matrix” (1961); A. Kotchoubey, “Numerical Calculation of the Energy and Wave Function of the Ground State of Beryllium “ (1966).
 15. Science: Satellite countermeasures. *Time*, May 3, 1954, available at <http://www.time.com/time/magazine>.
 16. John Teague, “The Two Center Problem in a Space of Constant Curvature” (M.A., 1972).
 17. Llewellyn Hilleth Thomas Papers, 1921-1989. MC210, Special Collections Research Center, North Carolina State University Libraries, Raleigh, N.C. An extensive guide to the collection is available online at <http://www.lib.ncsu.edu/findingaids/mc00210>.

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