NATIONAL ACADEMY OF SCIENCES

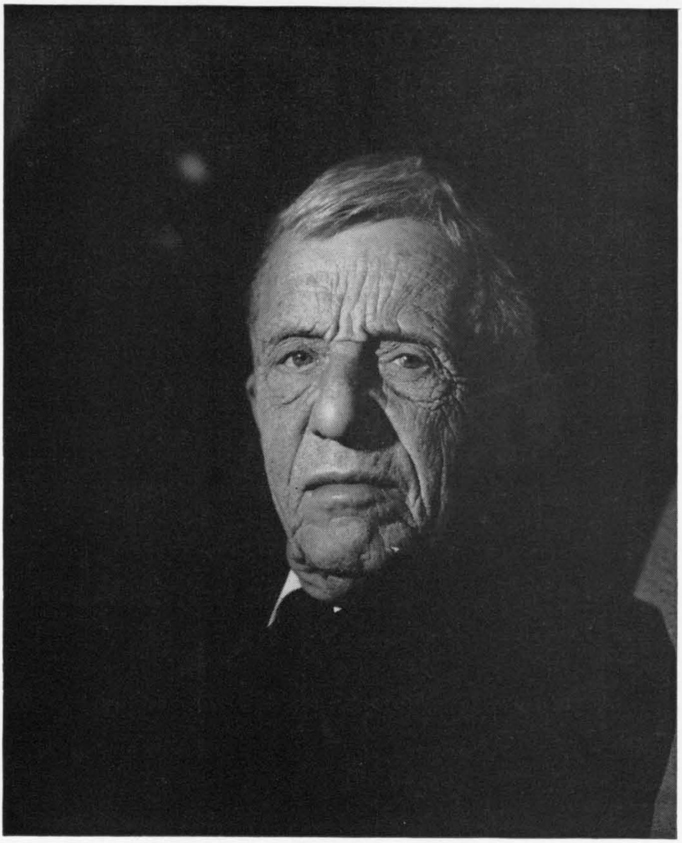
LUTHER PFAHLER EISENHART

1876—1965

A Biographical Memoir by

SOLOMON LEFSCHETZ

Biographical Memoir



L. P. Eisenhart

LUTHER PFAHLER EISENHART

January 13, 1876–October 28, 1965

BY SOLOMON LEFSCHETZ

LUTHER PFAHLER EISENHART was born in York, Pennsylvania, to an old York family. He entered Gettysburg College in 1892. In his mid-junior and senior years he was the only student in mathematics, so his professor replaced classroom work by individual study of books plus reports. He graduated in 1896 and taught one year in the preparatory school of the college. He entered the Johns Hopkins Graduate School in 1897 and obtained his doctorate in 1900. In the fall of the latter year he began his career at Princeton as instructor in mathematics, remaining at Princeton up to his retirement in 1945. In 1905 he was appointed by Woodrow Wilson (no doubt on Dean Henry B. Fine's suggestion) as one of the distinguished preceptors with the rank of assistant professor. This was in accordance with the preceptorial plan which Wilson introduced to raise the educational tempo of Princeton. There followed a full professorship in 1909 and deanship of the Faculty in 1925, which was combined with the chairmanship of the Department of Mathematics as successor to Dean Fine in January 1930. In 1933 Eisenhart took over the deanship of the Graduate School. For several years Eisenhart was also executive officer of the American Philosophical Society. Eisenhart was survived by his second wife, the former Katharine Riely Schmidt of York, Pennsylvania (his first wife, the former Anna Maria Dandridge Mitchell, died in 1913), by a son Churchill (by his first wife) of the National Bureau of Standards, by two daughters of the present Mrs. Eisenhart, and by six grandchildren. The following highly interesting and characteristically modest statement, left by Eisenhart, provides more complete information about his life:

"My parents, who were residents of York, Pennsylvania, had six sons, I being number two, born January 13, 1876. The only other survivor of the group is Martin Herbert, Chairman of the Board of the Bausch-Lomb Optical Company of Rochester.

