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A Biographical Memoir by

FRANK RAYMOND

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BY FRANK RAYMOND

RAYMOND LOUIS WILDER, loved by his family, friends, students, and colleagues, was a pioneer in the emerging discipline of topology and attained international acclaim for his creation and development of generalized manifolds. He also had a lifelong scholarly interest in anthropology and the foundations of mathematics. This erudition resulted in many articles and two important books on the cultural origins and development of mathematics.

He was born in 1896 in Palmer, Massachusetts, and as a youth attended schools in that town. His family was musical, and he played the cornet in the family orchestra at dances and fairs. His flair at the piano resulted in employment at the local movie house to accompany the silent films. Love for music making never left him, although later he usually stuck with the classics.

Wilder entered Brown University in 1914. This was interrupted by World War I, and he served two years in the Navy as an ensign. He returned to Brown and completed his bachelor's degree in 1920 and his master's degree in actuarial science in 1921. In the meantime, he had married the charming Una Maude Greene. They had three daughters: Mrs. Mary Jane Jessop of Long Beach, California, Mrs. Kermit Watkins of Altadena, California, Dr. Beth Dillingham of

Cincinnati, Ohio, and a son Dr. David E. Wilder of Pound Ridge, New York. At the time of Wilder's death in Santa Barbara in 1982 there were in addition 23 grandchildren and 14 great-grandchildren. His wife, Una, survived him for an additional 19 years, dying at the age of 100 in Long Beach.

The University of Texas at Austin was well known for its actuarial program, and Wilder decided to pursue further actuarial studies there. As he had enjoyed "pure math" as an undergraduate, he asked permission to participate in R. L. Moore's analysis-situs (the old name for topology) course. "No, there is no way a person interested in actuarial mathematics could do, let alone be really interested in, topology," said Moore. Wilder persisted and after Moore's extensive questioning and Moore's surprise to Wilder's answer to "what is an axiom?" he relented. He granted him admission but proceeded to ignore him. Moore's famous method of teaching was to begin with a few axioms and definitions. He would then state theorems, and it was up to the participants to find the proofs. Some of the propositions were quite difficult, and after Wilder had solved one of the more difficult ones, Moore began to take notice. He was also in the habit of posing unsolved research problems under the guise of homework. When Wilder found an elegant solution to one of the problems that had eluded J. R. Kline and Moore himself, Moore invited him to write this up as a Ph.D. thesis. So Wilder abandoned an actuarial career and became Moore's first doctoral student at Texas. Wilder and Moore maintained very cordial relations throughout the years.

He stayed an additional year at Austin as an instructor and in 1924 he moved with his family to Ohio State University to assume an assistant professorship. In the R. L. Moore archives at the University of Texas there is an exchange of letters discussing research and teaching along with Wilder's reluctance to sign a required loyalty oath at Ohio State University. Wilder's hostility to mindless patriotism and his predilection for liberal thought accompanied him throughout his life.

In 1926 he moved to the University of Michigan. Thus began a relationship that lasted 41 years, until his retirement from teaching in 1967. Wilder's first works in set-theoretic topology were well underway and had begun to gather international attention when a paper of J. W. Alexander was studied at the University of Michigan. This very important paper, in which Alexander proved his now famous duality theorem, was instrumental in turning Wilder's interest toward manifold theory and the use of algebraic techniques. For us, equipped with the systematic machinery of algebraic topology, Alexander's theorem does not seem so overwhelming today. However, we must remember at that time cohomology, relative homology, products, exact sequences, homotopy theory, etc., were still years in the future. Alexander had also produced his famous horned sphere, which meant the end of any hope for routine generalizations of plane topology to n-space. Yet the closed complementary domains of Alexander's sphere, while not both 3-cells, could not be distinguished from 3-cells by any homological means. It was this insight that led to Wilder's converse to the Jordan-Brower separation theorem in 3-dimensions.

In 2-dimensions the Jordan-Brower separation theorem says that a topological circle separates the plane into two uniformly locally connected complementary connected pieces of which the circle is the common boundary. The converse states that a connected closed and bounded subset of the plane that separates the plane into two uniformly locally connected pieces with the closed subset as the common boundary must be a topological circle. In 1930 Wilder introduced, in terms of homology, the analogue of uniform local connectedness in higher dimensions and determined when a closed subset of a 3-manifold was an embedded 2-manifold. Thus, the 2-manifold is determined by properties of its complement.

In 1933 the Institute for Advanced Study was founded in Princeton. Roaming the corridors of old Fine Hall were many topologists including Oswald Veblen, J. W. Alexander, Solomon Lefschetz, E. R. Van Kampen, A. W. Tucker, Leo Zippin, and Ray Wilder. Alexander, Eduard ¨ ech, L. Vietoris, P. S. Alexandroff, and Lefschetz had invented or were inventing various homology theories that could handle general spaces and their subsets. The notion of a generalized manifold was not unknown in the polyhedral category (Van Kampen, in 1929), and the formulation for topological spaces in more abstract homological terms was done in the early 1930s. The first proofs of dualities for generalized manifolds were by ¨ ech and Lefschetz.

