NATIONAL ACADEMY OF SCIENCES

DUNHAM JACKSON

1888—1946

A Biographical Memoir by

WILLIAM L. HART

Any opinions expressed in this memoir are those of the author(s) and do not necessarily reflect the views of the National Academy of Sciences.

Biographical Memoir

COPYRIGHT 1959
NATIONAL ACADEMY OF SCIENCES
WASHINGTON D.C.

Dunham Jackson

DUNHAM JACKSON

July 24, 1888–November 6, 1946

BY WILLIAM L. HART

MY FIRST MEETING with Dunham Jackson occurred in September, 1916, at Cambridge, Massachusetts, when I arrived to begin an interesting year as a Benjamin Peirce instructor in the Department of Mathematics at Harvard University. At the time, Jackson was an assistant professor in that department. He immediately made me feel as if we were old friends, and this was typical of his reactions to new acquaintances. Later, he and I served together briefly in mathematical work in the Army. Thereafter, starting in 1919, we were colleagues in the Department of Mathematics [1] at the University of Minnesota. Hence, except for an interruption lasting from 1917 to 1919, I had continuous intimate contacts with him from 1916 until his death. In writing this biography, I have no fear of failing to do justice to his mathematical achievements, which will be well documented by the record of his publications and the honors which he received. The main problem of the biography will be to weave into the mathematical record a vivid picture of Jackson's exceptionally warm personality, which endeared him to practically everyone who had contact with him in any capacity.

Before beginning a somewhat chronological account of Jackson's

[1] Meaning, in this memoir, the Department of Mathematics in the College of Science, Literature, and the Arts (SLA). At the University of Minnesota, in 1919 and for many years thereafter, mathematics was taught in the Institute of Technology by its Department of Mathematics and Mechanics. At present, the University has two Departments of Mathematics, one in the Institute and one in SLA, which combine for graduate work.

life, it appears appropriate to outline its main features, many of which will be described in more detail later, and to sketch in bold outlines his attitudes and general characteristics. He received the degrees A.B. and A.M., with mathematics as his major field, at Harvard University. His remaining mathematical training was obtained in Europe, and he received his Ph.D. degree at Göttingen University in Germany, with Edmund Landau as the adviser for his dissertation. Jackson was a member of the Department of Mathematics at Harvard from 1911 to 1919, with a leave of absence in 1918–19, to serve as a captain in the Ordnance Department of the United States Army, where he performed mathematical work. For the remainder of his life, from 1919 to 1946, he was a professor in the Department of Mathematics at the University of Minnesota, and hence a large part of this biography will refer to that period. He presented his first scientific paper [1] to the American Mathematical Society (A.M.S.) in 1909 while he was a student working for the degree A.M. His last paper [75] was presented to the A.M.S. posthumously on December 26, 1946. In all, he published 75 mathematical papers involving novelty in the results, or in the methods and organization. He was the author of two books [45 and 67]. Also, he published various articles of general mathematical interest intended for teachers at the secondary or college level, or for the general public. While he was in the Army, he wrote a brief textbook on numerical integration in ballistics in pamphlet form for the Ordnance Department [15]. He was noted for his exceptional ability as an expositor of mathematics in his published work, in lectures, and in teaching at all levels. His high productivity as a scholar becomes more impressive when account is taken of the fact that ill health after the age of fifty-two and death at fifty-eight cut off at least a dozen years of vigorous research by him.

The special abilities of Jackson and his many services to the field of mathematics were appropriately recognized during his lifetime. He was elected to the honor societies Phi Beta Kappa and Sigma Xi while he was a student at Harvard. He was a member of the A.M.S.,

Mathematical Association of America (M.A.A.) (one of its charter members), National Council of Teachers of Mathematics, American Physical Society, American Association for the Advancement of Science (A.A.A.S.), American Academy of Arts and Sciences, and the National Educational Association. For several years, starting while he was a member of the faculty at Harvard, he was a member of various committees on examinations in mathematics for the College Entrance Examinations Board. For an extended period, he was a member of the Committee on the Teaching of Mathematics of the North Central Association of Colleges and Universities. He was a member of the Council of the A.M.S. (1918–20), Vice-President (1921), and a member of the Editorial Committee of the Transactions of the A.M.S. (1926–31). He gave the colloquium lectures of the A.M.S. for 1925. He was a member of the Board of Governors of the M.A.A. (1923–29), Vice-President (1924–25), and President (1926). He was awarded the Chauvenet Prize for exposition of the M.A.A. for the period 1932–34. He was a vice-president of the A.A.A.S. (1927), and Secretary of its Section A (Mathematics) for many years. He was a fellow of the American Academy of Arts and Sciences and was elected a member of the National Academy of Sciences in 1935. He always maintained a lively interest in mathematical affairs at the secondary level, and frequently played an active role in joint actions of the fields of college and secondary mathematics.