"My father, after being a student at the York County Academy, taught in a country school until, acting upon the slogan by Horace Greeley—'Go West young man'—he went to Marshall, Michigan, and worked in a store. At the same time, he was apprentice to a local dentist. Being very expert with his hands, he soon acquired competence in the technique of dentistry. In due time he returned to York, set up a dentist office, and was married. He made sufficient income to meet the expenses of his growing family, but his intellect was too active to be satisfied by dentistry. Electricity appealed to him and he organized the Edison Electric Light Company in the early eighties. The telephone also made an appeal. He experimented with telephones and in the late nineties organized the York Telephone Company. Both of these companies are very active today.

"Mothers with such a large group of growing children would have felt that to provide for their well-being was enough. Not so with my mother. Not only did she do that to our satisfaction but she took part in our education before we went to school and helped in our 'home work.' In my case the result was that at the age of six years and five months I entered school and passed through the lower grades in three years instead of the normal six.

"We boys had a good time together. Each had his job to do in the family life, and when this was done we played baseball.

"While we were growing up my father was Secretary of the Sunday School of St. Paul's Lutheran Church and we all attended regularly. Also to an extent the social life of the family centered in the Church.

"Early in my junior year in the York High School the Principal, whom I recall as having been a very good teacher of mathematics, advised my father to have me withdraw and devote full time to the study of Latin and Greek, which I did in preparation for admission to Gettysburg College the next September (1892).

"In June 1893 I was awarded the freshman prize for excellence, greatly to my surprise, and the next year the prize in mathematics. By the middle of junior year, I was the only member of my class desiring to continue the study of mathematics. The professor, Henry B. Nixon, gave me books to study and report when I had any questions; there were no class sessions. The same plan on a more extensive basis was followed in senior year. This experience with the value of independent study led to my proposal in 1922 that the undergraduate curriculum for juniors and seniors in Princeton should provide for independent study, which was adopted and has continued.

"The next year, 1896, I taught mathematics in the preparatory school of the College and in October 1897 began graduate study at the Johns Hopkins. Professor Thomas Craig aroused my interest in differential geometry by his lecture and my readings in Darboux's treatises. Toward the close of 1900 I wrote a thesis in this field on a subject of my own choosing and in June the degree of Doctor of Philosophy was granted.

"In September 1900 my Princeton career began as Instructor in Mathematics, together with research and publication in the field of differential geometry. Five years later Woodrow Wilson instituted the Preceptorial System in Princeton with fifty Preceptors with the rank of Assistant Professor, to which group I was appointed. Four years later I was appointed Professor of

Mathematics. During 1925-1933 I was Dean of the Faculty and during 1933-1945 Dean of the Graduate School. Upon the death in 1928 of Henry Burchard Fine, who had been chairman of the department since 1902, I became Chairman and as such served until my retirement in 1945. Throughout the years I taught undergraduates and graduate students and conducted research in differential geometry, the results of which were published in books and journals."

EISENHART THE MAN

He was par excellence a family man and found in his family a great source of happiness and strength. Eisenhart was essentially a most modest man. The intimate atmosphere which surrounded him, its very serenity, was due in large measure to the care and devotion which he received from Mrs. Eisenhart. The Dean, as we all called him, did not seem to realize that he was an outstanding leader both in his field and in higher education. For outside his family he had two "loves": differential geometry (as research and study) and education. More about this below.

I remember my first association with Eisenhart at the 1911 summer meeting of the American Mathematical Society, then in its early years and still quite small. The meeting was held at Vassar College in an ordinary classroom, with hardly thirty participants. The usual dinner was a matter of perhaps fifteen participants. All those present were very young but included most of the coming leaders, among them Eisenhart. My close association with him dates from 1924 when I joined the Princeton faculty. One had already the firm impression of a most tolerant man, of the "live and let live" type, which I always found him to be.

EISENHART THE SCIENTIST

A true realization of what it meant to be a research scientist in the United States of 1900 requires a historical perspective rare among the members of the younger generation. Our scientific advance is so rapid and so all-absorbing that time is simply not available for the necessary (and wholesome) glance backwards.

When Eisenhart began his scientific career we had only a handful of scientific centers worthy of note and perhaps half a dozen creative mathematicians. Only those with tremendous energy, stamina, and scientific devotion such as Eisenhart had in abundance forged in time their way to the front. Isolation was the rule. There were very few avid graduate and postgraduate students from whom to draw inspiration and fervor. Nothing resembling the wave of very capable undergraduates was in sight, and indeed Eisenhart never saw any sign of it. His very few mathematical colleagues were occupied with their own work, none of it in Eisenhart's direction. We are all aware of the general effect of this "splendid isolation."

