NATIONAL ACADEMY OF SCIENCES

OSWALD VEBLEN

1880—1960

A Biographical Memoir by

SAUNDERS MAC LANE

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Oswald Veblen

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June 24, 1880—August 10, 1960

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OSWALD VEBLEN, geometer and mathematical statesman, spanned in his career the full range of twentieth-century Mathematics in the United States; his leadership in transmitting ideas and in developing young men has had a substantial effect on the present mathematical scene. At the turn of the century he studied at Chicago, at the period when that University was first starting the doctoral training of young Mathematicians in this country. He then continued at Princeton University, where his own work and that of his students played a leading role in the development of an outstanding department of Mathematics in Fine Hall. Later, when the Institute for Advanced Study was founded, Veblen became one of its first professors, and had a vital part in the development of this Institute as a world center for mathematical research.

Veblen's background was Norwegian. His grandfather, Thomas Anderson Veblen, (1818–1906) came from Odegaard, Homan Congregation, Vester Slidre Parish, Valdris. After work as a cabinetmaker and as a Norwegian soldier, he was anxious to come to the United States. This he reached in 1847, settling with his wife Kari first at Holland, Michigan, and later on a farm in Rice County, Minnesota. There, under difficult pioneering conditions, he raised a family of twelve children; one of them was the brilliant but maverick economist and social theorist, Thorstein Veblen (1857–1929). Another (Oswald's father), Andrew Anderson Veblen (1848–1932), was interested in Mathematics, which he taught in Decorah, Iowa, just after graduating from Carleton College in 1877; he later studied at Johns Hopkins (1881–1883) and then became Professor of Physics at the University of Iowa. On July 11, 1877, he married Kirsti Hougen (1851–1908) whose family (father Thomas A. Hougen) had emigrated from Hollingdal, Norway, in 1856. Their son Oswald (the first of eight children) was born in Decorah, Iowa, on June 24, 1880. He received his A.B. degree at the University of Iowa in 1898, taught as a laboratory assistant in physics there for one year, and then went to Harvard, where he received a second A.B. degree in 1900. Then came three years of graduate study at the University of Chicago under the dynamic leadership of E. H. Moore.

There was then great interest in the foundations of Euclidean geometry. Euclid's *Elements,* that model of logical precision, had been shown logically inadequate because of its neglect of the order relations between points on a line and its consequent inability to prove rigorously that the plane is separated into two halves by a line or into an "inside" and "outside" by a triangle. David Hilbert, the famous German Mathematician, had proposed a new and precise system of axioms which had great vogue, and which depended on the use of a large number of primitive concepts: point, line, plane, congruence, and betweenness. Veblen, in his thesis, took up the alternative line of thought started by Pasch and Peano, in which geometry is based directly on notions of point and order. Thus in Veblen's axiom system there are only two primitive notions: point and order (the points A,B,C occur in the order ABC); as was the fashion, he carefully studied the independence of his axioms and the relation of his geometry to Klein's Erlanger Program.

With the completion of his thesis in 1903, Veblen became an Associate in Mathematics at Chicago and immediately plunged into the active work of helping to train other young men. R. L. Moore wrote a thesis on the foundations of geometry under the stimulus of Veblen ("Set of Metrical Hypotheses for Geometry," Transactions of the American Mathematical Society, 9 [1908]:487–512). Veblen early recognized the central position of the Heine-Borel theorem in analysis (1904) and emphasized this in the pioneering text on real variable theory (1907) which he wrote in collaboration with N. J. Lennes. Here, as in many other cases, his mathematical insight led him to pick out an idea which later came to full generality and significance—in this case, the Heine-Borel theorem, which expressed the compactness of the interval and which appears today in the more general study of compact topological spaces. Veblen's study of the foundations of geometry naturally led him to consider various other finite geometries, as in his work with W. H. Bussey, "Finite Projective Geometries" (1906), and his work with J. H. M. Wedderburn, using algebraic results by L. E. Dickson, on non-Desarguesian and non-Pascalian geometries—a subject still of lively interest to algebraists centered at Chicago.

With this period, the direction, but not yet the development, of Veblen's scientific interests was determined. He was a geometer, with an especial interest in the foundations and with a particular flair for picking out and developing the important ideas which he found roughhewn in the literature. This he did for many other fields of geometry, covering projective geometry, topology, and differential geometry, and touching on mathematical logic, relativity theory, and ballistics. With this scope, geometry appears to be coextensive with Mathematics, and so Veblen viewed it. His famous definition of geometry (1932) reads:

> Any mathematical science is a body of theorems deduced from a set of axioms. A geometry is a mathematical science. The question then arises why the name geometry is given to some mathematical sciences and not to others. It is likely that there is no definite answer to this question, but that a branch of mathematics is called a geometry because the name seems good, on emotional and traditional grounds, to a sufficient number of competent people.
>
> As the words are generally used at present, a geometry is the theory of a space, and a space is a set of objects usually called points, together with a set of relations in which these points are involved. A space, therefore, is not merely a set of objects, but a set of objects with a definite system

of properties. These properties will be referred to as the structure of the space.