Jackson was a man of high ideals, extremely unselfish both in ordinary life and in regard to matters of priority in mathematical research. He was very conscious of his responsibilities not only in strictly personal affairs but also with respect to his profession as an educator and his duties as a citizen. He was intensely devoted to the field of mathematics, thought deeply of its place in advancing human welfare, and was fluent in expressing his convictions on such matters. He was reticent in personal matters, and was tolerant and lacking in undesirable curiosity about the opinions and actions of others. An outstanding characteristic was his enjoyment of close associations with people. In particular, his congenial wife and children

were a continuing source of pleasure to him, and he was deeply attached to his father, mother, and sister. It was typical of Jackson that after a few contacts he was likely to call a new acquaintance, including any student, by his given name as a sincere implication of permanent regard. At the University of Minnesota he transmitted this habit to members of his department, so that it became noted for its exceptionally friendly atmosphere, among both students and teachers. As a result of this attitude, his numerous office consultation hours always were heavily attended by students, who probably came as much for the intimate contacts with him as for mathematical assistance. Until his final illness restricted his activities, he was a regular attendant at meetings of the A.M.S., where his cheerful manner and cordiality, in addition to his wide knowledge and active interest in all mathematical programs, made him a great asset. One of his major regrets during his last few years was his inability to attend meetings of the A.M.S. to renew old friendships and to make new friends.

Jackson was apparently in excellent physical shape, and in a vigorous period in his mathematical research, in 1940 at the age of fifty-two, when he experienced a serious heart attack. After his recovery, for about two years, he was able to live a relatively normal life, except that extra precautions had to be exercised concerning physical exertion, and he was restricted as to travel away from home. Then his health began to deteriorate, with a succession of minor heart attacks, and he was able to teach only irregularly. After 1943, he was frequently confined to his room at home or in a hospital, and this was almost continuous for the last eighteen months of his life. Nevertheless, he maintained his scientific activity. During his last year, he gave approval to three[2] Ph.D. dissertations, the final one just two weeks before his death. His last research paper was finished

[2] In these cases, he was assisted by Professor Robert H. Cameron, who came to the Department of Mathematics at Minnesota in 1945. Another thesis, originally directed by Jackson, was not wholly complete when he died, and the Ph.D. degree for the student was not awarded until later, with Cameron as the adviser.

The parents of Jackson undoubtedly noticed early in his life that he had exceptional intelligence, as well as an unusual share of excellent characteristics. As soon as Jackson was of walking age, he became his father's companion on long hikes and his father's interest in science made these excursions essentially field trips for the study of botany, zoology, and geology. Thus, very early, Jackson acquired from his father a vast store of miscellaneous information and habits of observation that he never outgrew. At an early stage, he started reading the science textbooks in his father's library. Once, the family doctor became concerned over Jackson's health and prescribed a week's vacation from astronomy, chemistry, and physics. Jackson was heartbroken and pleaded, "Can't I even read astronomy? That isn't so exciting." During his years in high school, he showed unusual aptitude and liking for languages and literature. From both his father and his mother, he inherited a love of poetry and the habit of learning it by heart. He had an exceptional teacher of English in high school who motivated him to take the examination for exemption from freshman English when he entered Harvard University. He was less fortunate in his other teachers in high school. At that time, he took great pleasure in his three foreign languages—Latin, Greek, and German. He enjoyed learning Latin and German poetry by heart, and was fond of teaching it to his young sister when they worked together at household chores. As a consequence of the Puritan prohibition of study on Sunday, he once developed the bright idea of utilizing his father's collection of Bibles in foreign languages, and read the gospels in Greek as a legally religious and also intellectually advantageous Sabbath diversion. In later life he frequently read Greek literature for relaxation.