Still another observation must be made. By modern standards the faculties of the day were all tiny, so that each faculty member had to carry a very full teaching load, much of which was of doubtful inspiration. This is what Eisenhart had to face, and did face unflinchingly, for many years.

Eisenhart's scientific devotion, stimulated by Thomas Craig of Hopkins, turned early and remained forever directed toward differential geometry.

In differential geometry Eisenhart appears in the direct line marked by Gauss, Riemann, and Eisenhart's immediate predecessors, Gaston Darboux and Luigi Bianchi. All the major differential geometers of the time up to and including Eisenhart lacked the mathematical panoply provided much later by the intense modern development of algebraic topology. Therefore, they came too early to attack the fundamental problem of differential geometry: to determine the exact nature *in the large* of a manifold from the knowledge of its differential properties *in the small,* that is, its local differential properties. It may be said in parenthesis that even at this writing (1966) the problem has not been solved except in very special cases.

To return to Eisenhart, his life work was the study of the local differential properties of a manifold. As happens in many branches of mathematics, most of the problems that came up, and were accessible, were solved in time, leaving wide open the truly difficult core; as a consequence, research in the subject was discouraged, so that today its investigation is not assiduously pursued.

Eisenhart was certainly one of the most prolific investigators of the subject, as the large number of papers and books that he devoted to it shows. A chronological perusal of his books underscores his evolution, as well as that of differential geometry as a consequence of the intense intellectual excitement caused by the advent of relativity.

His first book, *A Treatise on the Differential Geometry of Curves and Surfaces* (Ginn and Company, 474 pp., 1909; 2d ed., Dover Publications, Inc., 1960), went deservedly through several printings. The treatment is gradual, thorough, and complete. It is one of the first high-grade mathematical books printed in the United States. It is excellent didactically and was well within reach of beginning graduate students of the time. The first chapter, about one tenth of the book, deals with the differential theory of curves in three-space. Although strictly introductory, it brings forward as early as possible some of the processes occurring later. The representation is parametric:

$$x_i = f_i(t), \qquad i = 1, 2, 3$$

where the f_i are continuous in some range and differentiable as far as may be required, or for that matter even analytical. Tangents, osculating planes, curvature, and torsion are carefully described. The first chapter is an all-important preparation to the rest of the book. The proper topic of the book begins with the second chapter. Surfaces are considered as subsets of Euclidean three-space R^3 given by a parametric representation

$$S\colon x_i = f_i(u, v), \qquad i = 1, 2, 3$$

in terms of which all the important characters are defined.

The successive chapter headings give a good idea of the scope of the work: I. Curves in space (= three-space). II. Curvilinear coordinates on a surface. Envelopes. III. Linear elements on a surface. Differential parameters. Conformal representation. IV. Geometry of a surface in the neighborhood of a point. V. Fundamental equations. The moving trihedral. VI. Systems of curves. Geodesics. VII. Quadrics. Ruled surfaces. Minimal surfaces. VIII. Surfaces of constant total curvature. *W* surfaces. Surfaces with plane or spherical lines of curvature. IX. Deformation of surfaces. X. Deformation of surfaces. The method of Weingarten.

The last two chapters cover the topic which particularly interested Eisenhart and on which his thesis was based.

Eisenhart's next book, *Transformation of Surfaces* (379 pp., 1923), appeared fourteen years later (first edition, Princeton University Press; second edition, largely a reproduction, Chelsea Publishing Co., 1962). This is deeply a research book, the last one from the prerelativity era by our author. It still deals with surfaces in R^3. Briefly speaking, most of the transformations that appeared in the preceding twenty-five years belonged to two types. They are systematically studied in the book, the aim being to bring some order in the related investigations. (The delay in the appearance of this book was due to World War I.)

We come now to the explosion caused in differential geometry by the appearance of Einstein's theory of general relativity. From the pre-Einsteinian, that is Riemannian, standpoint the universe as a three-dimensional manifold governed by a positive definite form:

$$ds^2 = \Sigma\, g_{ij}\, du_i\, du_j; \qquad i, j = 1, 2, 3.$$

Special relativity, and especially the 1918 model—general relativity—upset these Newtonian notions, replacing the above ds^2 by one reducible to the type

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

with light lines of zero length among other things. It so happened that the suitable mechanism for the new approach was already in existence, having been developed by Ricci as early as 1900. The day of tensor calculus, of the study of non-Riemannian geometry, had arrived, and Eisenhart was thoroughly prepared for it. This innovation affected all his later work and again is clearly evidenced in his later books. There are four of them and we shall say a few words about each.

