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UGO FANO
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A Biographical Memoir by

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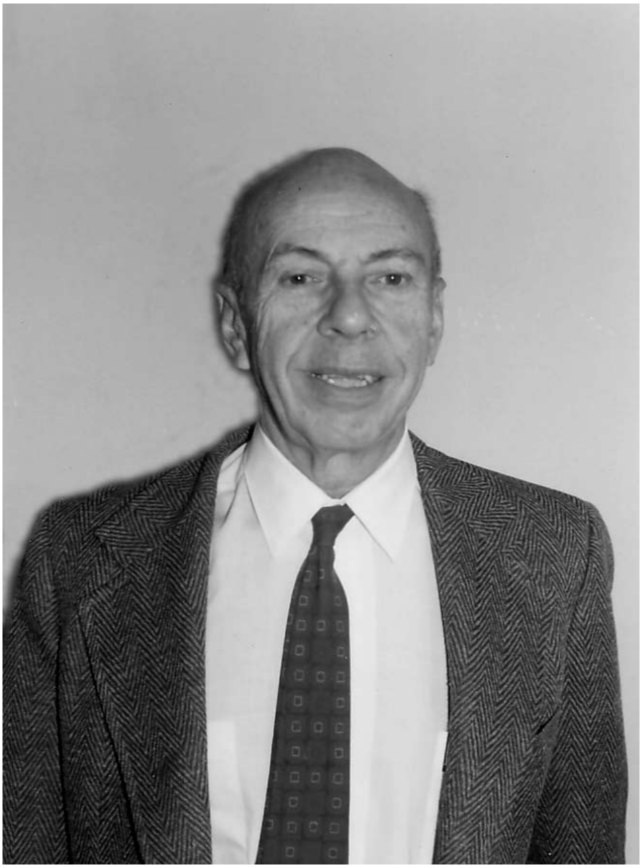
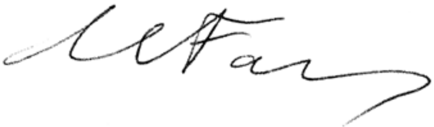
Biographical Memoir





UGO FANO

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UGO FANO WAS A LIVELY, challenging, and creative physicist, known for his wide-ranging studies in atomic physics, radiological physics and radiation biology, and statistical physics and relaxation. He stimulated and guided a cadre of students, postdoctoral associates, and colleagues whose admiration and affection for him continued, as did their interactions with him, throughout his life.

Ugo Fano was born in Torino, Italy, the elder son of Gino Fano, a distinguished mathematician, and Rosa, neé Cassin. He grew up in a well-to-do community in which there were no noticeable boundaries between their own Jewish family and their Catholic and Protestant friends. He was reading newspapers, following events of World War I, by the time he was four years old. He could recall, until the end of his life, the day that war ended. As a young boy, he was considered "delicate," a euphemism for sickly, and was home-schooled for the first three years of elementary school. He was not well coordinated, and had difficulty with writing, but was clearly very intelligent. His younger brother Robert (Tuccio) was stronger and better coordinated when they were young boys. Ugo did enter school at fourth grade and continued into junior high school and high school, where many of his teachers were priests. When he was 12, he received a bicycle, which gave him incentive to physical activity, something that he had previously shunned, largely because of his health. From cycling his athletic interests extended to hiking and rock climbing, especially in the mountains near the town of Colognola, near Verona, where his grandfather had bought a villa many years before. He continued to love mountains and mountain activities well into his older years.

He remembered well and with affection this paternal grandfather (also named Ugo) and the many stories he heard from him as a child. The grandfather was a citizen of Mantova before Italian unification. Indeed, he had served in Garibaldi's army and had a signed picture of Garibaldi, which Ugo Fano later kept on his desk but which was lost in the 1930s during the turmoil that led the family to flee from Italy for the United States. The grand "Villa Fano" in Colognola with its nearly 100 rooms, a huge frescoed ballroom, stables and orchards, and the interactions with his grandfather established a connection to a bygone era of Europe that clearly made a deep impression on him. He would often recall events from his childhood, including being taught to tie his shoelaces by his grandmother Angelica. (He described himself as being a child with very bad small-muscle coordination.) He had vivid memories of his grandmother's kitchen during Sunday morning distribution of food and money to the poor of the parish and the ritual nightly shuttering, with heavy iron bars and hooks, of the villa's ground-floor windows. Later-generation colleagues and students remember on their summer visits to the villa, which still remains in the family, that Fano himself performed that ritual.