It would be a mistake to imagine that Jackson spent a wholly or even primarily bookish childhood. It is a tribute to the intelligence of his parents that his intellectual interests, up to the time he went to college, were kept in proper balance in his activities. He shared all the regular amusements of boys of his age—baseball, tennis, skating, cycling, swimming, sailing, and hiking. He maintained these

athletic interests throughout his life. Thus, when he became a graduate student at Göttingen, he regularly played baseball, hockey, and tennis, and took long hiking and bicycle trips around the neighborhood. Even though an attack of polio (to be discussed later) curtailed his physical activities in later life, he continued to engage in all possible forms of his favorite athletic diversions. In fact, up to the time of his heart attack at the age of fifty-two, he gave evidence of a constant excess of physical as well as mental energy. His fondness for participation in sports carried over to a strong interest as a spectator at sports events. Thus, in his later life at the University of Minnesota, he rarely missed athletic contests of University teams in any of the sports. At any athletic competition, he always managed to become a strenuous partisan and enjoyed such affairs keenly.

Jackson was barely sixteen years of age when he entered Harvard University. His record as an undergraduate was unusually distinguished, not only in his field of concentration, mathematics, and in the related fields of astronomy, chemistry, and physics, but also in course work in various ancient and modern languages. As a sophomore, he received the Wendell Scholarship as the leading student of his year. He had highest second year honors in both mathematics and the classics. During his senior year, he received the Palfrey Exhibition and the Richard Augustine Gambrill Scholarship in recognition of his achievement as first scholar of the senior class. He received the A.B. degree *summa cum laude* in 1908, with distinction in mathematics. His record in the Graduate School, completed by the award of the degree A.M. in 1909, was equally distinguished. During 1908–09 he was one of seven students receiving George C. Shattuck Fellowships for "persons of superior merit in the study of mathematics or various languages." He also held an assistantship in astronomy. In 1909, after obtaining the degree A.M., he received the Rogers Fellowship (1909–10) and the Edward William Hopper Fellowship (1910–11) from the Graduate School at Harvard University, for study toward the Ph.D. degree at Göttingen.

It is interesting to note that Jackson's first published research in

mathematics [1], in the field of algebra, was written shortly after he received the degree A.B. His inspiration for this work undoubtedly came from his courses in algebra under Professor Maxime Bôcher, and from exceptionally close contacts with him outside of the classroom. These contacts continued in correspondence all during Jackson's absence in Europe. Thus, we find from his diary that he and Bôcher exchanged frequent letters, sometimes as often as once a week over long periods. The content of these letters is not disclosed in the diary, but undoubtedly they dealt largely with mathematics of joint interest, or with advice to Jackson concerning his mathematical life in Europe. In all, it is clear that Bôcher and Jackson were close friends, and that Bôcher exerted great influence on him in his early life as a mathematician.

Jackson arrived in Göttingen in September, 1909, shortly after he was twenty-one years old. The ensuing two years were an intensely interesting period in his life. He started a diary on his twenty-first birthday, and carried it forward meticulously, with rarely a day of omission for about five years. All specific details which I shall give concerning his life during his student days in Europe will be based on information presented by him in this diary.[4] He started it with the following statement: "I shall keep this record for my own convenience and pleasure and not for the purpose of constructing a model diary. If I sometimes write what I think, as well as what I do, it will be for the sake of comparison and not because the thoughts are of any value in themselves. I intend to tell the truth, and nothing but the truth, but not the whole truth." In the preceding statements, he distinctly was in error in certain respects. The diary, consisting of hundreds of pages, indeed is a model for such a composition, if it is to give a clear picture of the author. In addition to a listing of day-by-day events, Jackson's diary frequently contains a fascinating record of his most important thoughts. In reading them we gain a clear impression of the character and of the workings of the mind of a

[4] The diary was placed at my disposal recently by Jackson's sister.

brilliant young scholar, his philosophy about life in general, his evaluation of the psychology of his major field, and his reactions to problems in the field of education. Jackson also was wrong in estimating that his thoughts, as transmitted to his diary, were of no value. For instance, even at this late date, I believe that many of his philosophical musings about mathematics and its pedagogy could well form the basic themes for extended discussion. He adhered to his decision that he would "not tell the whole truth." He was very reticent about personal affairs in his diary, just as in later life he maintained reticence in such matters even in contact with his most intimate friends. Hence, a reader of the diary does not feel that he is intruding on the privacy of its author. It appears appropriate to quote a few passages from the diary, illustrating its scope outside of mere factual information. Also, the quotations give some indication of the unusual maturity of Jackson at this time, his philosophical introspective nature, his serious attitude about religion, and his well-organized opinions about the psychology of learning and teaching mathematics.