This description today would seem as apposite to Mathematics in general as to geometry, and Veblen's notion of structure is almost exactly that recently made popular by Nicholas Bourbaki—though current fashion would call for more emphasis upon the so-called "morphisms" of the structure.

But back to 1905. At this time Woodrow Wilson was establishing at Princeton a system of preceptors. He called Veblen to one of these positions. Veblen remained at Princeton as a preceptor till 1910, then as a professor (1910–1926) and Henry Burchard Fine Professor (1926–1932). In 1908 he married Elizabeth Mary Dixon Richardson; there were no children.

At this period Veblen's interest in the foundations of geometry led him to study the axiom systems for projective geometry. With John Wesley Young (1908) he worked out an effective system. This was used in the magnificent two-volume treatise *Projective Geometry*. (The first volume was by Veblen and Young together, the second by Veblen alone.) For elegance, completeness, and clarity this work was and is a model; perhaps too thorough and too leisurely to serve its officially intended function as a textbook for college courses, it sets forth lucidly the notions of axiom systems and of independence of axioms, the properties of projectivities, conics, inversive geometries, and non-Euclidean geometries, and the general process of classifying various geometries by the Klein Erlanger program. The two volumes are beautifully organized. The first volume deals largely with geometries over arbitrary fields, while the difficult continuity considerations required to prove the von Staudt theorem are postponed till the second volume, which then handles the special properties of the real projective spaces. At the end of this volume Veblen's earlier interest in order and separation of points comes to the fore in a long and careful chapter, "Theorems on Sense and Separation"—a chapter which in effect was an introduction to some of the ideas (convexity, orientation, incidence matrices) of analysis situs.

Analysis Situs is the title of Veblen's most influential book. The subject began in the description of the connectivity of manifolds by means of Betti numbers and torsion coefficients; the decisive ideas were introduced by the great French Mathematician Poincaré in a series of brilliant but difficult memoirs (1895–1904). Veblen had been interested in related topological questions since his early paper (1905) proving the Jordan curve theorem from his axioms for plane geometry. About 1912 he took up a more systematic study of the subject; when he was invited to give the 1916 Colloquium Lectures of the American Mathematical Society, he chose to lecture on analysis situs for manifolds. The resulting published volume (1922) is the first systematic treatment in a book of the basic ideas of analysis situs. It covers linear graphs, two-dimensional complexes and manifolds and their connectivity as calculated by chains, cycles, and boundaries, n-dimensional complexes, orientable manifolds, the fundamental group, intersection numbers, and the knot problem. For many years Veblen's book, as the best source, was assiduously studied by generations of topologists who have gradually wholly transformed the subject: Betti numbers have been replaced by homology groups, coefficients modulo 2 by arbitrary groups of coefficients, or even by sheaves of coefficients, simplicial chains by singular chains, while the fundamental groups are now just the first stage in the study of homotopy theory. With these changes "Analysis Situs" has been renamed, first "Combinatorial Topology," and then "Algebraic Topology"—so by now Veblen's expressed wish that his book would soon become obsolete has belatedly been fulfilled—thanks in substantial part to the very influence of this book.

Differential geometry next attracted Veblen's interest. In Riemannian geometry the parallelism of Levi-Civita (1917) provides a means of transporting tangent vectors along curves so as to remain "parallel" to themselves. Technically, this parallelism is determined by the 3-index Christoffel symbols $\left\{ {i \atop jk} \right\}$ as calculated from the components g_{ij} of the Riemann metric, and these symbols determine the geodesic paths. In 1922 Eisenhart and Veblen proposed the suggestive idea of a generalized geometry in which the $\{^{i}_{jk}\}$ are taken as basic and as determining a geometry of paths. This "Geometry of Paths" was vigorously developed at Princeton by Eisenhart, Veblen, and Veblen's student, T. Y. Thomas, in close parallel to related developments in Europe (Weyl's symmetric affine connections and the work of J. A Schouten and E. Cartan). Today this geometry finds its place in a more general setting, that of connections in fibre bundles—in a form reflecting especially the basic concepts of E. Cartan and the ideas of Veblen himself on the foundations of differential geometry (see below).