Riemannian Geometry (first edition, Princeton University Press, 251 pp. [index and bibliography not included], 1926; second edition, 1949, 306 pp., 35 pages of new appendices, a most extensive historical bibliography going back to Riemann's fundamental paper of 1854). This is a treatment of the general manifold possessing a form

$$ds^2 = \epsilon \, \Sigma g_{jk} \, dx^j \, dx^k.$$

The important assumption made is that the sum is not necessarily positive definite, and $\epsilon = \pm 1$ is to guarantee $ds^2 > 0$ wherever the sum is <0. This is suggested primarily by the ds^2 of relativity. Without going into details let me note that the well-known Levi-Civita parallelism is introduced early (in Chapter II, the first chapter being devoted to an introduction to tensor analysis which is used throughout the book). The extension to non-Riemannian manifolds is reserved.

Non-Riemannian Geometry (American Mathematical Society Colloquium Publications, viii + 184 pp., 1927). This volume deals basically with manifolds dominated by the geometry of paths developed by Eisenhart and Veblen, as well as being closely related to one due to Èlie Cartan. In Riemannian geometry parallelism is determined geometrically by this property: along a geodesic, vectors are parallel if they make the same angle with the tangents. In non-Riemannian geometry the Levi-Civita parallelism imposed *a priori* is replaced by a determination by arbitrary functions (affine connections). The main consequences of the deviation are investigated in the volume.

Continuous Groups of Transformations (Princeton University Press, 301 pp., 1933; Dover reprint, 1961). This represents a large excursion into a closely related but independent field, going back almost a century to Sophus Lie. Tensor calculus and recent Riemannian geometry are systematically applied in this major field.

An Introduction to Differential Geometry, with Use of the Tensor Calculus (Princeton Mathematical Series, 304 pp., 1940). As the title clearly indicates, this volume applies as many of the modern developments as possible, above all tensor calculus, to a number of the topics fully developed in the 1909 treatise of the author.

The last volume, which appeared five years before the author's retirement, underscores as strongly as possible his deep attachment to mathematical research and to the teaching of mathematics at the advanced level. During this very time Eisenhart was also fully occupied by his arduous administrative duties as Chairman of the Department of Mathematics and Dean of the Princton Graduate School.

EISENHART THE EDUCATOR

In education as in much else, Eisenhart was and remained a man of his time, an American of the beginning of the century, a citizen of a nation so fabulously rich that it could afford all sorts of luxuries, including Eisenhart's educational credo that education was something not merely for the gifted but also for the middle intellectual level. In less well-endowed nations economically—Europe by and large—this credo was untenable. Curiously Europe is in the process of veering in our direction while we may be moving in the opposite direction.

As it happens, Eisenhart's very moderate educational point of view enabled him to put through an important educational reform where more impetuous characters would certainly have failed. I refer to the famous "four-course plan" adopted by Princeton in the early twenties. It is of interest to dwell for a moment upon this event.

Doubtless as a consequence of the post-World War I era, unrest had developed at Princeton in regard to the undergraduate educational process in general. A committee was appointed to see what could be done. The report came back with the suggestion, promoted by Eisenhart, that a *four-course plan of study* be adopted for the junior and senior years. This was done; its operation began in 1923 and continues to this day. The general idea was to concentrate a student's work on his major field. This was accomplished by replacing the earlier five-course scheme by a four-course one, two of the courses to be in the student's major field. This went along with independent reading in a subject of the student's choice within his field and the writing of a serious terminal thesis with some research flavor. At the present time the level of students and their theses has risen exceptionally high, with resulting augmented work for the faculty! If I am not mistaken, this or an analogous scheme has been adopted in many places, but the originator of it is Eisenhart.

CONCLUSION

It is evident from all that has been said that Eisenhart was endowed with enormous energy: besides his considerable teaching load, he carried for years a full load as university administrator and as research scientist. During his deanship of the

Graduate School he was the *de facto* adviser to President Dodd on all scientific matters and was also chairman of the famous Research Committee of Princeton University—this at a time, 1930-1945, when there were very few such serious committees in the country.