In regard to church hymns, after attendance at church one day in Germany: "Most of the words were written by men who have been dead for one century, if not two. I have tried to make out what common characteristic it is that seems to make them all so remote from practical life. . . . The thing that is lacking in the hymns is perhaps any suggestion of our intelligently doing anything. We can only wonder, and worship, and pray for guidance. Precisely this way of thinking has accomplished great things."

Concerning college education: "I have been reading President Lowell's (Harvard) inaugural address. I went on to think about the problem of improving college education, as I do occasionally, and was struck by an idea which impressed me very much. (I must wait now for it to cool off.) The idea is to add to the present general courses for average students, and special courses for good students, a few general courses for good students, and make them open only to distinguished seniors."

Comment on address of President Nichols of Dartmouth, where it was stated that we need special knowledge in college teachers but not specialized men: "The question is, is it possible for a man to get the specialized knowledge to the extent that is necessary without himself suffering some specialization? Some men, surely, are successful in this, but I suspect that it is easier in some fields than in mathematics." (Any qualified observer would agree that Jackson himself furnished a good example of a specialist in knowledge who did *not* become a specialized or unnatural man.)

Mathematical teaching at advanced levels: "What is the purpose of a higher course in mathematics? To teach facts or methods? Should a course be simply a succession of theorems with proofs? Or, is it undesirable to try to prove everything? Does a mathematician read and make absolutely clear to himself all the work on which his own original work is based, so that he can assert, of his own knowledge, that the preceding work is true? Or, does he leave something to the accuracy of his predecessor? This is a point on which information ought to be gained by inquiry. Is the purpose of proofs in a higher course to teach methods which will be useful to the pupil or to assure him that every point is based on a rigorous foundation? If the answer to these questions is different for different courses or for different parts of a course, I think it would be well for the teacher to have in mind just what point of view he is adopting at each stage."

A basic attitude for any-school: "Every school ought to be a normal school teaching the pupils to teach themselves, actually telling them how to do it, how to observe the success of various methods, not just giving them the example and hoping they will observe." "I have been thinking again that a university ought to teach students to use their minds better than it does. . . . no instruction is given, not much anyway, about *how* to work."

The text-book level of presentation of mathematics to immature students: "Looked at Kowalewski's Calculus. The theoretical work with which it begins would be out of the question in America. The American student wouldn't be able to understand it. I don't believe

the German is either. But, the German learns it obediently for the time being, and comes to an understanding of it by and by. The American wouldn't do that. In the first place, he wouldn't keep on studying something he didn't understand. And, in the second place, with the best of intentions he wouldn't be able to get any grasp of it, even to the extent of remembering it, if he didn't understand it. That is the way he is made, judging by me."

Mathematics in general education: "I have been wondering whether it would be possible to give a course of lectures on descriptive mathematics so that seniors in college who had never heard of a sine when they began could tell an integral equation when they saw it at the end of the year. It would be hard to do it, but I am not sure it couldn't be done. I should want to emphasize the fact in every lecture that mathematics is a science of finding out what is so and what isn't and not a juggling with formulae according to mysterious and irresponsible rules."

How to work at mathematics: "It often happens that the ideas that are worth most come when I am just on the point of stopping work. I plug away for a while and don't get anywhere, and make up my mind to stop, and then as my mind wanders back involuntarily over what I have been pondering, some relation that I had overlooked takes form as if by itself."

In spite of numerous philosophical touches as just indicated, the diary does not make heavy reading. Thus, Jackson frequently included his own humorous interpretations of events, or amusing anecdotes or jokes of companions. His hearty sense of humor is well shown by such features, and they prepare one to expect the fondness which he showed in later life for jolly spinning of limericks or other verbal amusements with friends. The following lighter touches in the diary bear out the preceding remarks:

"When you have got a bad cold, and it is dark and rainy, and you have lost your umbrella, and can't make head nor tail out of what you are trying to do, you have something to look forward to when circumstances change."

