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HARISH-CHANDRA
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A Biographical Memoir by

ROGER HOWE

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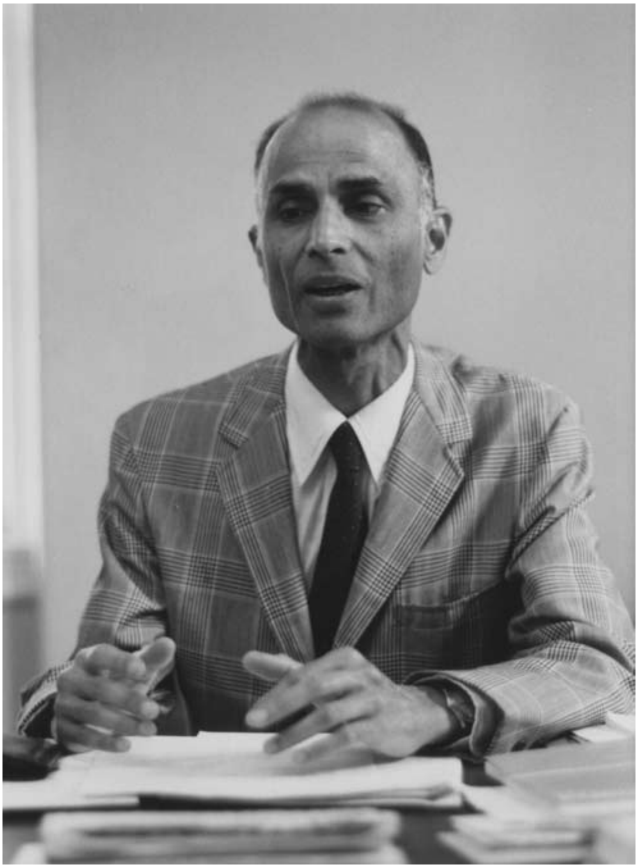

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Harish-Chandra

HARISH-CHANDRA

October 11, 1923–October 12, 1983

BY ROGER HOWE

He taught them the Kshatria code of honor:
that a warrior may never refuse a challenge….
The Five Sons of Pandu,
The Story of the Mahabharata
Retold by Elizabeth Seeger

HARISH-CHANDRA WAS, if not the exclusive architect, certainly the chief engineer of harmonic analysis on semisimple Lie groups. This subject, with roots deep in mathematical physics and analysis, is a synthesis of Fourier analysis, special functions and invariant theory, and it has become a basic tool in analytic number theory, via the theory of automorphic forms. It essentially did not exist before World War II, but in very large part because of the labors of Harish-Chandra, it became one of the major mathematical edifices of the second half of the twentieth century.

Harish-Chandra was born in 1923 in Uttar Pradesh, in northern India. His family belonged to the Kshatria (warrior) caste. Kshatria traditionally were rulers, landowners, and military leaders, and more recently have commonly been businessmen or civil servants. Harish-Chandra's father, Chandrakishore, was a civil engineer who monitored and maintained the dikes and irrigation canals that sustain agriculture on the North Indian plains. One of Harish-Chandra's two brothers was a successful businessman, the other a high civil servant. His sister's husband was an admiral. For someone of this background to choose an academic career was exceptional.

Harish-Chandra's childhood home was a large family compound maintained by his mother's father, Ram Sanehi Seth. The Seths had long been a family of distinction in the region. Ram Sanehi Seth himself had not been born into wealth, but made his fortune as an energetic and skillful lawyer. Harish-Chandra's mother, Chandrarani (maiden name, Satyawati Seth), seems to have imparted her father's energy to all her children.

About the name Harish-Chandra: Indian names do not necessarily follow the Western two-part pattern of given name, family name. A person may often have only one name, and this was the case with Harish-Chandra, who in his youth was called Harishchandra. The hyphen was bestowed on him by the copy editor of his first scientific papers, and he kept it. Later he adopted "Chandra" as a family name for his daughters. Given names in India are often those of gods or ancient heroes, and "Harishchandra" was a king, legendary for his truthfulness already at the time of the Mahabharata. I once saw an Indian comic book whose cover featured "Harishchandra—whose name is synonymous with truth."

Harish-Chandra was conspicuously precocious and successful in school. He graduated from Christ Church High School at 14, from intermediate college in Kanpur at 16, received the B.Sc. from the University of Allahabad at 18, and the M.Sc. at 20. On his M.Sc. exam in physics he placed first in the state of Uttar Pradesh, with a perfect written examination, which earned him gold medals. Besides his academic studies he was tutored at home in painting and music. His interest in painting lasted throughout his life, and his family retains some of his copies of old master paintings.

Along with his academic gifts Harish-Chandra's intense, high-strung personality was visible early. (George Mackey once likened Harish-Chandra to a thoroughbred horse, with all the attendant virtues and difficulties.) He had nightmares before examinations, and a long string of illnesses, including measles, paratyphoid, and malaria. His nerves sometimes made the turbulence usual to life in an extended family difficult to tolerate.

In the library at Allahabad, Harish-Chandra happened upon his future in the form of a book, *Principles of Quantum Mechanics* by P. A. M. Dirac. Its lucid style and powerful ideas inspired Harish-Chandra to become a physicist, the first step toward his life's work. Eventually, he went to England to study under Dirac himself, who pointed him towards representation theory.

Harish-Chandra's professor, K. S. Krishnan, and his M.Sc. examiner, C. V. Raman, vigorously encouraged him to pursue physics. They arranged a place for him at the Indian Institute of Science in Bangalore, in southern India (at that time the only research institute in India). He went to Bangalore in 1943 to study with H. J. Bhabha. His first papers, a short paper joint with Bhabha and a longer one by him alone, appeared in the 1944 *Proceedings of the Royal Society.*

These papers served to introduce Harish-Chandra to Dirac. They dealt with one of Dirac's research interests, the theory of point particles, and because of the slow wartime mails, Bhabha asked Dirac as a special favor to correct the proofs for the papers. Shortly after, Dirac accepted Harish-Chandra as a student. He sailed for Cambridge in the summer of 1945. Hiroshima was bombed and World War II ended while he was under way. On the boat Harish-Chandra's anticolonialist views made him reluctant to mingle with the English.