Veblen was asked to revise an early Cambridge Tract, *The Invariants of Quadratic Differential Forms.* Veblen's 1927 tract, under the same title, is a systematic treatment of the basic formal properties of Riemannian geometry and is distinguished by the great care and precision of its presentation. In particular, this tract displayed in full clarity the fact that the usual formal treatment of the Riemann metric g_{ij} and of tensors is strictly local—valid in a domain covered by one coordinate system. This brought to the fore the question of an effective treatment of a Riemann metric in the large—that is, in a manifold covered by several overlapping coordinate systems. This question was the subject of a second Cambridge Tract, *The Foundations of Differential Geometry,* this time written jointly by Veblen and his brilliant student, J. H. C. Whitehead. Here we find a clear definition of a differentiable manifold (in the large). Whitney, Chevalley, Steenrod, Cartan-Eilenberg, and others have subsequently refined and generalized this definition to apply to manifolds in the large which may be differentiable, analytic, or complex analytic, and to the various types of fibre bundles which may be constructed over them. The resulting definitions are a basic tool in modern geometrical researches (in particular, for the geometry of paths, as noted above). Veblen early saw the necessity of this type of development; thus, in his Rice Institute Pamphlet (1934) he says at one point that this "brings to light what I think is the most important unsolved problem of our epoch—the relation between differential geometry and topology." The subsequent progress in this direction has owed much to Veblen's steady interest in the foundations of geometry, as exhibited in his tract on the foundations of differential geometry in the large.

Since any physical theory rests upon an underlying geometry, Veblen found that his interests in the foundations of geometry carried over to an interest in physics. As he said before the American Mathematical Society in his retiring Presidential address (1924), "The foundations of geometry must be studied both as a branch of physics and as a branch of mathematics." Thereby Veblen was led to his extensive work on projective relativity theory. Riemannian geometry was already involved in the relativistic theory of gravitation. Einstein, Weyl, and others were searching for a more inclusive geometric structure which would yield a field theory unifying gravitation and electromagnetism. The Kaluza-Klein theory had achieved a remarkable formal unification by means of field equations in a suitable five-dimensional space. The fifth coordinate was restricted to a limited group of transformations; its physical interpretation posed a problem. Veblen showed that it could be regarded as a gauge variable and that the transformations then amounted to changes of a proportionality factor for homogeneous coordinates in projective tangent spaces. He was thus able to reinterpret the whole theory as one involving four-dimensional projective space-time. These ideas of Veblen influenced the research of Schouten, van Dantzig, Pauli, and others.

In the same connection Veblen's geometrical insight is well illustrated by his geometrical theory of four-component spinors. These objects had been thoroughly studied by many mathematical physicists; Veblen showed that the Plücker-Klein correspondence between the lines of a three-space and the points of a quadric in projective five-space could be used to treat these spinors by means of a conformal geometry which contained the geometry of projective relativity as a special case. Though this train of ideas is no longer so fashionable, Veblen's contribution was well summarized in his

Ergebnisse volume, *Projektive Relativitätstheorie* (1933) and in the lecture notes published as *Geometry of Complex Domains* (1936).

Parallel to this development of geometrical ideas runs Veblen's work as a professor in the development of younger Mathematicians at Princeton University. Among those whom he influenced we may note A. A. Bennett, H. S. Brahana, A. B. Brown, Phillip Franklin, and A. W. Tucker in topology and projective geometry; M. S. Knebelman, J. M. Thomas, and T. Y. Thomas in the geometry of paths; Wallace Givens, Banesh Hoffmann, and A. H. Taub in projective relativity theory. Veblen's influence upon algebraic topologists was especially fruitful. He collaborated with J. W. Alexander in early years, at the time when Alexander discovered his fundamental proof of the invariance of the Betti numbers and torsion coefficients; his student J. H. C. Whitehead went on to do fundamental research in homotopy theory; with his Princeton colleagues, Alexander and Solomon Lefschetz, he developed at Princeton that lively center for the study of algebraic topology which, then and now, is perhaps the leading world center in this field. Veblen's interest in the foundations of geometry included a concern for mathematical logic. In his American Mathematical Society Presidential address (1924) he wrote, "The conclusion seems inescapable that formal logic has to be taken over by mathematicians. The fact is that there does not exist an adequate logic at the present time, and unless the mathematicians create one, no one else is likely to do so." Veblen's own early interest in the role of ordinal numbers in logic was subsequently taken up by his student Alonzo Church in his 1927 thesis, "Alternatives to Zermelo's Assumptions" *Transactions of the American Mathematical Society* (1927), pp. 178–208; Church went on to found the presently active school of mathematical logic in Princeton.