". . . Hurwitz's pun last night: 'What is the shape of a kiss?—It's ellip-tical.' "

"A Berlin joke: He fell into the river and was drowned. How did that happen? I thought he could swim. Answer: But it is verboten to swim in the river."

"I have been trying to prove the uniform convergence of a series. It doesn't converge worth a pfennig!"

Several quotations ascribed to the mathematician Felix Klein show that he could be quite humorous and informal in the classroom, and that Jackson enjoyed this feature. The following are Kleinisms:

"You can make a hyperbolic geometry for yourself by twisting your neck."

"Die Mathematik ist die Wissenschaft der dinge die selbstverständlich sind" (quoted by Klein from Jacobi).

Jackson arrived at Göttingen in a golden age of that mathematical center, and made it his headquarters from 1909 to 1911. Its staff during this period included the mathematical giants David Hilbert, Felix Klein, Edmund Landau, and Ernst Zermelo, as well as others who were not so famous. In addition to attending lectures under members of the staff, Jackson also took part in seminars and attended the Mathematische Gesellschaft, at which the speakers were local mathematicians or, occasionally, prominent visitors. Jackson immediately received cordial attention from members of the mathematical staff. He credited some of the special interest in him to the fact that Bôcher had mentioned his name in the preface of the German translation of Bôcher's text on higher algebra, for a suggestion by Jackson which had come up in a course under Bôcher at Harvard. Bôcher's book was of interest at that time in Göttingen. For a few months in 1911, Jackson attended the University of Bonn, where he heard lectures by Felix Hausdorff and Aduard Study. Also, Jackson spent a few weeks of 1911 in Paris, where he met and attended lectures by Emile Picard, Edouard Goursat, and Jacques Hadamard. During Jackson's stay in Europe, we find from items in his diary

that he had personal contacts or correspondence with practically all of the famous German and French mathematicians of that period. Also, he initiated friendships with many young European mathematicians who became famous later. Jackson was fascinated by these contacts and, as might be expected, at times was somewhat awed by lectures given by various famous men. As a matter of general mathematical history, it is interesting to learn from Jackson's diary that he met as students in Europe many of his American contemporaries in mathematics, who later became leaders in the A.M.S.

At Göttingen, Jackson selected Landau as the adviser for his doctoral dissertation. I infer that advice from Bôcher partly dictated this choice, because Jackson's diary contains the following statement, during a visit made to Göttingen by Bôcher in January, 1910: "As a result of a talk of Professor Bôcher with Professor Landau, I went to see the latter this afternoon and got a list of possible subjects for a dissertation." Jackson's choice of a subject for his thesis permanently channeled his main future research into the field of approximation theory and orthogonal functions, which he enriched by many new results and outstanding expositions of fundamental content. In the preface of the book containing his colloquium lectures [45], delivered to the A.M.S. in 1925, he made the following remarks concerning his choice of a thesis topic: "Guided partly by natural inclinations, perhaps, and partly by recollection of a course on methods of approximation which I had taken with Professor Bôcher a few years earlier, I committed myself to one of the topics which Landau had proposed (for a thesis), an investigation of the degree of approximation with which a given continuous function can be represented by a polynomial of given degree.[5] When I reported my choice he said meditatively . . . Das ist ein schones Thema, ich beneide Sie um das Thema . . . Nein, ich beneide Sie nicht, aber es ist ein wunderschönes Thema."

[5] In considering this topic, Jackson enlarged it to include approximation by trigonometric sums as well as by polynomials.

The primary result of his thesis was as follows:

There exists an absolute constant K_1 with the following property: if $f(x)$ has the period 2π and satisfies a Lipschitz condition, $|f(x_1)-f(x_2)|\leqq\lambda|x_1-x_2|$, then, for every positive integer n, there exists a trigonometric sum $T_n(x)$ of order n at most so that

$$|f(x)-T_n(x)|\leqq K_1\lambda/n \tag{1}$$

for all values of x, and thus max $|f(x)-T_n(x)|=O(1/n)$.

