Eugene B. Dynkin

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A Biographical Memoir by Sergei Kuznetsov

Eugene Dynkin was a noted mathematician and educator who made significant contributions to the fields of algebra and probability. His early area of research was algebra, specifically Lie groups and Lie algebras, but later he became interested in various areas of stochastic processes and their applications, especially Markov processes. He returned to Lie groups at the very end of his career, in 2009. The Dynkin diagram, Dynkin systems, and Dynkin's lemma are all named for him. The zero-sum, stochastic stopping Dynkin game model that he developed is important in the field of economics.

Eugene Dynkin (a.k.a. Evgeniy Borisovitch to his Russian students and colleges and Zhenya to his relatives and closest friends) was born in Leningrad in the former Soviet Union, now St. Petersburg, Russia, on May 11, 1924. That was a very turbulent time in the nation's history. World War I (1914–18), the Russian Revolution (1917), and the Civil War (1918–22) were in the recent past, and Joseph Stalin's Terror (1930–53) and World War II (1939–45) were straight ahead. Zhenya's father, a lawyer, was born in 1888 in Belarus, graduated from a high school in Odessa, and eventually managed to get into the university, not an easy thing for a Jew under Czarist rule. During the Civil War, he worked at the Red Army offices but remained a civilian. Afterwards, he worked as a legal advisor at various Soviet establishments. Communists did not trust educated people, however, and in March 1935 the whole family was exiled to Kazakhstan. At the peak of Stalin's terror, in 1937, Zhenya's father was arrested and disappeared into the Gulag (Stalin's system of concentration camps). Zhenya's mother graduated from dental school but did not practice before her husband's arrest.

In Kazakhstan, they eventually managed to settle in Aktubinsk, a medium-sized city, and Zhenya was admitted to high school (later on, he stated that the school in Aktubinsk was the best in his experience). He graduated in 1940, at the age of sixteen, with highest possible honors. By that time, he had become very interested in mathematics. Despite the fate of his father, Zhenya was admitted to Moscow State University (MSU) to the Division of Mechanics and Mathematics (in the Soviet Union, students had to choose their major right away).

On June 22, 1941, German Nazi armies invaded the Soviet Union. Eugene was only seventeen and thus not eligible for the draft, but as the German forces approached Moscow, the university was evacuated. His stories about this time of chaos include, in particular, a month-long journey to Perm, a city east of Moscow, on trucks and freight trains. There, he was admitted to the University of Perm, where several other faculty and students from MSU had ended up. Each year, Eugene was exempt from the draft by medical commission, sometimes because of poor vision and sometimes because of consequences of bone tuberculosis, which he had contracted as a child. He was able to return to Moscow and to the university in 1944 and graduated from MSU in 1946.

At MSU, there were a number of research seminars. Eugene was especially interested in Israel M. Gelfand's seminar devoted to Lie groups and Andrey Kolmogorov's seminar on Markov chains. In Gelfand's seminar, he was asked to do a presentation on the structure of semisimple Lie groups, based on papers by Elie Joseph Cartan, Hermann Weyl, and Bartel Leendert van der Waerden. While working on that, he invented a new approach to the problem, based on so-called simple roots. Because the angle between two simple roots can be equal only to $\pi/2$, $2\pi/3$, $3\pi/4$, or $5\pi/6$, a system of simple roots can be represented by a simple diagram. Later on, this approach greatly simplified the classification of Lie groups and Lie algebras. His method of visualizing the structure of the simple roots is now known as the Dynkin diagram. At the same time, in Kolmogorov's seminar, he solved (jointly with Nikolai Dmitriev) a problem devoted to eigenvalues of stochastic matrices.

In 1946, Dynkin started his graduate studies, with Kolmogorov as advisor. Though he was indeed interested in probability, his thesis, defended in 1948, was devoted to Lie groups and Lie algebras. When he completed his graduate studies, he was hired as an assistant professor at MSU in the Department of Probability and Statistics. He taught probability courses as well as algebra courses. In 1952, he was awarded a doctor of sciences degree (advanced degree beyond the Ph.D.). His second thesis was also devoted to Lie groups and Lie algebras. After that, he was promoted to full professor.

On a couple of occasions, he had to teach topics courses on stochastic processes and became very interested in the area. Traditionally, Markov processes were defined as stochastic processes with Markov property, that is, conditional independency of the past and future behavior of the process, given their present state. For a long time, theory was focused on transition functions of Markov processes and related operators in functional spaces and was mostly analytical. Most publications were devoted to special classes of Markov processes that are related to second-order linear differential equations. In the late 1940s, Joseph L. Doob published a series of papers devoted to the properties of the trajectories of Markov processes with discrete state space, so-called Markov chains with continuous time. It was quite natural to try to study the properties of trajectories of a Markov process in a more general situation. In 1952, Dynkin established a sufficient criterion for the existence of a continuous version of a Markov process, as well as a criterion for existence of a right continuous process with left limits: the so-called Dynkin-Kinney continuity condition (a similar result was established independently by J. R. Kinney in 1953).