In 1930 Abraham Flexner established in Princeton a new type of postgraduate institution for the encouragement of research—the Institute for Advanced Study. Veblen, as an early adviser, played an important role in the selection of Mathematics as one of the first fields of activity for the Institute, and in the choice of the initial faculty (1932) of the School of Mathematics at the Institute: a faculty including J. W. Alexander, Albert Einstein, Oswald Veblen, John von Neumann, and Hermann Weyl. Veblen's own devotion to the Institute was legendary; he played a vital role in setting the Institute policy of assembling younger Mathematicians as temporary members of the Institute; he worked extensively with both the trustees and the faculty, in especially close collaboration with Alexander and with von Neumann. As the Institute grew, Veblen was called on more and more to provide advice, guidance, and encouragement to the young Mathematicians who flocked to the Institute for advanced training. Thus Veblen gradually became the elder statesman for Mathematics—a role in which he is fondly remembered by many young men whom he helped. This, too, was a natural continuation of earlier activities. As President of the American Mathematical Society (1923–1924), he played a vital part in improving the financial stability of the Society through the establishment of a Board of Trustees; as a member of the Division of Physical Sciences of the National Research Council (1920–1923) he helped in the establishment of the National Research Council Post-doctoral Fellowships.

During the First World War Veblen served as captain and later as major in the Ordnance Department of the United States Army; he was in charge of the range firing and ballistic work at the Aberdeen Proving Grounds. Though his work was largely experimental and administrative, he succeeded in bringing eight or nine other Mathematicians into the analytical work of this organization. In the Second World War he was called back to similar work in ballistics; from 1941 to 1945 he served as a section head at the Aberdeen Proving Grounds. There his wisdom was influential in bringing many Mathematicians to work on currently pressing problems; for instance, it is reported that he had an important hand in bringing von Neumann to his initial work on shock waves. The 1943 proposal to support ENIAC, the first electronic computer, had his hearty and decisive support, as did the later (von Neumann) electronic computer at the Institute for Advanced Study.

After the war international mathematical activity was formally resumed with the International Congress of Mathematicians in Cambridge, Massachusetts, in 1950. Veblen (who had earlier served as one of the American representatives to the old International Mathematical Union) was the President of this Congress. In the same year he became Professor Emeritus at the Institute for Advanced Study, but continued none the less his interest in the development of Mathematics. He had long enjoyed walks with his friends in the country about Princeton. In 1957 he and Mrs. Veblen gave eighty-one acres of their property near Princeton "for the development of a public arboretum, expressing the hope that the tract would be a place where you can get away from cars, and just walk and sit." About this time, Veblen, while engaged in some consulting work, suffered an accident which seriously impaired his eyesight, but which left no trace of resentment in his kindly and friendly spirit.

He died on August 10, 1960, in Brooklin, Maine. His long life had been devoted to substantially advancing the science of Mathematics, both through the development of other scientists and through the clarification of basic geometric ideas, notably in algebraic topology and in the foundations of differential geometry in the large.

ACKNOWLEDGMENTS

A biography of Veblen appears in the American Mathematical Society Semi-centennial Publications (New York, American Mathematical Society, 1938) I (History): 206–11. It contains, *inter alia,* a complete list of Veblen's doctoral students.

Very helpful information for the present article was provided by Mrs. Elizabeth M. D. Veblen and by Professor Banesh Hoffmann.

HONORS AND MEMBERSHIPS

Honorary D.Sc., Oxford, 1929.
Honorary D.Sc., Chicago, 1941.
Honorary Ph.D., Oslo, 1929.
Honorary Ph.D., Hamburg, 1933.
LL.D., Glasgow, 1951.
Knight of the First Order, Royal Order of St. Olaf (Norway).
Army-Navy Certificate of Merit, 1948.

Member, National Academy of Sciences, 1919.
Member, American Philosophical Society.
Foreign member, Kgl., Danske Videnskabernes Selskab.
Foreign member, Polska Akademia Nauk.
Foreign member, Accademia Nazionale dei Lincei.
Foreign member, Royal Irish Academy (Department of Sciences).
Foreign corresponding member, Academia Nacional de Ciencias Exactas, Fisicas y Naturales de Lima.
Honorary Fellow, Royal Society of Edinburgh.

KEY TO ABBREVIATIONS

Am. J. Math. = American Journal of Mathematics
Am. Math. Mo. = American Mathematical Monthly
Ann. Math. = Annals of Mathematics
Bull. Am. Math. Soc. = Bulletin of the American Mathematical Society
Proc. Nat. Acad. Sci. = Proceedings of the National Academy of Sciences
Trans. Am. Math. Soc. = Transactions of the American Mathematical Society

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1911

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Note: Veblen also wrote reviews of books by Bortolotti and Vahlen, published in the Bulletin of the American Mathematical Society sometime between 1905 and 1924.