However, he was deeply impressed by English civilization as embodied in Cambridge, and he enthusiastically adopted English customs. In later years he wondered whether his English breakfasts had aggravated his heart problems.

When Harish-Chandra arrived in Cambridge, Dirac had only two other students, a Ceylonese (Sri Lankan) and a Brazilian. Even so, Harish-Chandra seldom met with Dirac. He found that Dirac's lectures almost repeated his book, and stopped attending. He visited Dirac privately only once a term or so, to avoid being a bother to him. He later described Dirac as "gentle and kind, yet rather aloof and distant." He did attend the weekly seminar that Dirac ran, and also "discovered the exciting world of mathematics" in lectures of J. E. Littlewood and Philip Hall. He began to be bothered by the divergent integrals of quantum electrodynamics, and he was attracted by the security offered by the certainty of pure mathematics.

As a thesis problem Dirac suggested that Harish-Chandra compute the irreducible unitary representations of the Lorentz group. Harish-Chandra's paper on the Lorentz group, which earned him his Ph.D. from Cambridge, appeared in the *Proceedings of the Royal Society* in 1947. With closely related papers of Gelfand and Naimark in Russia and V. Bargmann in the United States, this work initiated the era of systematic study of the representation theory of semisimple groups, a study that continues vigorously today.

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TECHNICAL INTERLUDE: ABOUT LIE GROUPS AND REPRESENTATION THEORY

Although representation theory is a much more widely known subject now than it was when Harish-Chandra began his work, it is still not quite the household word that, say, complex analysis or vector space are; so a few words of description may be in order. I will also comment briefly on the semisimple Lie groups, Harish-Chandra's main object of study. The reader may wish to refer to this section only as seems required by the main narrative.

A. LIE GROUPS

Lie groups are continuous groups, such as the groups of rotations of two or three dimensional Euclidean space, or the group of all invertible linear transformations of n-dimensional space. The group $\mathbb{R}$ of real numbers under addition is a basic example of a Lie group, and a fundamental result says that Lie groups may be thought of as coherent collections of many copies of $\mathbb{R}$. Lie groups stand in contrast to discrete groups, such as the integers, or finite groups.

Lie groups may be studied by the methods of calculus. Just as we can think of differentiation as an infinitesimal version of translation on $\mathbb{R}$, there is a collection of infinitesimal operators attached to every Lie group. In nineteenth-century parlance these operators were called the infinitesimal group. Today they are known as the *Lie algebra*. If the Lie group acts on a manifold, the Lie algebra appears as first order differential operators, obtained by differentiating along copies of $\mathbb{R}$ contained in the group.

Simple Lie groups are the analogs for Lie groups of the finite simple groups. In fact, there are deep connections between them. The simple Lie groups are the ones that seem to be naturally connected with geometry: They occur as the groups that preserve distances or some analogous geometric structure.

B. REPRESENTATION THEORY

Representation theory may be thought of as an extension to noncommutative systems of Fourier analysis, or from a somewhat more abstract viewpoint, as a generalization of spectral theory. Spectral theory describes how a single linear operator can act on a vector space. Taking the finite dimensional case as paradigm (and ignoring complications such as generalized eigenvectors), we can say that spectral theory shows how to resolve general vectors into combinations of eigenvectors. Or more abstractly, single-operator spectral theory decomposes the whole vector space into a sum of minimal subspaces invariant by the operator. The minimal invariant subspaces are of course the eigenlines, the lines spanned by the eigenvectors. How the operator acts on a given eigenline is described by the associated eigenvalue.

Representation theory aspires to do the same, but for a system of operators forming a group rather than for a single operator. The main new feature is that these operators need not commute with each other. The problems addressed by representation theory thus fall into two general types: (1) irreducible representations and (2) harmonic analysis.

1) IRREDUCIBLE REPRESENTATIONS

a) Idea of an irreducible representation; classification problem.

If a group of operators on a vector space have no (proper) invariant subspace, the space is said to be *irreducible* for action of the group. It is also called an *irreducible representation* for the group. Irreducible representations are analogous to the eigenlines for a single operator. They form the building blocks, in an appropriate precise technical sense, of all representations. However, when G is noncommutative, irreducible representations need not be one-dimensional. Indeed, when G is a noncompact semisimple Lie group, an irreducible representation is typically infinite dimensional. In addition, the ways that G can act irreducibly on a space are not described by a single number, such as eigenvalue, and the description of the possibilities for irreducible representations is a challenging problem in its own right. This is the *classification problem.*

b) Characters and Classification.

Remarkably, there is a direct generalization of eigenvalues that describes irreducible representations of finite non-Abelian groups. Again, it involves a slightly unusual way of looking at the ingredients of standard spectral theory. We observe that if v is an eigenvector for an operator L, then the eigenvalue of v can be described as the trace of L acting on the one-dimensional subspace $\mathbb{C}$v. It turns out that if G is a finite group acting on a vector space V, then the function $\chi_v(g)$ = trace(g) on G turns out to determine uniquely the representation of G on V. The function χ_v is known as the *character* of the action of G on V.

Although noncompact Lie groups typically have irreducible representations that are infinite dimensional, it turns out that in many cases, including semisimple groups, one can again define the character of a representation. (This was done first by Harish-Chandra for semisimple groups and then extended to broader classes.)

For Lie groups the character of a representation is no longer a function but a potentially much more wildly behaved object, a distribution. Even for finite groups, associating characters to representations is not a solution of the classification problem, since characters themselves can be quite difficult to determine. It is merely a translation of the problem. Still, it is very convenient to have a concrete object on the group that completely characterizes an irreducible representation. For semisimple groups, Harish-Chandra used characters with decisive effectiveness.

2) HARMONIC ANALYSIS

a) General Description:

The second problem of representation theory is, given a representation of G on the vector space V, to decompose vectors in V into sums of vectors that belong to irreducible subspaces for G. Examples include Fourier analysis, where one decomposes a function of a real variable x into sums of complex exponential functions $e^{2\pi itx}$. From the point of view of group theory the reason that the exponential functions arethe right functions to use is that the functional equation $e^{2\pi i(s+t)x} = e^{2\pi isx}e^{2\pi itx}$ satisfied by the exponential functions can be interpreted as saying that they are the eigenfunctions for the group of translations of the real line. Precisely, we define the operator of translation by a number s by $T_s(f)(x)=f(x-s)$, for a function f on $\mathbb{R}$. Then the functional equation can be viewed as saying that $T_s(e^{2\pi itx}) = e^{-2\pi ts}e^{2\pi itx}$. If the function is periodic (with period 2π), then the complex exponential functions involved are also periodic, which means that t in the exponent must be an integer. This gives Fourier series, which are sums in the usual sense. If the function is not periodic, one needs to let t be an arbitrary real number, so the "sum" is the Fourier integral.

