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HAROLD HOTELLING
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A Biographical Memoir by

K. J. ARROW AND E. L. LEHMANN

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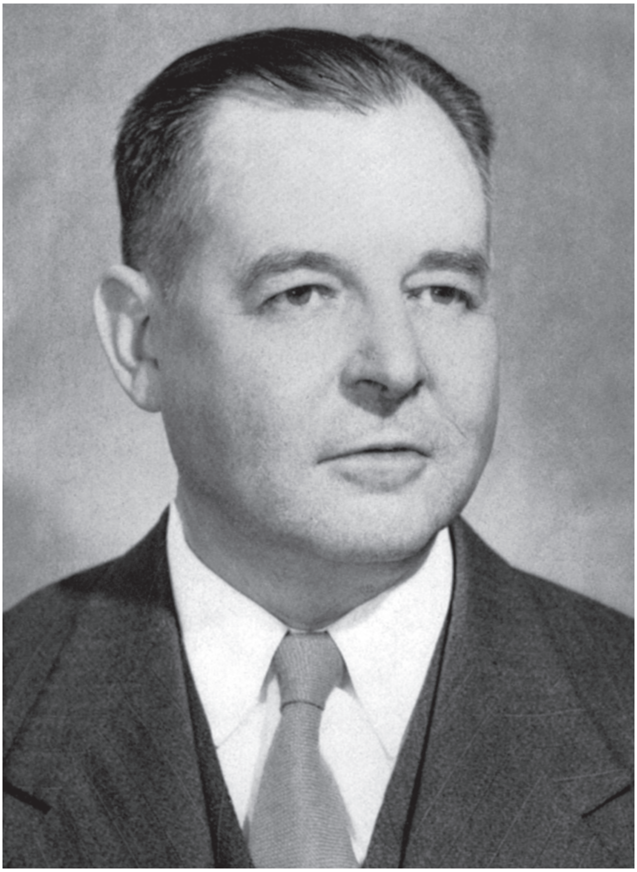

Harold Hotelling

HAROLD HOTELLING

September 29, 1895–December 26, 1973

BY K. J. ARROW AND E. L. LEHMANN

HAROLD HOTELLING WAS A man of many interests and talents. After majoring in journalism at the University of Washington and obtaining his B.A in that field in 1919, he did his graduate work in mathematics at Princeton, where he received his Ph.D. in 1924 with a thesis on topology. Upon leaving Princeton, he took a position as research associate at the Food Research Institute of Stanford University, from where he moved to the Stanford Mathematics Department as an associate professor in 1927. It was during his Stanford period that he began to focus on the two fields—statistics and economics—in which he would do his life's work. He was one of the few Americans who in the 1920s realized the revolution that R. A. Fisher had brought about in statistics and he spent six months in 1929 at the Rothamstead (United Kingdom) agricultural research station to work with Fisher.

In 1931 Hotelling accepted a professorship in the Economics Department of Columbia University. He taught a course in mathematical economics, but most of his energy during his 15 years there was spent developing the first program in the modern (Fisherian) theory of statistics. With the aid of a grant from the Carnegie Corporation, he was able to appoint a research associate who would also teach courses; among those who held this position were Abraham Wald and Henry B. Mann. He had as graduate students Samuel S. Wilks, W. Allen Wallis, Jacob Wolfowitz, Albert Bowker, and Meyer A. Girshick, who in turn set up similar programs at Princeton, Stanford, Chicago, and Cornell.

With the aid of the research associates the statistics program was indeed first-rate; it had a separate listing in the catalogue but there was no department and no degree associated with it. His intellectual leadership, kindness, and generosity to his students were legendary among them, and his house in Mountain Lakes, New Jersey, witnessed his and his wife's monthly open houses for them and other statisticians. He had none of the prejudices then still common; refugees from Europe and students from India could count on his warm support and very practical help.

His human and liberty-loving sympathies made him a bitter opponent of Hitler and an early advocate of intervention in World War II. When we entered the war, he persuaded the U.S. military to create a statistical research group at Columbia, of which he was made director. The group worked on many problems of quality control for munitions and other statistical issues relevant to the war effort.

The Columbia administration resisted his efforts to create an independent department of statistics with permanent faculty. It persisted with its refusal even in 1946, when Hotelling received an exceptionally attractive offer from the University of North Carolina to start a statistics program with strong external financial backing. As a result, Hotelling left Columbia for Chapel Hill, where he quickly built up a strong Department of Mathematical Statistics.

Hotelling married Floy Tracy in 1920, and they had two children. After her early death in 1934, he married Susanna Edmundson, with whom he had five sons. He retired in

1966 but remained at Chapel Hill, where he died in 1973 after a long period of ill health.