In his demonstration of this result, $T_n(x)$ was explicitly exhibited. Under similar hypotheses on a function $f(x)$ defined on an interval (a, b), omitting reference to periodicity, he exhibited polynomials $P_n(x)$ of degree n at most so that $|f(x)-P_n(x)\leqq G_1\lambda l/n$, where $l=b-a$ and G_1 is an absolute constant. Moreover, if $T_n(x)$ now represents the trigonometric sum of order n at most for which max $|f(x)-T_n(x)|\equiv\phi(n)$ has its least value, Jackson proved the important result that, under his hypotheses, in general $\phi(n)$ is not of lower order than $1/n$, with a similar result for the polynomial case. If $f(x)$ possesses an $(h-1)$th derivative satisfying a Lipschitz condition, Jackson obtained $K_h\lambda/n^h$ in place of the right-hand side of (1), and a parallel result for polynomials. He proved similar theorems for a function $f(x, y)$ with respect to approximation by polynomials $P_n(x, y)$. The key detail in Jackson's thesis was an expression for $f(x)$ by means of a new type of singular integral, probably suggested by those which had been employed previously in proofs of Weierstrass's theorem about polynomial approximation. Jackson's thesis answered a prize question which had been proposed earlier by the Göttingen faculty: "Would it be possible to improve on results (on approximation by polynomials) previously obtained by de la Vallée-Poussin and Lebesgue?" In a complimentary statement awarding Jackson the prize, we find the remark: "The author . . . has enriched the science with valuable results, all in competition with mathematicians of the first rank."

It is no reflection on Jackson's later achievements to remark that his thesis was one of his most important pieces of mathematical re-

search. The fact that his results improved on those of very famous mathematicians gives sufficient assurance of the quality of his thesis, and Landau was amply justified when he labeled its topic "ein wunderschönes Thema." Jackson thus had the rare good fortune to produce, in his first major effort, results which were fundamental for the development of a major field. Although much of his future research was more polished and more difficult than that in his thesis, only a few of his later results (as is usual in research) had the peculiarly basic importance of certain theorems in his dissertation.

The outstanding achievements of Jackson as a student in Europe become more impressive when we take note of the fact that, about halfway through this period, he experienced an illness leaving disturbing permanent effects, which could have injured the spirit of a man with less courage and intellectual balance. At some time during the spring of 1910, he suffered an attack of polio, which fortunately was only moderately severe. As evidence of Jackson's cheerful disposition, there is not a single omission in the daily entries of his diary, and there is no explicit mention of the illness, to indicate its exact date of occurrence. At that time, as in later life, he refused to complain about ill health, even in the privacy of his diary. The illness left him with a permanent abnormality in the muscles controlling one foot and caused considerable lameness for the rest of his life. Before the polio attack, his athletic interests at Göttingen had covered the whole gamut of usual games, and diversions such as bicycling and hiking. Suddenly, none of these activities were permissible for him, although eventually he was able to resume participation moderately in some sports. With typical reticence, he avoided giving details of his illness in his diary. Also, with a similar attitude, he kept immediate information about the matter from the members of his family. Their first knowledge of it came fifteen months later when he returned to Bridgewater. His innate cheerfulness and good morale is shown by the following entry in his diary several months after the polio attack: "Rode my bicycle in the garden. Doesn't go very well, but I am glad to be able to do it at all." As far as one can tell from

his diary, Jackson allowed the illness to keep him from lectures for only a few days. If anything, one senses increased vigor at this time in his devotion to his mathematical activities.

On the nonmathematical side, Jackson enjoyed his two years in Europe to the fullest extent. He made friends very easily, in Europe as in America, and deeply enjoyed his contacts with them, in person or through correspondence. Thus, his day-by-day record in his diary of letters sent and received gives evidence of exceptionally heavy correspondence, where the list of correspondents at first was composed only of American acquaintances and gradually was enlarged by the addition of many people whom he met in Europe. His diary gives extensive accounts of his social contacts in Göttingen, and later in Bonn and in France, with both the faculty and students of universities which he attended. One obtains a picture of frequent extracurricular activities in the colony of American students, from all fields, at Göttingen. We learn that, in Jackson's second year in Europe, he became "Patriarch" (the administrator) of the colony. During his vacations, he travelled extensively in Germany, and less widely in France and Italy.