After 1954, Markov processes became Dynkin's primary research area. He organized a seminar on stochastic processes that attracted a large number of graduate students and eventually became one of the leaders of Moscow probability group. He published two monographs[1,2] devoted to the theory of Markov processes based on his results as well as on the results of other participants in the seminar [see also[3,4]].

A number of his results had a major impact on the theory of Markov processes. Dynkin started with the general concept of a Markov process. For advanced theory of Markov processes, it was necessary to be able to start a process at any time instant t and from any starting point x. Also, in order to be able to work with, say, diffusions in bounded domains, processes with random lifetimes were also needed. As a result, a Markov process was defined not as a probability measure on the space of trajectories, but as a family of measures $P_{t,x}$ corresponding to different starting points. The complete definition includes random lifetime ζ and σ-algebras $F_{s,t}$ of events that are related to the time interval (s, t).

Dynkin's next topic was related to the so-called strong Markov property. Intuitively, this means that, for certain classes of random moments τ, the process $\eta_t = \xi_{\tau+t}$ is also a Markov process with the same transition function as the original process ξ_t. On a number of occasions, this property was used in the past without rigorous justification. It was first established by Doob, sometime in 1940s, for Markov chains with continuous time. Later, Doob extended this property to n-dimensional Brownian motion. Dynkin, jointly with Yushkevich, showed that the strong Markov property holds for right-continuous Feller processes.

Later, Dynkin becomes interested in the boundary behavior of Markov processes and so-called Martin boundaries. Originally, Martin boundaries were invented by R. S. Martin in the early 1940s; they were related to non-negative harmonic functions in a domain. If the domain is smooth, such functions are in one-to-one correspondence with finite measures on the boundary of the domain and can be represented as Poisson integrals. For a non-smooth domain, a geometric boundary must be replaced by a Martin boundary. For Markov processes in a domain, a class of non-negative (super)harmonic functions corresponds to a class of so-called excessive functions, and the corresponding Martin boundary is related to the boundary behavior of Markov processes.

Early in his research, Dynkin studied a class of Markov processes that behave in the same way within a certain domain and therefore could be considered as continuations of a process after it leaves the domain. Sometime later, he focused on the decomposition of excessive functions into extreme ones. Following the ideas of Gilbert A. Hunt, he published a number of papers on the subject, among them one about Markov chains with continuous time and a second about general Markov processes. Dynkin was not satisfied with the outcome, however, because some of the assumptions definitely looked technical. After several attempts to get rid of technicalities, Dynkin completely changed his strategy. First, he considered inhomogeneous Markov processes and inhomogeneous excessive functions and later on obtained a homogeneous result as a corollary. Next, he started with the decomposition of excessive measures and then reduced a decomposition of excessive functions to a decomposition of excessive measures for a dual semigroup. To this end, Dynkin introduced the concept of a Markov process with a random birth and death time, described such processes in terms of their transition function and some excessive measure, and constructed an entrance space for a Markov process in terms of limits of supermartingales with random birth moments. It turned out that such an approach did not require almost any technical assumptions, even the topological assumptions such as right continuity of the process and so on.

Using similar ideas, Dynkin created a concept of regular Markov processes. A process is called regular if its transition function is right continuous along the trajectories (again, no topological assumptions were used). Regular processes are strong Markov, and excessive functions are also right continuous along trajectories. Among other publications, let us mention a couple of papers devoted to the optimal stopping problem. Dynkin developed a solution to this problem in terms of the smallest excessive function, which dominates the payoff function. He also considered a game version of this problem with two players. Nowadays, models based on Dynkin's ideas are known as Dynkin games, and they are heavily used in economics applications.

In 1976, Dynkin finally decided to emigrate from the Soviet Union. He accepted an offer from Cornell University and remained there until his retirement in 2011, at the age of eighty-six. At the end of 1970s, Dynkin switched to random fields and related questions. Some of his publications are related to so-called Dirichlet spaces and Dirichlet forms. At the beginning, Dirichlet forms and spaces were associated with Brownian motion and the Laplace operator; later on, these concepts were extended to right-continuous strong Markov processes with symmetric transition density. Dynkin designed an alternative version of the theory for regular Markov processes. Another group of Dynkin's results is related to self-intersections of trajectories of a Brownian motion. In the late 1980s, Dynkin became interested in so-called branching measure-valued processes. Intuitively, such processes can be viewed as limits of branching particle systems. Namely, suppose we have a large number of small Brownian particles, each of which has a short life range and produces a random number of offspring when it dies. We assume that the mass of the particles as well as the lifetime goes to zero, but the total mass distribution at the starting points converges to a certain limit. Then, under certain assumptions, the mass distribution of the particles at time t converges to a measure-valued stochastic process: super-Brownian motion as conceived by Donald A. Dawson and Shinzo Watanabe (originally, it was constructed analytically, not as the limit described above).