HONORS AND DISTINCTIONS

HONORS

Sc.D., Gettysburg College, 1921; Columbia University, 1931; University of Pennsylvania, 1933; Lehigh University, 1935; Princeton University, 1952

L.L.D., Gettysburg College, 1926; Duke University, 1940; Johns Hopkins University, 1953

Decorated Officer of the Order of the Crown of Belgium, 1937

Luther Pfahler Eisenhart Arch, Princeton University, named by the Board of Trustees of Princeton University, 1951

PROFESSIONAL SOCIETIES

American Association for the Advancement of Science (Vice President and Chairman of Section A, Mathematics, 1916-1917)

American Mathematical Society (Vice President, 1913-1914; President, 1931-1932)

American Philosophical Society (Executive Officer, 1942-1959)

Association of American Colleges (President, 1930)

National Academy of Sciences, 1922 (Chairman, Section of Mathematics, 1931-1934; Vice President, 1945-1949)

National Research Council (Chairman, NRC Division of Physical Sciences, 1937-1946; Chairman, Advisory Committee on Scientific Publications, a joint committee with the NAS, 1940-1947)

Phi Beta Kappa

EDITORSHIPS

The Annals of Mathematics, Editor, 1911-1925

Transactions of the American Mathematical Society, Editor, 1917-1923

BIBLIOGRAPHY

KEY TO ABBREVIATIONS

Accad. Lincei. Rendiconti = Rendiconti della Reale Accademia dei Lincei
Am. J. Math. = American Journal of Mathematics
Annali di Mat. = Annali di Matematica Pura ed Applicata
Ann. Math. = Annals of Mathematics
Bull. Am. Math. Soc. = Bulletin of the American Mathematical Society
Circolo Mat. di Palermo. Rendiconti = Circolo Matematico di Palermo. Rendiconti
Phys. Rev. = Physical Review
Proc. Nat. Acad. Sci. = Proceedings of the National Academy of Sciences
Trans. Am. Math. Soc. = Transactions of the American Mathematical Society

1901

A demonstration of the impossibility of a triply asymptotic system of surfaces. Bull. Am. Math. Soc., 7:184-86.

Possible triply asymptotic systems of surfaces. Bull. Am. Math. Soc., 7:303-5.

Surfaces whose first and second forms are respectively the second and first forms of another surface. Bull. Am. Math. Soc., 7:417-23.

1902

Lines of length zero on surfaces. Bull. Am. Math. Soc., 9:241-43.

Note on isotropic congruences. Bull. Am. Math. Soc., 9:301-3

Infinitesimal deformation of surfaces (thesis). Am. J. Math., 24: 173-204.

Conjugate rectilinear congruences. Trans. Am. Math. Soc., 3:354-71.

1903

Infinitesimal deformation of the skew helicoid. Bull. Am. Math. Soc., 9:148-52.

Surfaces referred to their lines of length zero. Bull. Am. Math. Soc., 9:242-45.

Isothermal-conjugate systems of lines on surfaces. Am. J. Math., 25:213-48.

Surfaces whose lines of curvature in one system are represented on the sphere by great circles. Am. J. Math., 25:349-64.

Surfaces of constant mean curvature. Am. J. Math., 25:383-96.
Congruences of curves. Trans. Am. Math. Soc., 4:470-88.

1904

Congruences of tangents to a surface and derived congruences. Am. J. Math., 26:180-208.
Three particular systems of lines on a surface. Trans. Am. Math. Soc., 5:421-37.

1905

Surfaces with the same spherical representation of their lines of curvature as pseudospherical surfaces. Am. J. Math., 27:113-72.
On the deformation of surfaces of translation. Bull. Am. Math. Soc., 11:486-94.
Surfaces of constant curvature and their transformations. Trans. Am. Math. Soc., 6:473-85.
Surfaces analogous to the surfaces of Bianchi. Annali di Mat.(3), 12:113-43.

1906

Certain surfaces with plane or spherical lines of curvature. Am. J. Math., 28:47-70.
Associate surfaces. Mathematische Annalen, 62:504-38.