In Fourier analysis, since the group involved is just the additive group of real numbers, which is commutative, the irreducible representations are all one-dimensional, so we are not so far from standard spectral theory. The classical theory of spherical harmonics involves a noncommutative group, the group of rotations in three dimensions. This theory shows how to decompose a function on the unit sphere in $\mathbb{R}^3$ as a linear combination of the standard spherical harmonics P^m_ℓ . Here ℓ is a non-negative integer, and m is an integer with $|m| \leq \ell$. The P^m_ℓ are essentially uniquely defined by their symmetry properties. In particular, the $(2\ell +1)$-dimensional space of functions on the unit sphere spanned by the functions P^m_ℓ for fixed ℓ, is invariant and irreducible under the group of rotations.

From this example the general problem of decomposing a representation into its irreducible subrepresentations is called *harmonic analysis*. Cases like the periodic functions, or the functions on a sphere, when one can decompose a general function into a series of functions transforming by irreducible representations, are said to exhibit *discrete spectrum*. Cases like the nonperiodic functions of a real variable, in which one needs to integrate over a continuous family of eigenfunctions in order to reproduce an arbitrary function, are said to exhibit *continuous spectrum*.

b) Plancherel Formula:

One obvious set for a group to act on is itself, by group multiplication (on the left or on the right). If we take a reasonable space of functions on the group, like L^2, the space of square integrable functions, then translation by g in G also defines an operator on this space, so we get a representation of G. This is called the *regular representation* of G. The regular representation for a semisimple Lie group G was Harish-Chandra's main object of study. The goal is to decompose the regular representation explicitly into a direct integral of irreducible representations. For G = R this decomposition amounts to the Plancherel theorem:

$$\int_{-\infty}^{\infty} |f(t)|^2 \, dt = \int_{-\infty}^{\infty} |\hat{f}(s)|^2 \, ds$$

where $\hat{f}$ indicates the Fourier transform of a function f. In allusion to this case the general problem is called the *Plancherel formula*.

It turns out that the set $\hat{G}$ of all irreducible unitary representations has a structure as a reasonable topological space. (Although it is not usually Hausdorff, the set of points where it fails to be Hausdorff is relatively small.) Further, there is a notion of Fourier transform taking a function f on G to an (operator-valued) function on $\hat{G}$, and there is a natural measure on $\hat{G}$ such that the analog of the Plancherel formula holds. This is known abstractly for very general groups G. Harish-Chandra's chosen problem was to explicitly describe the Plancherel measure. Implicit in this was the need to describe $\hat{G}$ well enough to do this: to describe at least a subset of $\hat{G}$ that supports the Plancherel measure. Unlike classical cases, which exhibited either discrete spectrum exclusively or continuous spectrum exclusively, the regular representation of a semisimple Lie group can have both types of spectrum at once. One of Harish-Chandra's guiding principles was what he later formulated as the "Philosophy of Cusp Forms," which for the Plancherel formula effectively meant that the crucial case was the discrete spectrum, from which the continuous spectrum could be constructed by well-established techniques.

3) ENVELOPING ALGEBRA

We have said that the Lie algebra of a Lie group is like a first derivative approximation to the group. Continuing this analogy, the universal enveloping algebra of a Lie group can be thought of as a Taylor series approximation to the group. If a group is acting on a manifold, then the Lie algebra is realized as first order differential operators. If we apply the operators of the Lie algebra one after another, we will get higher order differential operators. The linear span of all operators obtained by multiplying elements of the Lie algebra in any order is called the *enveloping algebra* of the Lie algebra. Actually, the detailed structure of the algebra of operators generated by the Lie algebra may depend on the specific realization of the Lie algebra but there is a largest such algebra, which is referred to as the *universal enveloping algebra*. For a semisimple Lie group it turns out that the center—the set of elements that commute with all operators in the enveloping algebra—has a very nice structure that can be described quite explicitly. This was done by Harish-Chandra in his first major paper and served as a key tool throughout his long pursuit of the Plancherel formula.

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Later in 1947, after Harish-Chandra had submitted his thesis, Dirac traveled to the United States to visit the Institute for Advanced Study in Princeton, New Jersey, accompanied by Harish-Chandra as his assistant. Harish-Chandra stayed two years at the institute and in this time he turned from physics to mathematics. He once remarked on the transition: "In Princeton I learned that not every function is analytic, and after that I couldn't be a physicist anymore." In particular, he learned that from a mathematical point of view his paper on the Lorentz group was less satisfactory than those of Bargmann and Gelfand-Naimark. It shared a common feature of essentially mathematical papers written by physicists: overly formal treatment or dismissal of technically delicate points. When he lamented to Dirac on the mathematical shortcomings of his paper, Dirac responded, "I am not interested in proofs, but only in what nature does." This exchange increased Harish-Chandra's feeling that he lacked the sixth sense required for success in physics, and he switched to mathematics soon after.

It was a hard decision. He had already stretched tradition by choosing a career in science. To change fields after several years of effort might be taken for failure or lack of seriousness. In retrospect, we can be confident that he chose correctly, and applaud the strength of character that carried him through. Despite his personal choice, he retained throughout his life a deep respect for physics and physicists, and especially for Dirac.

In Princeton Harish-Chandra began what became lifelong friendships with several other young mathematicians, including G. D. Mostow and I. E. Segal. Two anecdotes from that period—one from Mostow and one from Segal—give glimpses of Harish-Chandra's mathematical personality.

C. Chevalley was lecturing on Lie groups at Princeton University. Harish-Chandra and Mostow both attended and walked together from the institute to the university for the lectures and back afterward. Chevalley normally gave polished lectures, but one day he got confused. He turned to the board to draw a small diagram, hiding it with his body and erasing it before turning again to the audience. He announced, "My assertion is certainly correct, but I don't see at the moment how to prove it." He promised the proof for the next lecture. On the walk back Harish-Chandra mused, "How can one know a mathematical statement is true without actually knowing how to prove it?"