In his accounts of his travels, we find evidence of the breadth of his interests. He took great pleasure in viewing beautiful scenery, foreign cities, and works of art. The diary contains elaborate descriptions of features of interest. He philosophizes about art and sculpture, about their appreciation, and about the people whom he observed. All during his stay in Europe, and particularly on his travels, he took every possible opportunity to attend musical performances. Throughout his life, he maintained his interest in the fine arts and the theater. As evidence that he was normal in his reactions and by no means was a "specialized man," he even extended these outside interests to movies, and had many favorite actors and actresses in them.

Jackson started his Göttingen dissertation in the spring of 1910, and finished the job in April, 1911. He passed his doctoral examination and received his degree *magna cum laude* on July 12, 1911. His thesis was published separately as a "Gekrönte Preisschrift" of 98

pages, by Göttingen University in 1911, and never appeared in exactly its original form in any mathematical periodical. Later he published refined and simplified forms of the main results of the thesis in the *Transactions* of the A.M.S. He left Europe for the United States in the summer of 1911.

Early in 1911, Jackson was offered an instructorship in the Department of Mathematics at Harvard, and he returned there to start the school year in September, 1911. In the inspiring mathematical atmosphere of Harvard, his closest contacts were with Maxime Bôcher, George D. Birkhoff, Julian Coolidge, William F. Osgood and, later, the young geometer Gabriel Green, whose untimely death in the influenza epidemic of 1917–18 cut short a brilliant career. Jackson's research from 1911 to 1918 consisted of progress in his own major field [3–8, and 13]; a pair of articles relating to differential equations [9 and 12], and three papers dealing with the theory of functions of complex variables, where we sense the influence of Osgood [10, 11, and 14]. During this period, Jackson also directed the research of his first two doctoral candidates. I was a close companion of Jackson at Harvard during 1916–17; at that time, he and Green were the youngest permanent members of the Department of Mathematics at Harvard. Although the whole department was a great inspiration to a young mathematician, Jackson and Green were the foci of the mathematical life and much of the social life of the younger men outside of their lectures and seminars. In a remarkable degree, both Jackson and Green possessed, first, natural ability to establish cordial friendships with their associates and, second, unusual power to encourage and assist young men in performing mathematical research. A request for mathematical information from one of them never was met with an attitude leading the questioner to be embarrassed over his ignorance. The request always was treated as if it were an indication of special understanding and commendable intellectual honesty. This attitude of Jackson in disseminating mathematical knowledge was one of his most valuable char-

acteristics as a teacher, as a director of dissertations, and later as a mathematical leader in his department at Minnesota.

In April of 1917, the First World War broke up the community of graduate students and young mathematicians at Harvard. Many of us left immediately for officers' training camps. About one year later, Jackson was commissioned [6] as a captain in the Ordnance Department and was assigned to the Ballistic Unit of the Technical Staff of the Ordnance Department in Washington, D. C., directed by the mathematical astronomer Forest Ray Moulton (then Major, Ordnance Department). At various times, many mathematicians [7] were attached to the unit, either as civilian employees or as Army personnel. Although the routine calculation of range tables for the artillery services was a major function of the Unit, its work was carried on in a scholarly atmosphere, as far as was possible under wartime conditions. From 1918 to 1920 [8] various advances were made in ballistic theory and actual artillery practice through cooperative actions of the members of the Washington Unit and members in a corresponding Unit at the Aberdeen Proving Grounds in Maryland, under the direction of Oswald Veblen (then Major, Ordnance Department). At Washington, Jackson became one of the most useful members of Moulton's staff. As part of the work, Jackson prepared an excellent pamphlet [15], suitable as a brief textbook, presenting fundamentals of the small arc method of numerical integration for differential equations as developed by the Ballistic Unit for the com-

[6] By special action, his physical handicap was disregarded, in view of the noncombat nature of his intended activities in the Army.

[7] In particular, J. W. Alexander, A. A. Bennett, Arnold Dresden, Lester R. Ford, Phillip Franklin (before he had had graduate work), H. T. Gronwall, William L. Hart, W. D. MacMillan, William E. Milne, and J. F. Ritt.

[8] The Ballistic Unit was kept active between the First and Second World Wars by the devoted service of the physicist Robert Kent, who preceded Moulton in the work at Washington, and who remained as a civilian employee of the Ordnance Department after the end of the First World War. In the Second World War, the work expanded tremendously to become the Ballistic Research Laboratory of the Army, at Aberdeen, with Kent as the director.

putation of trajectories in ballistics. Jackson resigned his Army commission in 1919.