Dynkin began with the construction of such a process. Naturally, the original process could be a general right-continuous strong Markov process, the number of offspring may have a distribution that depends on the location of the particle, and so on. This way, Dynkin arrived at a general class of measure-valued processes that could be obtained as limits of the branching particle systems described above.[5]

If the original process is a Brownian motion, then the corresponding measure-valued process is called a super-Brownian motion. Super-Brownian motion is related to a nonlinear equation $\Delta u = \psi(u)$ where the nonlinear term $\psi(u)$ is related to branching. The family of possible functions $\psi(u)$ includes, in particular, the family $\psi(u) = u^\alpha$, $1 < \alpha \leq 2$. For instance, a solution to the equation $\Delta u = u^\alpha$ in a domain D with given boundary conditions can be written in terms of the corresponding super-Brownian motion.

Dynkin started with a characterization of so-called polar sets. For a stochastic process, a set is called polar if the process hits it with a probability of zero. For a Brownian motion, polar sets can also be characterized as sets of certain capacity zero, as well as so-called removable singularities for the equation $\Delta u = 0$. Dynkin, jointly with Sergei Kuznetsov, established a similar result for a super-Brownian motion in a smooth domain. This result was also extended to subsets on the boundary of the domain (for a Brownian motion, this is not a question).

Next, Dynkin switched to characterization of the class of all non-negative solutions to the equation $\Delta u = u^\alpha$ in a smooth domain; his results were summarized in two monographs.[6,7] The first result in this direction was obtained by Jean-François Le Gall in 1995; he studied non-negative solutions to the equation $\Delta u = u^2$ in a planar domain and established a one-to-one correspondence between such solutions and pairs (ν, Γ), where Γ is a closed subset of the boundary where the solution explodes at a certain rate, and ν is a σ-finite measure on the rest of the boundary that serves as a boundary value for the solution. The tools used by Le Gall (so called

Brownian snake) could not be extended to the case α < 2 or higher dimensions, however.

Dynkin, in partial collaboration with Kuznetsov, found a general answer to the problem. To begin, they introduced a concept of moderate and σ-moderate solutions. A solution is called moderate if it is dominated by a harmonic function. For a moderate solution, its boundary value is a finite measure that does not charge polar sets. Next, a solution is σ-moderate if it can be represented as a limit of a monotone-increasing sequence of moderate solutions. σ-moderate solutions can be uniquely characterized by their fine trace: a pair (ν, Γ) where the set Γ is closed with respect to certain topology related to the process (fine topology), and the σ-finite measure ν on the rest of the boundary does not charge polar sets. Moreover, corresponding solutions can be expressed in terms of the corresponding super-Brownian motion. Finally, Dynkin proved that all non-negative solutions are σ-moderate (for α = 2, this was first established by Benoit Mselati; Dynkin proved this for all 1 < α ≤ 2). Similar analytic results, applicable to α > 2 as well, were established by Moshe Marcus and Laurent Véron a few years later.

In 2000, a large number of Dynkin's publications, both in algebra and in probability, were collected in the volume *Selected Papers of E. B. Dynkin with Commentary.*[8] As Dynkin said in his foreword,

> A few times I was lucky to find a new approach which simplified an important theory. One of them is related to the celebrated Campbell-Hausdorff theorem claiming that the formal series $\log(e^x e^y)$ can be expressed in terms of commutators. In 1947 I found a simple explicit expression: it is sufficient to replace all multiplications by commutators and then to divide each monomial by its degree. …
>
> Some of my results demanded rather lengthy computations but most exciting was to find from time to time a simple new connection between apparently unrelated phenomena. For instance, the existence of certain kind of sufficient statistics for a convex cone C of probability measures implies that every element of C can be decomposed, in a unique way, into extremal elements.

Other than doing research, Dynkin was very interested in mentoring. He started that in 1943 in Perm, where he organized a small seminar for a few students from Moscow that happened to be in Perm during the war. Back in Moscow, he organized a seminar for high-school students. The seminar was devoted to various mathematical problems that can be formulated in an elementary way (later, he published a book, jointly with Vladimir Uspenskii, based on the topics discussed there).[9] For many years, he led a research seminar at Moscow University; for a large number of mathematicians, their first publications were related to this seminar. A number of Cornell students got their Ph.D. under his supervision as well; however, as a rule, they did not maintain any long-term relationship with Dynkin.

In 1963, Dynkin, together with his graduate and undergraduate students, organized an "Evening Math School," weekly seminars for mathematically gifted high-school students. In 1964, he organized a section in a specialized high school for mathematically gifted students and taught some calculus and linear algebra there (his graduate and post-graduate students did the seminar work). Based on this, he eventually published two more books, jointly with Stanislav A. Molchanov, Alexander L. Rosental, and Alexei K. Tolpygo.[10,11]

Dynkin was honored by many organizations during his life. He was awarded the Prize of the Moscow Mathematical Society (1951) and the Leroy P. Steele Prize for Total Mathematical Work from the American Mathematical Society (1993). He was named a Fellow of the Institute of Mathematical Statistics (1962), the American Academy of Arts and Sciences (1978), and the American Mathematical Society (2012), a member of the National Academy of Sciences (1985), and an honorary member of the Moscow Mathematical Society (1995). He was also was awarded honorary degrees from the Pierre and Marie Curie University (1997), the University of Warwick (2003), and the Independent Moscow University (2003).

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