Segal suggested that Harish-Chandra read A. Weil's book *L'integration sur les groupes topologiques et ses applications.* At that time Harish-Chandra's mathematical background was quite limited. Yet meeting Segal two or three weeks later, Harish-Chandra remarked that he had read the book and found it interesting but that it had several errors. One of the mistakes was in the proof of Pontrjagin duality. Harish-Chandra communicated the errors to Weil, who corrected them in later editions.

Harish-Chandra's transition to mathematics caused no lengthy gap in his scientific output. His last physics papers were published in 1948, and his first mathematical papers appeared in 1949. From the outset his mathematics dealt with Lie theory. He apparently grasped immediately the fruitfulness of the universal enveloping algebra as a means of understanding Lie algebras. He seems to have spent about two years gaining a deep understanding of universal enveloping algebras. This early work culminated in his 1951 paper in *Transactions of the American Mathematical Society*. This is in many ways characteristic of his papers: It is long (69 pages), it is relentless (8 theorems, 51 lemmas), it gives full details, it considers only the general case and gives no examples, it makes heavy use of mathematical induction as a method of proof, and it contains results of lasting importance, including a general construction of a semisimple Lie algebra from first principles without any case-by-case computations. The treatment (which was inspired and established independently by Chevalley, who strongly influenced Harish-Chandra in this early period) has become standard. It contains the essential aspects of J. P. Serre's presentation of semisimple Lie algebras in terms of generators and relations, which in turn opened the way to construction of general Kac-Moody Lie algebras and more recently to quantum groups. The paper also establishes the Harish-Chandra isomorphism, which gives a natural description of the center of the universal enveloping algebra as a polynomial ring. This isomorphism and its variants have been ubiquitous tools in representation theory. Foreshadowing his main work, the last sections make a preliminary foray into infinite-dimensional representation theory. This part was apparently inspired by F. Mautner. It is directed toward the question of whether semisimple Lie groups were of Type I in the sense of the theory of von Neumann algebras. Type-I-ness is a regularity condition on the representation theory of a group, guaranteeing that its representations will be amenable to harmonic analysis in a conventional sense. The impact of this paper can be appreciated by reading the five-column review given to it by Godement in *Mathematical Reviews*. It won the Cole Prize of the American Mathematical Society. George Mackey referred to it as "Harish's big breakthrough." It can be taken as the end of his mathematical apprenticeship and his debut as a master.

In this paper and throughout his work on semisimple groups, mathematical induction was Harish-Chandra's hallmark method of proof. In the study of Lie groups there are many quantities with respect to which one may argue by induction: dimensions, ranks, degrees, and so on. Harish-Chandra took advantage of all of them. Indeed, it was part of his technical genius to be able to use induction in places where it was not obvious that it could be applied.

He sensed the power of induction at the very outset of his mathematical career. On one of his walks with Mostow in 1948 he pronounced, "You know, the best way to prove theorems is by induction." Many years later he offered the following simile: "Induction is like high finance: If you don't borrow enough, you have cash flow problems; if you borrow too much, you can't pay the interest." Induction was ideally adapted to Harish-Chandra's mathematical style. Although he dealt with complex, intricately structured objects, and always argued in terms of the general case, his mathematics was basically very concrete. Induction allowed him to take a complex situation and isolate the essential features, which in intricate combinations produced the range of phenomena he was studying. Many of his proofs are constructive, essentially algorithms for determining the quantity of interest. The inductive step then constitutes the basic "Do-loop" in the algorithm.

Harish-Chandra spent 1949-1950 at Harvard on a Jewett Fellowship. He went there ostensibly to learn algebraic geometry from Oskar Zariski. He never published research on algebraic geometry, but he did gain some sophisticated understanding of the subject (see Langlands [1985] for an anecdote about Harish-Chandra's knowledge of number theory). Apparently, however, the lure of Lie groups was too strong. A letter to Segal in late 1949 remarks that an encounter with Felix Mautner had revived Harish-Chandra's interest in representation theory. He must have spent a good deal of the time thinking about enveloping algebras.

At the end of the year the International Congress of Mathematicians was held in Cambridge. It gave Harish-Chandra the chance finally to meet André Weil. Each had already influenced the other mathematically, and their mutual influence continued throughout their careers. Their friendship became especially strong after they became colleagues at the Institute for Advanced Study. During Harish-Chandra's last illness, Weil was one of his most faithful visitors (see Langlands [1985] for several nice anecdotes from Weil).

Harish-Chandra extended his stay in Cambridge into the fall of 1950. At this time he got to know George Mackey. Although they had quite different perspectives on representation theory, their friendship was warm and long-lasting, and later broadened into a cordial relationship between their families. Talking with Mackey provided great comfort to Harish-Chandra at crucial periods in his life.

In 1950 Harish-Chandra accepted a position at Columbia University, where he stayed from the spring of 1951 until 1963. During these years, he laid the foundations for semisimple representation theory, fixed on the Plancherel formula as his main goal, and traveled the harder part of the way toward it. At the very end of the Columbia period he announced the regularity theorem for invariant eigendistributions and the construction of the discrete series. These two results are generally cited as the deepest parts of his work, and are the cornerstones of his Plancherel formula. These were years of intense labor, darkened in the middle by profound frustration, but ending in shining success.

The Columbia position was Harish-Chandra's only teaching appointment. As an instructor he was not an overwhelming success. He was almost never assigned undergraduate classes unless his graduate course had been canceled for lack of students. Even with graduate students he had a reputation as a tough lecturer. When he was in danger of having zero enrollment his colleague Richard Kadison would round up an audience for him. One of these attendees thanked Kadison years after, calling the course a formative experience. Harish-Chandra also found grading papers oppressive and sometimes nearly impossible, and once when Harish-Chandra was in a particularly tight situation, Kadison helped with that. However, hints of Harish-Chandra's research prowess seem somehow to have circulated at Columbia. A 1962 issue of the Columbia alumni magazine touted him as "possibly the finest mathematician that India has produced since Srinivasa Ramanujan."