In Wilder's famous paper in the *Annals of Mathematics* (1934), Wilder characterizes which closed subsets of a generalized n-manifold are (n-1)-generalized manifolds in terms of properties of their complements. This includes a generalization of the converse of the Jordan-Brower separation theorem in higher dimensions. That the setting in terms of generalized manifolds was appropriate can be discerned today because the analogous results for classical locally Euclidean manifolds are still not known in their full generality.

It is instructive to turn to Wilder's Symposium Lecture delivered to the American Mathematical Society (point sets in three and higher dimensions and their investigation by means of a unified analysis situs) in Chicago in 1932. Wilder had been concerned over the separation that had developed between the two schools of American topologists typified by the Texas (set theoretic or local) and Princeton (combinatoric or global) schools. Wilder was a successful "rebel" from the Texas school. It annoyed him to see criticism raised against unified methods. Actually, he was criticizing the dogmatism of both schools. By combining the methods of both schools he had been able to obtain generalizations of theorems of the plane whose extensions to n-space by means of set-theoretic methods alone had heretofore been unsuccessful. He was not alone of course in these successes, for Alexandroff, ¨ech, and Lefschetz, to name a few, had no qualms about combining methods. However, Wilder was brave to expose so much of his point of view to his contemporaries and to future generations, as he did in this Symposium Lecture.

Much of Wilder's work in topology can be said to center on placement problems and associated positional topological invariants. These essentially mean properties of a space M, in a space S, which are independent of M's embedding in S. For example, the uniform local connectedness of complementary domains of the (n-1)-sphere in the n-sphere is preserved by different embedding of the (n-1)-sphere in the n-sphere. These positional invariants manifested themselves in the plane as thoroughly investigated set-theoretic concepts. However, to obtain generalizations to higher dimensions required the introduction of homological (and later homotopical) concepts and techniques.

In 1942 Wilder delivered the American Mathematical Society Colloquium Lectures. World War II intervened and consequently it was not until 1949 that "Topology of Manifolds" was published by the society as volume 32 of its colloquium series. In the first portions of this 400-page book Wilder presents much of the topology of the plane that will be generalized to higher dimensions in the later portions of the book. ¨ech homology and cohomology theory and the lc^n and $colc^n$ properties are developed. At that time of writing and research, exact sequences and other functorial notions, which were to alter the point of view of topology, had not been introduced. Consequently, the reader will not find exact sequences, diagrams, and sheaves explicitly mentioned in the text. Wilder by that time had settled upon a modification (deletion of a superfluous lc^n axiom) of E. Begle's definition of generalized manifold as well as Begle's proof of the duality theorems. This definition is essentially equivalent to the one most popular today. The book contained much of Wilder's previously unpublished research and many generalizations of his previous research. It was a summing up of all that was known about generalized manifolds at the time.

Generalized manifolds are really the class of spaces for which Poincaré duality holds for every open subset. About 10 years later another explosion in the interest of generalized manifolds occurred. Powerful new machinery had been introduced into algebraic topology resulting in new proofs of the duality theorems. In 1957 C. T. Yang showed that Smith manifolds were really generalized manifolds, and so generalized manifolds began to play a very important role in topological transformation groups. In fact, they became the natural setting for the subject.

In 1957 Wilder in his monotone-mapping theorem gave sufficient homological conditions for a map of a manifold (or generalized manifold) to have a generalized manifold as an image. S. Smale, who was a student participant in Wilder's seminar at that time, was inspired to find the analogous homotopical setting. The results play an important role in modern research in generalized manifolds.

A generalized manifold cannot be distinguished from a classical topological manifold by purely homological means even though it may be far from a topological manifold. Today, finding the properties that will force a generalized manifold to be locally Euclidean is a major concern of geometric topologists. To avoid some of the pathology not detected by homology, one must assume that the generalized manifold is locally contractible. This enables homotopy theory and surgery methods to become active tools in this search. It is now known that these restricted generalized manifolds explain some of the mysteries behind topological surgery theory. Several homotopy theoretic ideas and the underlying basic technology for working with these more restrictive generalized manifolds goes back to works of Wilder and Eilenberg-Wilder in the 1930s and 1940s. Wilder's work and virtual creation of the theory of generalized manifolds has played a significant role in geometry and topology. Undoubtedly its influence will last long into the future.

Equal to Wilder's commitment to research in topology was his interest in teaching and in the foundations of mathematics. I shall quote from an article by Lucile Whyburn,[1] who wrote about Wilder's teaching.

> Thinking back to his student days he created his famous course, "Foundations of Mathematics." It is interesting that he began this course in the early thirties with a class of approximately thirty students whose central interest was actuarial mathematics. Much later he was to write a textbook for such a course and in the preface he says, "The reason for instigating such a course was simply the conviction that it was not good to have teachers, actuaries, statisticians, and others who had specialized in undergraduate mathematics, and who were to base their life's work on mathematics, leave the university without some knowledge of modern mathematics and its foundations."