1907

Transformations of minimal surfaces. Annali di Mat.(3), 13:249-62.
Applicable surfaces with asymptotic lines of one surface corresponding to a conjugate system of another. Trans. Am. Math. Soc., 8:113-34.
Certain triply orthogonal systems of surfaces. Am. J. Math., 29:168-212.

1908

Surfaces with isothermal representation of their lines of curvature and their transformations. I. Trans. Am. Math. Soc., 9:149-77.
Surfaces with the same spherical representation of their lines of curvature as spherical surfaces. Am. J. Math., 30:19-42.

1909

A Treatise on the Differential Geometry of Curves and Surfaces. Boston, Ginn and Company. xi + 474 pp. (2d ed., Dover Publications, Inc., 1960, 474 pp.)

1910

The twelve surfaces of Darboux and the transformation of Moutard. Am. J. Math., 32:17-36.

Congruences of the elliptic type. Trans. Am. Math. Soc., 11:351-70.

Surfaces with isothermal representation of their lines of curvature and their transformations. II. Trans. Am. Math. Soc., 11:475-86.

1911

A fundamental parametric representation of space curves. Ann. Math.(2), 13:17-35.

1912

Sopra le deformazioni continue delle superficie reali applicabili sul paraboloide a parametro puramento immaginario. Accad. Lincei. Rendiconti, 211 (5):458-62.

Ruled surfaces with isotropic generators. Circolo Mat. di Palermo. Rendiconti, 34:29-40.

Minimal surfaces in Euclidean four-space. Am. J. Math., 34:215-36.

1913

Certain continuous deformations of surfaces applicable to the quadrics. Trans. Am. Math. Soc., 14:365-402.

1914

Transformations of surfaces of Guichard and surfaces applicable to quadrics. Annali di Mat.(3), 22:191-248.

Transformations of surfaces of Voss. Trans. Am. Math. Soc., 15:245-65.

Transformations of conjugate systems with equal point invariants. Trans. Am. Math. Soc., 15:397-430.

1915

Conjugate systems with equal tangential invariants and the transformation of Moutard. Circolo Mat. di Palermo. Rendiconti, 39:153-76.

Transformations of surfaces Ω. Proc. Nat. Acad. Sci., 1:62-65.

One-parameter families of curves. Am. J. Math., 37:179-91.

Transformations of conjugate systems with equal invariants. Proc. Nat. Acad. Sci., 1:290-95.

Surfaces Ω and their transformations. Trans. Am. Math. Soc., 16:275-310.

Sulle superficie di rotolamento e le trasformazioni di Ribaucour. Accad. Lincei. Rendiconti 242, (5):349-52.

Surfaces with isothermal representation of their lines of curvature as envelopes of rolling. Ann. Math.(2), 17:63-71.

1916

Transformations of surfaces Ω. Trans. Am. Math. Soc., 17:53-99.

Deformations of transformations of Ribaucour. Proc. Nat. Acad. Sci., 2:173-77.

Conjugate systems with equal point invariants. Ann. Math.(2), 18:7-17.

Surfaces generated by the motion of an invariable curve whose points describe straight lines. Circolo Mat. di Palermo. Rendiconti, 41:94-102.

Deformable transformations of Ribaucour. Trans. Am. Math. Soc., 17:437-58.

1917

Certain surfaces of Voss and surfaces associated with them. Circolo Mat. di Palermo. Rendiconti, 42:145-66.

Transformations *T* of conjugate systems of curves on a surface. Trans. Am. Math. Soc., 18:97-124.

Triads of transformations of conjugate systems of curves. Proc. Nat. Acad. Sci., 3:453-57.

Conjugate planar nets with equal invariants. Ann. Math.(2), 18:221-25.

Transformations of applicable conjugate nets of curves on surfaces. Proc. Nat. Acad. Sci., 3:637-40.

1918

Darboux's contribution to geometry. Bull. Am. Math. Soc., 24:227-37.

Surfaces which can be generated in more than one way by the motion of an invariable curve. Ann. Math.(2), 19:217-30.

Transformations of planar nets. Am. J. Math., 40:127-44.

Transformations of applicable conjugate nets of curves on surfaces. Trans. Am. Math. Soc., 19:167-85.

1919

Triply conjugate systems with equal point invariants. Ann. Math.(2), 20:262-73.

Transformations of surfaces applicable to a quadric. Trans. Am. Math. Soc., 20:323-38.