His father, Gino Fano, did not go into the military as the grandfather wished but, with the support of the grandmother, joined the engineering school of the University of Torino. His interest soon turned to abstract geometry and he studied under Guido Castelnuovo. He became a distinguished geometer, attracting the attention of Felix Klein and an offer to join the faculty at Goettingen. But, as Fano himself would put it, moving to a foreign country seemed too drastic, and his father stayed in Italy, occupying a chair of geometry at Torino that was specially established for him. The work of his father no doubt influenced a strong geometrical way of thinking in Fano's own work in physics.

A favorite anecdote of Fano's was about hearing one evening at dinner, when he was a boy of 12, from his father about the Bohr-Sommerfeld model and the analogy of atoms to the solar system. Fano said he was shocked because to him the solar system was mostly empty unlike matter, which is full. "I recall demonstrating such fullness by touching my nose." The physical basis of that fullness emerged from Heisenberg and Schroedinger's work a few years later, but Fano would recall with satisfaction his own early intuition on the matter. On his style of reducing physical phenomena to a few key parameters that theory can then concentrate on (more on this below), he said on occasion that it may go back to the early influence of his father, who when he was a boy of nine, taught him fractions, setting numerous exercises every day on reducing them to the smallest denominator. Fano's mother, Rosa, was an accomplished artist, landscape painter, and musician with exquisite social manners. Just as with his father and that family, she also came from a family of engineers and technocrats and Fano himself would observe later in life that "so much of our interest was in technology rather than nature."

Fano did his studies in Torino, receiving his doctorate in mathematics in 1934, working with Enrico Persico. He then went to Rome, joining the group of Enrico Fermi, with whom he interacted very productively—but in Fano's own reminiscences, he writes of Edoardo Amaldi as his "main mentor." Amaldi gave him the nickname "Urango Fanoide," or "Fanoid Ape."

Fano's first very influential work resulted from his work with Fermi, the interpretation of shapes of spectral lines that appear in a spectral continuum, and have shapes like traditional dispersion curves, with falling as well as rising intensities. Now usually called "Beutler-Fano" line shapes, these were observed by Beutler and then interpreted by the young Ugo Fano. The concepts embodied in this work reappeared in very striking and influential studies he carried out in the 1960s on electron energy losses and far ultraviolet spectra, to which we shall return.

Many anecdotes connect Fano and Fermi. One concerns a mountain hike he took with his father when he was 12. They stopped so Gino Fano could talk a bit with a group of young men. When they moved on, Gino said to young Ugo, "You see that young man; he is called Fermi, and people say he will go far." Fano often recalled that he retained an almost photographic image of the back of Fermi as that group hiked on in front of them.

One story Fano himself enjoyed telling concerns his work on line shapes. At the very start of his stay with the Fermi group, "one of Fermi's lieutenants, Emilio Segre," brought him a sheaf of papers and figures of experimental line shapes observed by Beutler, suggesting that there was something to be explained about their strange shapes. Fano set to work to understand them, realizing fairly quickly that they looked more dispersive than absorptive in appearance. Fermi, to whom he reported this, peeped into his office next day saying, "Fano, you were right." In later days he consulted occasionally with Fermi and in a few weeks had the solution. When he asked Fermi if it was time to write a joint paper, Fermi said, "No, you did the work, you publish it yourself, a regular acknowledgment will do." Fano did just that. Not long after the paper appeared he was at Fermi's house, waiting for Fermi. As Fano put it, "I'm a nosy guy."

He thumbed through Fermi's working notebook and found the pages of the day they had first discussed the problem. Fermi had solved it entirely. The only difference between Fano's solution and Fermi's was not in the notation at all; Fano had solved an integral by partial fractions (again an echo of his early training by his father), while Fermi had done a numerical five-point integration.

One other Fermi anecdote Fano would tell particularly reveals his candor about his own feelings of his talents: he once came into Fermi's office to ask whether Fermi knew of any fellowships for which he could apply. "No," was the reply. As he was about to leave Fano then asked, "If I find out about any, would you write a recommendation for me?" Answer: "No, not for you; for Majorana, perhaps."