The foundations course continued until his retirement at the University of Michigan in 1967. In the classroom Wilder went beyond topology and foundation of mathematics. He saw mathematics not only as a beautiful technical edifice but also as a product of the cultures that had created it. He felt that a knowledge of mathematics and its methods should be a part of the intellectual and cultural background of all well-trained people; whether they be teachers, businessmen, legislators, public servants, or housewives. Over the years I have met people from different walks in life who related to me that they had taken Wilder's Foundation of Mathematics course at the university. They remember it as one of the important things they did for themselves at the university.

Wilder had lifelong friendships with the leading philosophers and anthropologists at the university. Close to 40 of his publications concern themselves with mathematics' role in society and world cultures. He viewed mathematics as having a cultural basis and believed that recognition of this would clear the air of most of the mystical and philosophical arguments offered in support or defense of theories of the foundations of mathematics.

Wilder held that mathematics develops from two kinds of cultural stress. Mathematics arising from environmental cultural stress is a response to a perceived need to facilitate certain societal interactions, whereas inherited cultural stress is the response to internal mathematical problems. For example, development in an old culture of a symbolic nomenclature for recording numbers as in the number of bushels of wheat a farmer owes in taxes is an environmental stress. A response to one of Zeno's paradoxes would be an internal cultural stress. His book *Evolution of Mathematical Concepts* (1968) convincingly lays out his thesis in terms accessible to a layperson.

Wilder took a very active role in the development of research at the University of Michigan. In 1927 Wilder and G. Y. Rainich founded a somewhat secret research club called "The Small C." They felt that the Department Club, which met monthly, was not accomplishing very much in the development of interests in research. The Small C met every

Tuesday evening to present a scientific paper by a member of the club. Usually it was on a member's own research, but sometimes it was a report on a new mathematical result of great importance. At the beginning there were eight members from the mathematics department, one from philosophy, and three from physics. Later, others active in research, including some research students, were invited to join. In 1947 the Small C was disbanded, because research now was expected of all faculty members. In 1981 Ray and Una Wilder endowed the G. Y. Rainich lecture series of the University of Michigan Mathematics Department to honor the memory of their friend who was an important figure in the development of mathematics at the university.

Wilder was very good at discovering and encouraging talent. He interested Norman E. Steenrod in mathematics in the 1930s. Steenrod did his first research under Wilder's direction. When Steenrod finished his undergraduate training he returned to Ohio and worked for a year and a half while Wilder arranged for him to study at Harvard and later with Lefschetz at Princeton. He managed to find a place at Michigan for the young Polish topologist Samuel Eilenberg in the late 1930s despite opposition from some quarters. The famous collaboration of Eilenberg and Steenrod began when Wilder was able to get Steenrod a position at Michigan.

Wilder directed 25 doctoral dissertations including those of Leon Cohen, Paul Swingle, Sam Kaplan, Ed Begle, Morton Curtis, Alice Dickinson, Joe Schoenfield, Tom Brahana, J. P. Roth, Kyung W. Kwun, and myself. His advanced graduate classes and seminars were intimate and stimulating. He enjoyed talking about the people, many of whom he knew personally, behind the ideas and theorems. I found myself often staying after his class. Our conversations would follow up some of the items in the classroom but would soon drift to other areas of his expertise. He was a devoted student of southwestern Native American culture. One day he told me that after retiring he would like to be a bartender in a rural area of Arizona or New Mexico, because he found the stories of the folk that he met in those bars so fascinating.

Among all the great mathematicians I have known, Wilder was the most approachable. He had a wonderful sense of humor and his wisdom made him a father confessor to many of his colleagues. With his wife, Una, they made their home a center of hospitality. My children called them—and still do—Grandpa and Grandma Wilder. Every year at Christmas we still hang up by the chimney the stockings that Mrs. Wilder made for the kids—over 30 years ago!

After retiring from Michigan in 1967 he moved in 1969 to Santa Barbara and joined the mathematical activities there as an emeritus researcher. However, when time and health permitted he did visit some of his favorite haunts in the West.

Wilder's accolades were many. He became the first person at the University of Michigan to hold a University Research Chair (1947-67). Respected throughout the university he used his influence to fight for intellectual integrity. The university honored him with the Russell Lectureship in 1958-59 and an honorary doctor of laws degree in 1980. He also received honorary degrees from Bucknell University (in 1955) and Brown University (in 1958).

He was president of the American Mathematical Society in 1955-56 and president of the Mathematical Association of America in 1965-66. For the American Mathematical Society he delivered a number of special lectures including the Josiah Gibbs Lecture in 1969 and received the award for distinguished service to mathematics by the Mathematical Association of America in 1973. He was elected to the National Academy of Sciences in 1963.

NOTE

1. Lucille Whyburn (R. L. Moore's first doctoral student at Texas). Lecture notes in mathematics. In *Algebraic and Geometric Topology*, vol. 64, ed. K. C. Millet, pp. 33-37. Springer Verlag, 1978.

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