Harish-Chandra's first papers in representation theory are titled simply "Representations of Semisimple Lie Groups n," where n = 1, 2, 3, … In fact, these titles are appropriate. The papers contain foundational results basic to his later work and to the subject as a whole. In a characteristic pattern Harish-Chandra announced his results in a series of notes in the *Proceedings of the National Academy of Sciences* and then followed with detailed arguments over a period of years. The first paper contains a technical result that allowed him to apply algebraic results effectively to study even infinite-dimensional representations. In particular, this result provided after-the-fact justification for the results in his thesis.

The next few papers give an improving sequence of finiteness results, culminating in the celebrated "subquotient theorem." This remarkable result says that an arbitrary abstract irreducible representation of a semisimple Lie group can be found inside one of a particular family of explicitly constructed representations. These representations had been introduced by Gelfand and Naimark who called them the *principal series*. Since the principal series themselves are constructed quite concretely, and since they have been intensively studied, one might hope that the subquotient theorem more or less solves the problems of semisimple representation theory: All one has to do is describe the constituents of a given principal series. However, this has proved to be a remarkably thorny problem, and the principal series still harbor mysteries. Some of the most important advances in the subject following Harish-Chandra have amounted to improvements in understanding of the structure of principal series. In particular, in 1974 R. P. Langlands refined later work of Harish-Chandra to give what is in some sense a classification of irreducible representations, in terms of how they sit inside principal series representations.

The subquotient theorem established the Type-I-ness of semisimple Lie groups in a very strong sense. More important for Harish-Chandra's future work, it set the stage for extending the notion of character of a representation from finite groups to semisimple Lie groups. The definition of character for semisimple Lie groups is rather technical, and involves distribution theory. However, characters provided the basis for Harish-Chandra's construction of the discrete series, which in turn were the key to his Plancherel formula (as well as to later developments, such as the Langlands classification. Indeed, characters defined in the same technical way have been an important tool in representation theory of general Lie groups.

Crucial to all these early results was Harish-Chandra's control over the universal enveloping algebra through its center. The center of the enveloping algebra was his enchanted sword throughout his long campaign. When he later abandoned his original algebraic tactics and brought to bear the weapons of analysis, the center appeared in a more concrete form, as a system of differential operators controlling the behavior of the objects he sought to master. One could say the Harish-Chandra's main discovery was that although general harmonic analysis appears to be about integral operators, the deepest facts about representations of semisimple Lie groups are controlled by differential operators.

Among the early announcements are descriptions of the Plancherel theorem for a special class of semisimple groups (the complex groups) and for $SL(2,\mathbb{R})$, the smallest noncompact semisimple group.

For a complex group G, $L^2(G)$ is completely decomposed by the principal series of representations, parameterized by a continuous parameter. However, for $SL(2,\mathbb{R})$ there are discrete summands as well as a continuous spectrum. These have come to be known as the *discrete series.* The form of the Plancherel formula for $SL(2,\mathbb{R})$ led Harish-Chandra to write, "In fact, it seems likely that there is a close connection between classes of conjugate Cartan subgroups and the various 'series' of unitary representations which occur in the Plancherel formula."

The main problem was to produce the discrete series, because there was an easy inductive (in both the usual sense and a technical, representation theoretic sense) procedure to produce all the representation from the discrete series (of the group and of certain of its subgroups also). Harish-Chandra later christened this theme—in a more mature form influenced by I. M. Gelfand and R. P. Langlands—the *philosophy of cusp forms.* Thus, here very early in his investigations we see at least in nascent form five key themes in his decades-long project: (1) the center of the enveloping algebra; (2) characters; (3) the Plancherel formula; (4) a key tool formula termed the limit formula; and (5) the discrete series. These were later supplemented and bound together by his general analysis of orbital integrals (F_f) and his study of asymptotics of matrix coefficients (the constant term and "c-functions"), which filled the second half of the 1950s, by his stunning use of the Abelian Fourier transform, and by the creation of the Schwartz space, a lovely analog for semisimple Lie groups of the famous space of test functions on $\mathbb{R}^n$ introduced by Laurent Schwartz. But it is striking how much of the final picture was in place from the start.

It took several years to consolidate this initial burst of activity. In the meantime, Harish-Chandra married.

When he was first in Bangalore, regular dormitory space was not available, and Harish-Chandra lodged with a family, the Kales. Mrs. Kale had been his French teacher in Allahabad. She was a Polish Jew. As a girl she had pestered her parents until they let her go to Paris to study, where she met Mr. Kale, come from India to learn botany. In Bangalore Mr. Kale served as librarian for the Institute of Science. The Kales had a lively nine-year-old daughter, Lalitha, who enjoyed distracting their studious lodger. On the day the gold medals from his M.Sc. exams arrived in the mail Harish-Chandra apparently needed to show them to *someone*. He beckoned Lalitha to his room. Soon Mrs. Kale was proclaiming a dinner in his honor. Harish-Chandra professed to be very cross.

Harish-Chandra moved into regular rooms after about six months, but Mrs. Kale kept an open household, and he visited often. After he left India, he corresponded with the Kales, and apparently he remembered Lalitha, because in 1953 he returned to Bangalore to propose marriage. It was a whirlwind courtship, and the stress of it showed in one of Harish-Chandra's only episodes of public intoxication. (In later years at cocktail parties at the Institute for Advanced Study, his normal drink was water, and he could make one glass last all evening.) His proposal was terse and oblique—

Did Lalitha have a passport? “Because you’ll need one if you come back with me”—but it worked.

Within his family Harish-Chandra’s marriage added to his reputation for unconventionality. His family and the Kales were of very different backgrounds, and Harish insisted on a civil rather than a traditional wedding (thereby forgoing his chance to don the Seth family sword). Nevertheless, on the crucial matter—his bride—Harish had shown his characteristic judgment. Lily Harish-Chandra, as she has become known, was a partner of ideal skill and dedication. She had some college in India and during her early years in New York, she earned a B.A. in zoology. If born a generation later, she would very likely have had her own professional career, as both their daughters, Premala (a physicist) and Devaki (an economist) have done. However, after obtaining her degree, she devoted herself to the care of Harish-Chandra, and to their daughters as they were born (Premala in 1959, Devaki in 1963). Many who know the family agree that Harish could not have accomplished what he did without the strong, loving, and expert support furnished by Lily. Although it is not directly germane to mathematics, I cannot pass without commenting, as have many colleagues over the years, that they made a strikingly handsome couple.