Fano remained in Rome until 1936, when he went to Leipzig to work with Werner Heisenberg for two years and then returned briefly to Rome as an assistant and lecturer. The turmoil had reached a point at which Fano and his fiancée, Lilla Lattes, (who had also studied physics) decided they would leave Italy and eventually go to America. Lilla's father urged that they marry as soon as possible, which they did, by getting baptized and married by a priest in February 1939 (reflecting the easy interactions between Jews and Catholic culture in the Italy of those times). Almost immediately Lilla went directly to Argentina and Fano to Paris, where he was working to enable his family to escape from Italy. He managed to get to Argentina, and from there the Fanos were able to get visas and come to New York in June 1939.

In the United States Fano first worked as a research associate at the Washington Biophysical Institute but soon, in 1940, moved to the Cold Spring Harbor Laboratory of the Carnegie Institution, in the group of Milislav Demerec. This was a time he was primarily involved in biological problems, especially radiation biology. They studied the effect of X rays, especially on *Drosophila*. His publications during that period include many with Demerec but do not reflect the many interactions he had with Max Delbrück. Delbrück's work on bacteriophages stimulated Fano to suggest to Demerec that they also study phages, especially their reproductive genetics. This work resulted in the discovery of virus-resistant *E. coli* mutants, which are still valuable genetic material.

It was Fano who introduced Delbrück to Salvador Luria, the two of them later sharing a Nobel Prize. During the summer of 1941, Nicholas Rashevsky offered Fano a position in his group at the University of Chicago, which Fano declined in order to stay at Cold Spring Harbor.

His world then changed.

When the United States entered World War II, Fano's wife, Lilla, was pregnant, so he was exempted from the draft. Instead he went to work at the Ballistic Research Laboratory at Aberdeen Proving Ground, first just as a consultant, then as a staff member. There he worked on effectiveness of weapons and developed documents for Air Force personnel, specifically on how to choose the best-suited weapons for specific purposes. After some time living apart from Lilla and their new daughter, Mary, they did obtain housing at Aberdeen and lived together.

In order to gain a deeper understanding of the physics underlying the biological effects he had been studying before World War II—stimulated to do so by Fermi—Fano took a leave from Cold Spring Harbor after the war ended to work for a year at Columbia University. There he decided he was really interested in understanding the details of the complex elementary processes associated with the radiation itself—photoionization, secondary radiation, the Compton effect, and the eventual degradation of high-energy radiation.

While he was at Columbia, Fano was offered a position at the National Bureau of Standards by Lauriston Taylor at the Bureau. He accepted in 1946 and began a very productive and varied career that continued in Washington until 1966. The Fanos found a house within a 10-minute walk from the Bureau. A few months after they arrived, their second daughter, Virginia, was born, not in Washington but back in Cold Spring Harbor, Mary's birthplace, because the family went there to continue work with Demerec and to interact with scientists gathered there for the summer.

For several years Fano's parents lived part-time with Ugo and Lilla, going back to Italy the rest of the time. In 1948 Ugo and Lilla made their first return trip to Italy, and to the old family manor at Colognola. They went back again for a year in 1956 when Ugo received a Fulbright award. It was in that year that he and Lilla wrote their first book, *Basic Physics of Atoms and Molecules.*[2] Although educated to be a professional physicist, Lilla, after spending some time in research, found her true calling in teaching science. This book is an elegant work of pedagogy, providing a clear, rigorous basis of atomic quantum physics in the context of the kind of macroscopic physics likely to be familiar to the student reading the book. They later wrote an undergraduate version, *Physics of Atoms and Molecules: An Introduction to the Structure of Matter.*[3]

At about the time *Basic Physics of Atoms and Molecules* appeared, Fano and his cousin Giulio Racah published a very useful (and very technical) book on angular momentum and its application, *Irreducible Tensorial Sets,*[4] essentially a treatise on the group theory of angular momentum and how to use it in contexts such as nuclear and atomic collision physics. That book, too, was the basis for another that Fano later wrote with his former graduate student Ravi Rau, *Symmetries in Quantum Physics*[5]; this was published in 1996. Symmetries and unitary transformations were central to Fano's thinking and at least subconsciously reflected his father's work as in Klein's "Erlangen Program," which saw all geometries in terms of symmetry transformations.