CONTRIBUTIONS TO ECONOMICS

(For some of the information on Hotelling's career as an economist, we draw on the work of Adrian Darnell [1990].)

Harold Hotelling wrote six major papers on economics. Four of them have had a profound effect on the field, though frequently only after a long period of time. The other two are equally ingenious but deal with matters on which others were also working. Hotelling was interested in economics from his undergraduate days. Though his undergraduate degree was in journalism, he did take some courses in economics and in later years seemed to regard them as equally important to his journalism. His Ph.D. thesis in mathematics concerned topology, but his economic interests led to a paper, published almost contemporaneously with one based on his dissertation, on the true economic meaning of depreciation (1925). (Incidentally and possibly irrelevantly, the uncle of his thesis advisor, Oswald Veblen, was a very well-known though highly unorthodox economist, Thorstein Veblen.) The topic of depreciation is somewhat specialized, but Hotelling's treatment is at once highly original and fully in accord with standard economic reasoning. This paper became the standard for all subsequent work in the field. It did require a reasonable amount of mathematics, including calculus of variations and solving an integral equation in the most general case. Although there had already been a tradition of the use of mathematics in economics, indeed with predecessors in the late eighteenth century the use of mathematics had not spread far at this point. The continuous use of mathematics and economics really begins with Hotelling and some European contemporaries.

Another paper, whose inception appears to date from the same period but was published somewhat later (1929) in a major economic journal, dealt with a topic that he introduced, that of spatial competition. He recognized that when establishments are spatially separated, the cost to the consumer includes not only the price but also the transportation costs. As a result, he concluded that firms will tend to be concentrated in the middle. He noted that spatial differentiation was not only interesting in itself but could also be regarded as a metaphor for quality differentiation in products. His method of analysis was to find the Nash equilibrium point of a two-stage game; in the first state the players locate themselves; in the second they choose prices. (This was, of course, long before game theory was defined.) He noted that the analysis could also be applied to competition between political parties; they would both tend toward the center. Thirty years later and with additional contributions by Howard Bowen (Bowen, 1943), Duncan Black (Black, 1948), and Anthony Downs (Downs, 1957), the idea entered political science as the median voter theorem and has remained a staple of formal political theory to this day.

Unfortunately, Hotelling's work on the economics of spatial location was vitiated by an error (he used local conditions for a maximum, which were not sufficient in this case), as first pointed out 50 years later (d'Aspremont et al., 1979), an error that, however, did not affect the political implications.

The most influential of all his papers, at least in recent years, was that on the economics of exhaustible resources (e.g., minerals, such as oil) (1931). He gave a formal argument that the price of a commodity (more precisely, the excess of the price over the cost of extraction) would have to rise over time at the same rate as the rate of interest. The argument, like some parts of the paper on depreciation, used the calculus of variations, well beyond the capacity of the great bulk of economists then, and the paper was rejected by one journal before being published. With the post-World War II concern over resource exhaustion, the paper became acknowledged and forms to this day an important component of the analysis of the future of resource scarcity.

The two papers on the theories of the firm and of the consumer (1932, 1935) were the best expositions of their subjects. He connected them soundly to the general mathematical theory of unconstrained and constrained maxima and put especial emphasis on the symmetries in the cross elasticities of demand and supply implied by the first-order conditions and on the implications of the second-order conditions. The first paper was a systematic treatment for any number of commodities of standard results derived graphically for two commodities. The second broke ground new to the general economics profession, but it turned out that its basic ideas had been anticipated by Evgenii Slutsky, the Russian probability theorist, in an Italian actuarial journal 21 years earlier. These papers were the basis of his Columbia course in mathematical economics, at least in 1941-1942.

The paper that received the most immediate recognition was his presidential address to the Econometric Society on the economic criteria for judging whether a policy change represents an economic improvement (1938). Though the general idea had been adumbrated over the history of economics, the precise statement and interpretation of the welfare criteria had never been stated clearly. He not only clarified the interpretation and gave a precise and simple proof but raised new ideas for measurement of welfare improvements, especially in a world of many commodities.

This paper and some remarks in earlier papers also made clear Hotelling's inclinations to some kind of market socialism. Despite the fact that his work was based on standard economic principles, he pointed out that they did not necessarily lead to *laissez-faire* implications.

Finally, one of the most used of Hotelling's economic insights is contained not in a paper but in a letter to the director of the National Park Service (Hotelling, 1947). The director had asked him and others how to measure the economic value of national parks. Since the entrance free is relatively small, it is clear that people are probably getting satisfaction from the park well in excess of the fee charged. Hotelling noted that people usually have to travel considerable distances and thereby incur significant money costs. If we find the largest distance traveled, then those individuals can be thought of as getting zero net satisfaction (the value of the park less the transportation costs). Then all nearer individuals get a surplus that can be computed. By integrating over the population, we can get a total measure of satisfaction. This method, called the "transport cost method," has been used numerous times to evaluate the social gain to parks and similar geographically defined sites.