In the mid-1950s Harish-Chandra’s fires were burning high, his forge white-hot. Volume II of his Collected Papers covers only the years 1955-1958. He worked up to 18 hours a day, day after day, at a small table next to the small kitchen of their small Columbia apartment. Sigurdur Helgason remembers visiting the Chandras in the late 1950s, and marveling at the cramped conditions in which Harish-Chandra created his amazing succession of papers. He would sing as he worked, and the tone gave clues to his progress.

After an initial attempt to construct discrete series representations explicitly met with only partial success, Harish-Chandra began to study characters in more depth. (The kind of explicit construction of representations that Harish-Chandra was seeking was finally achieved only in the 1980s, through the work of many researchers, and still only in an algebraic setting.) He showed that the characters of a semisimple Lie group, a priori highly singular objects, behaved quite well at most points (the regular points) on the group. He also established strong analogies between their behavior at these points and the characters of compact groups, described by the celebrated Weyl character formula. He already asks in (1955b) the crucial question whether his formulas at regular points actually determine the character.

At this time he hit on the idea of constructing discrete series characters via Fourier transforms of orbital integrals in the Lie algebra. He announced that this procedure works for compact groups in (1956e), thus anticipating by four years the celebrated orbit method developed in the context of nilpotent groups by A. A. Kirillov in 1960 and extended in the years following into an approach to representations of general Lie groups. His proof of the orbital version of the Weyl character formula in (1957b) is fascinating to read from the perspective of several decades of research. In it one can see not only the seeds of his own deepest work but also intimations of the two other major themes in representation theory over the next 30 years, the orbit method and the oscillator (or Segal-Shale-Weil) representation. The center of the universal enveloping algebra appears again in proving the restriction formula, which became his major weapon for dealing with questions of invariant harmonic analysis (e.g., characters, orbital integrals). For example, the simple form of characters at regular points is a consequence of the restriction formula.

In 1957-1958 he opened another front in his attack on the Plancherel formula with a study of spherical functions.

Spherical functions are the group theoretic context for understanding the theory of special functions. Using the differential equations coming from the center of the enveloping algebra, Harish-Chandra developed asymptotic expansions for spherical functions. Inspired by Weyl's work on the spectral theory of ordinary differential operators, he suggested that certain coefficients (the c-functions) appearing in the asymptotic expansion should give the Plancherel measure. Although this work seems very different in flavor from the immediately preceding work on invariant harmonic analysis, its descendant, the theory of the constant term, dovetails neatly with the results on orbital integrals and characters to produce the full argument for the Plancherel formula.

A short paper in 1957 proves what is usually called the *Bruhat decomposition* for a general semisimple Lie group. This describes the double coset decomposition with respect to the minimal parabolic subgroup. (For the general linear group GL_n, this is the subgroup of upper triangular matrices. The Bruhat decomposition for GL_n amounts to a refinement of the L-U decomposition of basic linear algebra, and is also the basis for the Schubert calculus in algebraic geometry.)

Harish-Chandra spent the year 1955-1956 at the Institute for Advanced Study, and 1957-1958 in Paris on a Guggenheim Fellowship. During the 1950s the theory of algebraic groups developed rapidly, and toward the end of the decade Harish-Chandra began to absorb the algebraic point of view. During the year in Paris, Harish-Chandra attended lectures by Weil on arithmetic groups (a natural family of discrete subgroups of Lie groups) and became interested in the question of finiteness of volume for G/Γ for a general arithmetic group Γ in a semisimple group G. This had been proved by C. L. Siegel for several classes of classical groups. Harish-Chandra continued thinking about the problem after returning to New York. He studied Siegel's proofs intensively, rewriting them

10 or more times. He finally saw how to prove the general result while out on a walk. In his elation he confided to Lily that he had done something that Poincaré would have liked to do.

During this time he continued to try to show that his formula for a character at regular points completely determines it. Because of the potentially singular nature of a distribution, this seemingly small extension presented formidable technical difficulties and caused him extreme frustration. In the late spring of 1960 he had a nervous breakdown. Throughout the summer he was depressed and could not work. He felt his world was collapsing. Drastic treatments, including shock therapy, were considered but fortunately were never carried through. The attention and support of Ellis and Kate Kolchin were of great comfort to Harish and Lily during this period.

While Harish-Chandra was recuperating, Armand Borel visited several times. Their conversations included Harish-Chandra's results on the finite volume of arithmetic quotient spaces. Borel wrote these up and incorporated them with other results in a joint paper. This is Harish-Chandra's only joint work in mathematics. Shortly before his crisis Harish-Chandra also had written two papers on differential equations, with a refined treatment of the asymptotic behavior of matrix coefficients. These remained unpublished until they were included in his Collected Works, edited by V. S. Varadarajan, and published by Springer-Verlag in 1984. However, R. P. Langlands knew of the results and used them to establish his classification of representations.

Gradually Harish-Chandra recovered. To prevent relapse Harish-Chandra's doctors prescribed regular work breaks in the form of summer vacations. Taking his relaxation as seriously as he did everything, Harish-Chandra every year spent long summers with his family in quiet resorts. They went many times to Eaglesmere, Pennsylvania, but they also vacationed in New England, in Oregon, and many places in between. Entertainment consisted mainly of reading, often communal reading of plays. After the girls were grown, Harish and Lily continued the routine by themselves.

Harish-Chandra resumed work with renewed vigor. The proof of regularity of characters now came smoothly. The construction of discrete series characters, essentially as Fourier transforms of orbital integrals, followed quickly. The relation between the condition that a representation should contribute to $L^2(G)$, and the asymptotic behavior of its matrix coefficients, probably learned from the spherical function studies, also fit neatly into the picture, and led to a notion of temperedness, and an associated Schwartz space. In announcing these results (1963) he described the proofs as "rather long." Indeed, they stretched over 7 papers occupying 359 journal pages.