At the National Bureau of Standards he held the position of chief of radiation theory and became a senior research fellow there. From his work in radiation biology he recognized the inadequacy of the target theory of radiation action, which viewed it in terms of "hits" and was only statistical in description. Instead he focused on the detailed atomic and molecular processes that occur when an energetic charged particle enters a medium. He addressed many topics, among them, the reasons some chemical modifiers such as oxygen enhance while others protect against radiation damage, the pathways along which super-excited molecules along the tracks of secondary electrons dispose of their excess energy, and the time for recovery of cells from nonlethal damage.

In 1947 he developed a way to predict not only the average yield of ionization from radiation but also its fluctuations, measured by what is now called the "Fano factor." In 1954 he and L. Spencer carried this further and provided an analysis of the energy spectrum that results from fast electrons slowing down in a medium. He also established the "Fano theorem," that the flux of secondary charged particles crossing a cavity is independent of the density variations in a region uniformly suffused by primary radiation. This extension of the Bragg-Gray relation is fundamental to radiation dosimetry. Throughout the 1950s his National Bureau of Standards group produced valuable tables and graphs of charged-particle stopping powers, using desk calculators as well as early digital computers.

He made a major contribution to the use of density matrices in atomic and molecular science with his article in *Reviews of Modern Physics* (1957). The power of density matrix methods and Liouville representations began to emerge under the influence of this paper, one of the most influential in physics. This work, which was roughly contemporaneous with his work with Racah on invariant tensors and their algebra, is also related in subject matter, falling under the umbrella of describing correlated systems in physics, including applications to correlated decays of atoms and nuclei.

In 1961 Fano again addressed the general problem of line shapes for excitation of states at energies that lie high enough to be in a continuum. This was, of course, intimately connected with his 1934-1935 analysis of Beutler's experiments. In that work Fano had introduced a parameter q to supplement the intensity and width, which indicates the asymmetry and degree of dispersion-like shape that the line has. This is now called the "Fano q-parameter." Not long thereafter, in 1963, to check the characteristics of a new synchrotron light source for the far ultraviolet, R. P. Madden and K. Codling filled the chamber of the light source with helium, expecting it to be entirely transparent. To their surprise they found spectral lines with those strange, dispersion-like shapes. Almost immediately after that J. W. Cooper, Ugo Fano, and F. Prats were able to interpret the spectral lines as due to doubly excited electronic states of helium atoms, states with two excited electrons, with energies well above the first ionization limit of the helium atom. Hence, these were discrete-like states buried in the continuum of states with one bound and one ionized electron.

This phenomenon was, in effect, a stimulus for much of the work Fano and his coworkers carried out in subsequent years—but that work took a number of interesting and unexpected directions. The "Fano 61" paper is one of the most frequently cited papers to this day, with extensions of "Fano resonances" to Kondo systems and quantum dots in condensed matter as well as systems in quantum optics.

The study of doubly excited states led also to another of Fano's favorite themes: the use of collective quantum numbers and even coordinates, such as the hyperspherical coordinates for two or more electrons, in terms of which the system particles can be described, rather than with independent electron coordinates and quantum numbers. This theme played a large role in his work in the Chicago years, to which we will return below. The study of the spectral region in which most multiply excited states of atoms and molecules lie may also be seen as an extension of his interest in radiation penetration of matter. He saw most clearly that the region between the near ultraviolet and X rays, which had been left unexplored till the 1960s, spoke to a deep point of physics.

Besides the lack of good sources of radiation that was remedied with the advent of synchrotron light sources, a primary reason for the neglect was the strong absorption of most materials, reflecting that this energy region lies in the ionization continuum of atoms. To overcome that problem of absorption, new techniques for windowless spectroscopy, with the sample gas held by fast differential pumping but no intervening windows between it and the light source, had to be developed. But the very fact that most substances absorb most strongly in this vacuum ultraviolet region means that this is where most of the oscillator strength of atoms lies. So, precisely the main part of the spectrum was the one that had been left unstudied. Simultaneous excitation of more than one electron, which also occurs in this region, "unfreezes" degrees of freedom that are qualitatively different from those with excitation of just one electron, however high in energy. Qualitatively new "physico-mathematical" (a favorite phrase of his) concepts, such as collective quantum numbers, are needed in their study.