Transformations of cyclic systems of circles. Proc. Nat. Acad. Sci., 5:555-57.

1920

The permanent gravitational field in the Einstein theory. Ann. Math.(2), 22:86-94.

Sulle congruenze di sfere di Ribaucour che ammetteno una deformazione finita. Accad. Lincei. Rendiconti, 292 (5):31-33.

Conjugate systems of curves *R* and their transformations. Comptes Rendus, 1st International Congress of Mathematicians, Strasbourg, pp. 407-9.

Darboux's anteil an der geometrie. Acta Mathematica, 42:275-84.

1921

Transformations of surfaces applicable to a quadric. Journal de Mathematiques, 4:37-66.

Conjugate nets *R* and their transformations. Ann. Math.(2), 22:161-81.

The Einstein solar field. Bull. Am. Math. Soc., 27:432-34.

Sulle trasformazioni *T* dei sistemi tripli coniugati di superficie. Accad. Lincei. Rendiconti, 302 (5):399-401.

Einstein static fields admitting a group G_2 of continuous transformations into themselves. Proc. Nat. Acad. Sci., 7:328-34.

1922

With O. Veblen. The Riemann geometry and its generalization. Proc. Nat. Acad. Sci., 8:19-23.

Ricci's principal directions for a Riemann space and the Einstein theory. Proc. Nat. Acad. Sci., 8:24-26.

The Einstein equations for the solar field from the Newtonian point of view. Science, n.s., 55:570-72.

Fields of parallel vectors in the geometry of paths. Proc. Nat. Acad. Sci., 8:207-12.

Spaces with corresponding paths. Proc. Nat. Acad. Sci., 8:233-38.

Condition that a tensor be the curl of a vector. Bull. Am. Math. Soc., 28:425-27.

1923

Affine geometries of paths possessing an invariant integral. Proc. Nat. Acad. Sci., 9:4-7.

Another interpretation of the fundamental guage-vector of Weyl's theory of relativity. Proc. Nat. Acad. Sci., 9:175-78.

Orthogonal systems of hypersurfaces in a general Riemann space. Trans. Am. Math. Soc., 25:259-80.

Symmetric tensors of the second order whose first covariant derivatives are zero. Trans. Am. Math. Soc., 25:297-306.

Einstein and Soldner. Science, n.s., 58:516.

The geometry of paths and general relativity. Ann. Math.(2), 24:367-92.

Transformation of Surfaces. Princeton, Princeton University Press. ix + 379 pp. (2d ed., Chelsea Publishing Company, 1962, 379 pp.)

1924

Space-time continua of perfect fluids in general relativity. Trans. Am. Math. Soc., 26:205-20.

Geometries of paths for which the equations of the paths add a quadratic first integral. Trans. Am. Math. Soc., 26:378-84.

1925

Linear connections of a space which are determined by simply transitive continuous groups. Proc. Nat. Acad. Sci., 11:246-50.

Fields of parallel vectors in a Riemannian geometry. Trans. Am. Math. Soc., 27:563-73.

1926

Einstein's recent theory of gravitation and electricity. Proc. Nat. Acad. Sci., 12:125-29.

Riemannian Geometry. Princeton, Princeton University Press. vii + 262 pp.

Geometries of paths for which the equations of the path admit n(n + 1)/2 independent linear first integrals. Trans. Am. Math. Soc., 28:330-38.

Congruences of parallelism of a field of vectors. Proc. Nat. Acad. Sci., 12:757-60.

1927

With M. S. Knebelman. Displacements in a geometry of paths which carry paths into paths. Proc. Nat. Acad. Sci., 13:38-42.

Non-Riemannian Geometry (Colloquium Publications, Vol. 8). New York, American Mathematical Society. viii + 184 pp.

1929

Affine geometry. In: *Encyclopaedia Britannica,* 14th ed., Vol. 1, p. 279.

Differential geometry. In: *Encyclopaedia Britannica,* 14th ed., Vol. 7, pp. 366-67.

Contact transformations. Ann. Math.(2), 30:211-49.

Dynamical trajectories and geodesics. Ann. Math.(2), 30:591-606.

1930

Projective normal coordinates. Proc. Nat. Acad. Sci., 16:731-40.