From 1961 to 1963 Harish-Chandra was a Sloan fellow. He spent 1961-1962 at the Institute for Advanced Study. On this visit he met R. P. Langlands, the colleague with whom he had perhaps his deepest mathematical interaction. Of Harish-Chandra at that time, Langlands later wrote (1985): "The carriage was erect and aristocratic. The magnificent features…were austere and chiseled, the outward expression of an almost unbearable inner intensity. But the flashing smile and the sparkling laugh, often triumphant, sometimes mischievous, were more frequent than they later became."

In 1963 Harish-Chandra returned to the Institute for Advanced Study, this time as a permanent member. Although members have no fixed duties, Harish-Chandra felt best when visibly contributing to scholarship, and he lectured regularly, on Tuesdays from 10 to 12 unless prevented by poor health. He nearly always spoke on his latest work. Weil described the process (by way of explaining why he stopped attending) as "Harish reading from his next paper." However, as the 1960s progressed and his work gained a following, his audience grew. Usually the first few lectures had a large attendance of people seeking an overview, but then the numbers would dwindle to a hard core. Faithful attendees developed a strong esprit de corps and followed a modest ritual, beginning with the lecture in the morning, with conversation in the break between hours, and followed by lunch at the institute cafeteria (without the master, who went home), usually a long lunch with discussion of the morning's material, a broader review of the current lecture series and Harish-Chandra's opus, and of course normal mathematical banter. I was lucky enough to enjoy this routine in 1971-1972. Paul Sally was there that year and several other years. Nolan Wallach came faithfully from Rutgers for many years. These lectures provided Harish-Chandra with a forum to expose his ideas when they were fresh, and in slightly less concentrated form than in his papers. His audiences over the years absorbed the ideas and helped spread them to the larger mathematical community. In 1967-1968 the lectures were on Langlands's theory of Eisenstein series. Notes were taken by J. M. G. Mars and published as Springer-Verlag Lecture Notes. Notes from his 1969-1970 lectures on p-adic groups were written by G. van Dijk and published similarly, and his 1971-1973 lectures were written up by Allen Silberger and published as Princeton Mathematical Notes. Garth Warner was a visitor for two years in the late 1960s and produced a two-volume Springer Grundlehren treatment of Harish-Chandra's work. What most mathematicians accomplish through students Harish-Chandra did with these lectures.

In the late 1960s, influenced by Langlands and perhaps also by Weil and probably confident of the Plancherel formula, Harish-Chandra branched out. He had already thought about automorphic forms in the 1950s, and published a basic finiteness theorem for them in 1959. In the meantime, the algebraic group point of view continued to spread, and in particular the formalism of adeles, and the exegesis of the structure underlying the arithmetic properties of automorphic forms, especially Hecke operators and L-functions, had been understood. From this viewpoint the real field $\mathbb{R}$ takes it place beside $\mathbb{Q}_p$, the p-adic numbers, as just one of the infinitely many completions of the rational numbers $\mathbb{Q}$, and all these completions figure equally in the structure of automorphic forms.

Accordingly, along with the representations of semi-simple Lie groups, one should equally study the representations of reductive groups with coefficients in the p-adic numbers for any prime number p. Harish-Chandra took up this challenge, and from 1968 until his death he spent a substantial portion of his efforts on p-adic groups. The theory he built strongly resembled his theory for real groups. He formulated the parallel explicitly, calling it the Lefschetz principle, an echo of his early training in algebraic geometry. The parallel did not arise simply from an effort to mimic the already established real theory for p-adic groups; cross-fertilization enriched and clarified both theories and brought them closer. The theory of automorphic forms, in which they were both united, also suggested ways of thinking about representation theory of groups over real or p-adic fields. Thus, in Harish-Chandra's final formulation of the Plancherel formula for real groups, he writes of the Eisenstein integral and the Maass-Selberg relations, recalling terms for automorphic forms. Both these terms reflect his philosophy of cusp forms, which he attributed to Gelfand. This philosophy focuses attention on the cuspidal representations, the analog in this context of discrete series, and produces all other automorphic forms from the cusp forms. This philosophy is as relevant to p-adic groups as it had been to Lie groups, and in (1969b) he showed it could also be used to organize the representations of groups over finite fields. In 1972 at the summer symposium of the American Mathematical Society in Williamstown, Massachusetts, Harish-Chandra delivered a series of lectures. (His vacation spot that summer was conveniently near the conference site.) After explaining the Lefschetz principle, he told a story of a conversation between God and the Devil shortly before creation. The Devil offered to take the job off God's hands, and God thanked him kindly and agreed. The Devil asked whether God had any specific instructions. God handed the Devil a short list and said, "Here are a few things I want to be sure are done right. For the rest, you can suit yourself." Harish then expressed hope that the Lefschetz principle was on God's short list. He used this story several times to highlight various aspects of his mathematical philosophy.

In 1966 Harish-Chandra delivered a plenary lecture at the International Congress of Mathematicians in Moscow. There he met and was warmly received by I. M. Gelfand, the central figure in the Russian school of representation theory. In 1968 the institute named him IBM-von Neumann Professor of Mathematics. In 1969 he gave the colloquium lectures at the summer meeting of the American Mathematical Society in Eugene, Oregon. It was in these lectures that he first announced the Plancherel formula for semisimple Lie groups. In 1973 he was elected a fellow of the Royal Society. The Indian Mathematical Society awarded him the Srinivasa Ramanujan Medal in 1974, and he was elected a fellow of the Indian National Science Academy and the Indian Academy of Sciences in 1975.

In 1969, at age 46 and as trim as he was in his twenties, Harish-Chandra had a heart attack. A second one came in 1970. Thereafter his regimen included not only summer rest but also daily walks. His diet was severely restricted, and it took all of Lily's skill to provide meals that were satisfactory from both health and aesthetic viewpoints. With her attentive care, however, Harish enjoyed more than another decade of productive work. Walking presented no problems; he had always enjoyed it. The Institute woods and the Delaware-Raritan Canal to the north of Princeton provided pleasant venues. He and Lily would sometimes walk for hours along Lake Carnegie, which bounds Princeton on the east, and further along the canal, for roundtrips of 10 or more miles. Sometimes he might walk with a colleague and discuss mathematics. Paul Sally remembers some of his walks with Harish-Chandra as the best mathematics lessons in his life.