Another effect generally associated with Fano's name and that of W. Lichten is the consequence of atomic shells forced to overlap during collisions, especially very close collisions. That consequence is the promotion of some electrons to higher-energy shells so that kinetic energy of the collision is transformed into energy of electronic excitation. This has become known as the "Fano-Lichten promotion mechanism." They carried out their analysis in the last period that Fano was still in Washington, and Lichten had just moved from Chicago to Yale. This mechanism for high excitation has general consequences for chemical transformation. Throughout his work, including the one on two electron states and the role played by a saddle in the hyperspherical potential surface, Fano saw connections to transition states and the crucial part they play in chemical transformation.

In 1966 the National Bureau of Standards was moving from Washington to the suburb of Gaithersburg, Maryland. At the same time a group of people at the University of Chicago had become interested in wooing Ugo Fano to Chicago, despite his earlier refusal to come. Robert Platzman, then newly at Chicago himself, was the prime mover; he had interacted extensively with Fano (but never published with him) and was a strong admirer. Platzman easily found a group who shared his views. This group approached Fano, and quickly learned that he was not at all enthusiastic about going to suburban Maryland, and that "if there was ever a time for me to move, this was it." He joined the Physics Department at Chicago that fall and remained an active member until 2000, even though he officially became Professor Emeritus in 1982. He was chair of the department from 1972 until 1974.

While he had collaborators at the National Bureau of Standards, he was able to have a real group of students and postdoctoral research associates at Chicago. In a sense the academic environment allowed him to blossom, in other words, to explore many more of his rich flow of ideas. It was also very evident how much he enjoyed working with students, conveying to them his combination of enthusiasm and concern for precision and accuracy. He mentored approximately 30 doctoral students in his years on the Chicago faculty, and of course had a steady flow of postdoctoral associates, while continuing to work with former colleagues as well.

One former student, Shinichi Watanabe, characterized Fano with his vivid tale of their first meeting, when Watanabe came in to discuss possibilities of doing research in the Fano group. After some description of research topics, Fano seated at his desk bent toward Watanabe, cupped his right hand with fingers pointed upward, and said in his Italian accent, "Well, physics is conceptualization." That won Watanabe to the Fano team. He saw that statement as a testimony of Fano's style, confidence, and trustworthiness.

The Fano group had constant and close interactions with their mentor. Fano would come regularly into his students' offices in the morning to ask, "So, what's new?" The group would lunch together to discuss physics and whatever else interested them at the moment. The group moved from the old Eckart-Ryerson complex to the Research Institute building a block away when the entire Physics Department moved from its original home. The two changes for the Fano group were that the students' offices were now down the hall from Fano's instead of next door, and lunch moved from local restaurants to the Research Institute. At the group lunches they would discuss the most recent papers in *Physical Review A* and *Physical Review Letters* (not restricting themselves at all just to the atomic physics papers). Besides the informal discussions, for many years the Fano group and that of R. S. Berry held a weekly chemical physics and atomic physics seminar together. But even these were informal, and it was usual for people to question speakers during the talks.

Students and postdoctoral associates of Fano have sometimes been said to constitute a "school," and there is something substantive to that designation. Partly it is the way they approach problems much as Fano himself did—conceptualizing and then choosing a mathematical means to express the concepts. At the same time, again in Fano's style, the approach should be as free as possible of jargon, as clear as possible to anyone familiar with basic physics. The goal is always to reduce the description of the physics to a minimum set of parameters to allow one to relate experimental results to their theoretical interpretation.

One elegant example is the extensive use the Fano group (and of course Fano himself) has made of quantum defect theory as a powerful phenomenological and theoretical analysis of complex spectra. Depending on different regions of its motion, a photoexcited electron may be described appropriately by different basis states, coupling schemes, etc. An entire series of spectral levels, resonances and associated continua with all other quantum numbers in common, energy alone a running index, constitutes a channel. During its motion, when the electron of interest is close to the residual core, it is strongly coupled to those other electrons in one of the various strong potentials. However, with those potentials setting the energy scale the relevant parameters there cannot be a determinant for states that differ only in small changes in asymptotic energy and are therefore common to all the energy levels. A numerical computation in this region can also be adequately done on a coarse energy mesh. It is the region of asymptotic, long-distance motion of the electron, however, one with weak potentials (often just a Coulomb tail), where the sensitive changes from one high Rydberg level to the next or phenomena in the continuum just above threshold are set. This region is best handled analytically. One must adopt such differing descriptions and connect them through appropriate unitary transformations to get a meaningful and economical description. Analytical, numerical, and graphical descriptions of such a quantum defect theory were developed by the Fano group.