CONTRIBUTIONS TO STATISTICS

Hotelling's statistical work falls into two categories. On the one hand, he made deep and highly original research contributions; on the other, he was in the broadest sense the teacher of a generation of American statisticians.

In one of his earliest statistical papers, written with Holbrook Working (1929), Hotelling obtained confidence bands for regression curves several years before Neyman developed his theory of confidence sets. His best-known and most influential paper (1931) extended Student's t-test (in both the one- and the two-sample cases) from univariate to multivariate distributions. The resulting test is known as Hotelling's T^2-test. As in the earlier paper, he points out that the tests can be converted into confidence statements, this time for the unknown multivariate mean. He also introduces invariance considerations to simplify determining the null distribution of the test statistic. This is an idea that entered the mainstream only much later. Hotelling continued his contributions to multivariate analysis with, among others, two basic papers on principal components (1933) and canonical correlations (1936).

A paper pointing in quite a different direction was "Rank Correlation and Tests of Significance Involving No Assumption of Normality" (1936). Of it, Richard Savage, in his 1953 "Bibliography of Nonparametric Statistics," writes that "papers related to nonparametric problems were published in the nineteenth century, but the true beginning of the subject may be taken as 1936, the year in which Hotelling and Pabst published their paper on rank correlation."

Again, invariance considerations are central and are used to motivate the reduction to the ranks of the observations.

It may seem surprising that Hotelling, having conceived of such basic ideas as confidence sets and invariance and having used them more than once, did not take the additional step of formulating them abstractly and developing their properties. But that was not his style. Mathematicians distinguish between problem solvers and system builders. Hotelling (at least in his statistical work) was emphatically the former. He took on a difficult problem and after having solved it, moved on to tackle the next one. As a result, confidence sets are credited to Neyman, who discovered the idea independently a few years later and developed it into a general theory; a general concept of invariance was first formulated in the late 1940s by Hunt and Stein.

Hotelling's role as an educator may be said to have begun when—it seems on his own initiative—he reviewed the first edition of Fisher's *Statistical Methods for Research Workers*, published in 1925, for the *Journal of the American Statistical*

Association (JASA). His review ended by noting that "the author's work is revolutionary and should be far better known in this country." Hotelling considered the task of making American statisticians aware of Fisher's work so important that he went on to review the next six editions of the book as well as the first two editions of Fisher's *The Design of Experiments.*

In the same vein, after spending six months with Fisher in Rothamstead, Hotelling published two survey papers in *JASA*: "British Statistics and Statisticians Today" (1930) and "Recent Improvements in Statistical Inference" (1931). They summarized what he had learned in England.

During the 1930s, knowledge of the new methodology developed by Fisher and expanded by his successors spread in America into many different fields of application, and led to the setting up of statistics courses in the corresponding university departments. However, these courses were not taught by statisticians, since that profession did not yet exist. Instead, they were typically assigned to the youngest, most recently hired member of the department, who often knew nothing of the subject and had to learn it one lecture ahead of the students.

The situation dismayed Hotelling, and in 1940 he analyzed it in a paper (1940,2), "The Teaching of Statistics," that raised a question basic to the future of the discipline.

> The growing need, demand and opportunity have confronted the educational system of the country with a series of problems regarding the teaching of statistics. Should statistics be taught in the department of agriculture, anthropology, astronomy, biology, business, economics, education, engineering, medicine, physics, political science, psychology or sociology, or in all these departments? Should its teaching be entrusted to the department of mathematics, or a separate department of statistics, and in either of these cases should other departments be prohibited from offering duplicating courses in statistics, as they are often inclined to do?

Hotelling continued this discussion five years later with a talk on the slightly broader topic—"The Place of Statistics in the University" (1949). He concluded the paper by recommending that "organization of the teaching of statistical methods should be centralized and should provide also for the joint functions of research and advice and service needed by others in the institution and possibly outside it, regarding the statistical aspects of their problems of designing experiments and interpreting observations."

In the following decades, the issues raised by these two papers were hotly debated in universities across the country, but in most cases were gradually settled along the lines suggested by Hotelling, with independent departments of statistics offering not only centralized teaching of the subject but also consulting services to fill the needs of researchers in other departments.

The most central aspect of Hotelling's educational activities was, of course, his development of two outstanding statistics groups at Columbia and Chapel Hill and the training of statisticians he carried out at those institutions. It is no exaggeration to state that during the 1930s and early 1940s, Hotelling nearly single-handedly brought American statistics into the modern age and laid the foundation for the extraordinary development of the subject after the Second World War.

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