In the summer and fall of 1970 a recuperating Harish-Chandra took his only sabbatical leave from the Institute, taking his family to the Institut des Hautes Etudes Scientifiques in Bures-sur-Yvette, outside Paris. George Mackey was also there with his family, and the two families enjoyed each other's presence and joint activities.

Returning to Princeton in 1971, Harish-Chandra was back on the job. To us in his seminar that year he looked healthy, although extremely thin (Peter Trombi quipped that Harish "had to take three steps before his pants started to move")—far from the stereotypical heart attack patient. As many others have, I found him extremely generous with his time, always willing to talk about mathematics, to explain his current or older work, to answer questions or compare ideas.

Although he worked steadily during the 1970s, it was not with the same intensity as before. He finished the Plancherel formula for semisimple Lie groups, reworking and synthesizing in three long and beautiful papers the lessons culled from his years of hard struggle. On p-adic groups, however, he published few complete papers. Most of his papers on p-adic groups are only announcements or summaries of results. As

I mentioned above, the results summarized in his Williamstown lectures were written in detail by Allen Silberger. Full accounts of some of his later results still have not appeared. He announced the Plancherel formula for p-adic groups in (1977b). However, it is only a relative Plancherel formula, in the sense that it does not explicitly describe the discrete series. For p-adic groups the discrete series are bound up with the number theory of the base field and still present mysteries. Fortunately for Harish-Chandra's program they were not needed to carry out the analysis of the continuous spectrum required for his formulation of the Plancherel formula.

In 1980 Harish-Chandra took U.S. citizenship. At the next opportunity, in 1981, he was elected to the National Academy of Sciences. In that year he also received an honorary degree from Yale University, as the same ceremony in which his daughter Premala graduated, with a major in physics.

A third heart attack came in 1982. It seems clearly to have been brought on by intense work. In pursuing the discrete series for p-adic groups Harish-Chandra considered the spectral analysis of a certain space of functions on G (the Whittaker functions). He had worked out what he considered to be a satisfactory theory, and had agreed to lecture on it in Paris and in Toronto, at the summer meeting of the American Mathematical Society. Then he found an error. Working furiously, he fixed it, but then he was stricken. During a visit from Paul Sally in the hospital, Harish-Chandra complained, "I don't understand why this happened. It worked out!" V. S. Varadarajan delivered the Toronto lecture on Harish-Chandra's behalf.

He never really recovered. His treatment lowered his energy level and his capacity to work, which he found frustrating. He had a custom of not shaving while ill, and he developed a distinguished beard.

A fourth attack came early in 1983 while he was out walking. He would not have survived it except for an alert neighbor who saw him fall and had him rescued. Thereafter he mostly stayed home and kept largely to bed. He was isolated from the life of the institute except for the news that André Weil brought on his regular visits.

A group of admiring colleagues led by Paul Sally and V. S. Varadarajan decided to organize a conference to mark Harish-Chandra's sixtieth birthday. Armand Borel, Harish's longtime colleague, was also turning 60, and conferences were organized for both. Since Borel was the elder by six months, his conference was scheduled first, October 13-15, which was in fact the week of Harish's birthday.

At Borel's conference I was delighted to find Harish, handsome in his pepper-and-salt beard, sparkling and sociable. He and Lily had planned a party for Borel on Sunday. After the last lecture on Saturday, we lingered on the terrace outside the lecture hall. We discussed recent developments in representation theory, Harish's intuitions about harmonic analysis, his early days, the institute then and now, mathematicians past and present, a rich blend of mathematical shoptalk. A circle of young mathematicians gathered in the slanting afternoon sun, listening to Harish reminisce, asking questions to draw out more detail. The shadows lengthened and the air grew cool. The crowd thinned, until just Harish, Paul Sally, and I were left. We went into Fuld Hall. As the conversation wound down, Paul and I were astonished to hear Harish-Chandra doubting the value of his work, wondering if it would last. We stumbled over each other to remind him of his many fundamental results. Without seeming very convinced he stopped protesting and took his leave. Fuld Hall suddenly seemed chilly to me. I stood and shivered, to think that even Harish-Chandra, who had carved a broad path deep into a forest where many famous mathematicians had gotten tangled in thickets at the borders, whose results supplied the toolkits of some of the best mathematicians of a new generation, could so doubt his achievements. The poignancy of his doubt remains fresh—a reminder of the fragility of all our efforts.

Harish-Chandra died the next day. After the party for Borel, a large one at which Harish was a gracious and animated host, he went out for his daily walk, and collapsed under a final attack. His ashes were scattered in Princeton and in the Ganges at Allahabad. The conference planned for his sixtieth birthday was held as a memorial, April 23-27, 1984, with reminiscences by V. S. Varadarajan, G. D. Mostow, S. Helgason, and R. P. Langlands. Varadarajan was trained in statistics but had changed fields after meeting Harish at Columbia in 1960. He became Harish's close friend and most ardent disciple, and edited his Collected Papers, published in four volumes by Springer-Verlag. In October 1993 a bronze bust of Harish-Chandra was unveiled at a memorial ceremony held at a new Mathematics and Physics Institute in Allahabad. His family was represented by his wife, Lily, and daughter Premala, who read remarks acknowledging the honor for the family.

All mathematicians are unique, but I always felt, and believe that the feeling was widely shared, that Harish-Chandra was more unique than most. Like the bard, his formal (mathematical) schooling contained little Latin and less Greek, but he used it to create his own language, and with it conjured a brave new mathematical world. Harish-Chandra migrated to representation theory when it was still virgin territory, just after World War II. He was a pioneer of awesome intensity. He built powerful technical machinery to bring the field under cultivation and left it a well-developed, central area of mathematics, linked by broad highways to analysis, number theory, algebra, geometry, and physics. He literally gave his life to the subject. Without excessive exaggeration, he can be called the hero of the heroic era of representation theory.

THIS ACCOUNT HAS benefited from several others: S. Helgason, R. P. Langlands, G. D. Mostow, and V. S. Varadarajan, as shown below. In addition, I am grateful to these four and to Richard Kadison, George Mackey, Paul Sally, Nolan Wallach, André Weil, and Gregg Zuckerman for sharing their memories of Harish. I especially thank Lily Harish-Chandra and her two daughters for lengthy conversations and for comments on drafts of this work.

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