Another example is the characterization of scattering phenomena, such as the angular distribution of electrons. This kind of process lends itself elegantly to simple parametric representation, so that the goal of the theory, from Fano's perspective, is to show how the values of the key parameters vary with the controllable experimental value, such as the energy of the collision, and then how the parameter governs the observable results of the experiment. In describing angular distributions he developed the powerful concept of angular momentum transfer from the projectile to the target, its advantage lying in that amplitudes for different values of this parameter add only incoherently without introducing interference terms in the cross-sections.

Fano liked to work with physical, almost tangible models, such as the one he and his group used extensively to describe electron scattering by molecules. In this picture when the electron is close to the molecule, its behavior and hence its quantum characteristics are closely associated with the shape of the molecule. When the electron is far from the molecule, it "sees" the rotating molecule but not its detailed structure. Hence, one can use two different representations for how the electron interacts with the molecule, one when the electron is close, another when it is far, and Fano showed how to connect those two representations through frame transformations. In this as in the work on quantum defect theory generally, an experimentalist once remarked that Fano describes as if he were on the electron, moving with it.

Fano devoted substantial efforts to activities aimed at influencing the scientific community to move into directions he thought were important and timely. One of the goals of such activities, which he liked to call campaigns, concerned radiological physics and radiation biology. Most of his scientific work at the National Bureau of Standards laid down the fundamentals of this topical area, and Fano later maintained strong interest in it and in effect served as a senior statesman in attending meetings and in advising funding agencies and organizations, such as the International Commission on Radiation Units and Measurements and the National Council on Radiation Protection and Measurements. In this connection he used to say, "Radiological physics is an applied science. Remember that a result in an applied science is good only when it is respectable in the eyes of basic scientists." In other words, he kept warning against a frequent tendency to be content with low standards of work. His dictum turns out eminently valid in the recent advance in medical diagnosis exemplified by computerized tomography and other techniques.

The second goal of his campaign since the early 1960s and in various countries was the use of electron synchrotron radiation as a light source for spectroscopy of atoms, molecules, and solids. The success is obvious in the number and variety of such sources now in operation spread around the world. Fano himself remarked on several occasions on the decisive role that experiments, including the first synchrotron spectra of helium, played in orienting his own research. In his writings, as in the book (with Ravi Rau) *Atomic Collisions and Spectra*,[6] he gave top billing in the first chapter to a summary of key experimental techniques and apparatus that have led to our understanding, as a way of emphasizing to students the importance of experiments and this phenomenological approach to physics.

The third goal concerns improvement of communications in physics. Since the late 20th century the growth of the number of scientists, the internationalization of scientific work, and the diversification of the scope of scientific research have led to a general feeling of difficulties of communications, often expressed by complaints such as "Recent papers are hard to read" or "Talks nowadays are not understandable." Fano felt strongly that conscientious efforts are needed to improve the quality of communications in science; he often discussed what to do, and also worked hard to comment and edit manuscripts written by his associates and friends. He served seriously as an associate editor for *Reviews of Modern Physics* for 1990-1995, and innovated the Colloquium section to provide a forum for a short readable review of a topic in the style of a colloquium talk at a university. He even organized a small meeting to discuss ways to improve communications, as summarized by B. Bederson in *Physics Today*.[7] Another aspect was his cultivating relationships with Soviet physicists already in the 1960s when such contacts were few, and he hosted several visits to his Chicago group by colleagues from Leningrad and Moscow.

His extraordinary accomplishments earned him a number of honors. In addition to membership in the National Academy of Sciences (elected in 1976), he was a foreign member of the Accademia Nazionale dei Lincei (Rome) and of the Royal Society (London). He received honorary doctorates from the Queen's University of Belfast and from Université Pierre et Marie Curie (Paris), as well as the Davisson-Germer Award of the American Physical Society and the Enrico Fermi Award of the U. S. Department of Energy. The Fermi Award made him the most visibly happy, obviously because of his utmost respect and lifelong affection for Fermi.

NOTES

1. Mitio Inokuti, died on June 4, 2009, prior to publication of this memoir.
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