1932

Intransitive groups of motions. Proc. Nat. Acad. Sci., 18:195-202.

Equivalent continuous groups. Ann. Math.(2), 33:665-76.

1933

Spaces admitting complete absolute parallelism. Bull. Am. Math. Soc., 39:217-26.

Continuous Groups of Transformations. Princeton, Princeton University Press. ix + 301 pp.

1934

Separable systems in Euclidean 3-space. Phys. Rev., 45:427-28.

Separable systems of Stäckel. Ann. Math.(2), 35:284-305.

1935

Stäckel systems in conformal Euclidean space. Ann. Math.(2), 36:57-70.

Groups of motions and Ricci directions. Ann. Math.(2), 36:823-32.

1936

Simply transitive groups of motions. Monatshefte für Mathematik und Physik, 43:448-52.

With M. S. Knebelman. Invariant theory of homogeneous contact transformations. Ann. Math.(2), 37:747-65.

1937

Riemannian spaces of class greater than unity. Ann. Math.(2), 38:794-808.

1938

Fields of parallel vectors in Riemannian space. Ann. Math.(2), 39:316-21.

1939

Coordinate Geometry. Boston, Ginn and Company. 297 pp.

1940

An Introduction to Differential Geometry, with Use of the Tensor Calculus (Princeton Math. Series, No. 3). Princeton, Princeton University Press. x + 304 pp. (Translated into Serbo-Croatian.)

1948

Enumeration of potentials for which one-particle Schroedinger equations are separable. Phys. Rev., 74:87-89.

Finsler spaces derived from Riemann spaces by constant transformations. Ann. Math.(2), 49:227-54.

1949

Separation of the variables in the one-particle Schroedinger equation in 3-space. Proc. Nat. Acad. Sci., 35:412-18.
Separation of the variables of the two-particle wave equation. Proc. Nat. Acad. Sci., 35:490-94.

1950

Homogeneous contact transformations. Proc. Nat. Acad. Sci., 36: 25-30.

1951

Generalized Riemann spaces. Proc. Nat. Acad. Sci., 37:311-15.

1952

Generalized Riemann spaces. II. Proc. Nat. Acad. Sci., 38:506-8.

1953

Generalized Riemann spaces and general relativity. Proc. Nat. Acad. Sci., 39:546-50.

1954

Generalized Riemann spaces and general relativity. II. Proc. Nat. Acad. Sci., 40:463-66.

1956

A unified theory of general relativity of gravitation and electromagnetism. Proc. Nat. Acad. Sci., 42:249-51.
A unified theory of general relativity of gravitation and electromagnetism. II. Proc. Nat. Acad. Sci., 42:646-50.
A unified theory of general relativity of gravitation and electromagnetism. III. Proc. Nat. Acad. Sci., 42:878-81.

1957

A unified theory of general relativity of gravitation and electromagnetism. IV. Proc. Nat. Acad. Sci., 43:333-36.

1958

Spaces for which the Ricci scalar R is equal to zero. Proc. Nat. Acad. Sci., 44:695-98.

1959

Spaces for which the Ricci scalar R is equal to zero. II. Proc. Nat. Acad. Sci., 45:226-29.

Generalized spaces of general relativity. Proc. Nat. Acad. Sci., 45:1759-62.

1960

The cosmology problem in general relativity. Ann. Math., 71:384-91.

The paths of rays of light in general relativity. Proc. Nat. Acad. Sci., 46:1093-97.

Fields of unit vectors in the four-space of general relativity. Proc. Nat. Acad. Sci., 46:1589-1601.

Generalized spaces of general relativity. II. Proc. Nat. Acad. Sci., 46:1602-4.

Spaces which admit fields of normal null vectors. Proc. Nat. Acad. Sci., 46:1605-8.

1961

The paths of rays of light in general relativity of the nonsymmetric field V_4. Proc. Nat. Acad. Sci., 47:1822-23.

1962

Spaces in which the geodesics are minimal curves. Proc. Nat. Acad. Sci., 48:22.

The paths of rays of light in generalized general relativity of the non-symmetric field V_4. Proc. Nat. Acad. Sci., 48:773-75.

1963

Generalized Riemannian geometry. II. Proc. Nat. Acad. Sci., 49:18-19.

The Einstein generalized Riemannian geometry. Proc. Nat. Acad. Sci., 50:190-93.