During the spring of 1919, Jackson was offered an appointment as Professor of Mathematics in the College of Science, Literature, and the Arts at the University of Minnesota. Undoubtedly, even with a large increase in academic rank, it was hard for him to think of leaving Harvard University, where his future progress seemed assured. At this time, I was working with him in Washington and also was considering an offer of an assistant professorship at Minnesota. With typical reticence about his own affairs and, perhaps, with the desire to avoid influencing my decision, he never acquainted me with the details of his thoughts as he considered the offer from Minnesota. However, I suspect that he was swayed largely by a description of future possibilities in the position as pictured by the brilliant President Marion LeRoy Burton [9] of the University of Minnesota, and by the frank, effective salesmanship of William H. Bussey, then chairman of the Department of Mathematics at Minnesota. It was admitted that, on arriving at Minnesota, Jackson would find essentially negligible graduate work in mathematics. But he had assurance that there would be a large potential demand for advanced mathematics by itself and also in cooperation with the physical sciences and technical departments. In 1919, the University of Minnesota was starting a period of vigorous growth, at both the undergraduate and graduate levels, which has continued unabated to the present date. I am sure that Jackson came to Minnesota because he visualized a golden opportunity to plan and develop advanced mathematics almost from scratch, in a well-situated university which was young and virile academically, with an explosive potential for future expansion. He was promised freedom from routine administration and a free hand in planning graduate work if he came. Both Jackson and I accepted the offers of positions at Minnesota. We arrived there in September, 1919, and were colleagues until Jackson's death in 1946. I am certain

[9] Later president of the University of Michigan, where he died after very brief service.

that he never regretted this move. He soon took on the reactions of a native Minnesotan in his pride in the state, and in the intelligence of its citizens in demanding and obtaining a state university of the best quality, immune to politics, regardless of the high cost in comparison with state resources. Also he had great respect for the human qualities of the typical student at Minnesota and his potentiality for intellectual development.

On June 20, 1918, Jackson married Harriet Spratt Hulley, whom he had met in 1917, while she was a graduate student in English at Radcliffe College. Her father was president of John B. Stetson University. The Jacksons had two daughters, Anne Hulley (Mrs. William F. Byess) and Mary Eloise (Mrs. William J. Thorpe). Jackson and his family quickly made a place for themselves in the friendly environment at Minnesota. He frequently became vocal in expressing his satisfaction with the cordial social contacts which were typical of the relations between members of both the academic and non-academic staffs at the University. Also, he felt that the general situation in the state of Minnesota, and in Minneapolis in particular, was ideal for living and for rearing a family. At a later date, his appreciation of living and working conditions at Minnesota led him to decline several very enticing offers of positions elsewhere. Jackson was in the habit of taking frequent brief trips by himself to small towns in Minnesota, especially when he felt the need for relaxed concentration on some piece of research. By the end of his life, there were few Minnesota towns possessed of a reasonably decent hotel, and accessible by rail, which Jackson had not visited. On any one of these excursions, it was his habit to hike over the neighboring countryside, while letting his thoughts crystallize in regard to whatever problem was uppermost in his research. In all, Jackson and his family led a well adjusted and very happy life at Minnesota, with ample cultural advantages, innumerable friends for social contacts, and very congenial associates for Jackson in his work in his department and in the University at large. As a minor light observation, Jackson particularly enjoyed the fact that his home was sufficiently

near a city fire engine station that he could hear the engine sirens when the firemen went out on a call, and thus have the opportunity to pursue them to the fire, which he was likely to do at almost any time of the day or night.

In accordance with early agreements, Jackson was kept free of administrative duties at Minnesota. However, although the Department of Mathematics had a succession of chairmen [10] who commanded substantial University respect, the Department and the University in general recognized Jackson's unique claim to certain phases of mathematical leadership. Also, it was quickly learned on the campus that he had an unusually sound viewpoint about matters of general educational policy. Hence he was frequently sought as a member of committees dealing with important questions at both the undergraduate and graduate levels. He had considerable influence on the development of matters affecting fields related to mathematics. In all, it is no exaggeration to state that, aside from Jackson's activities as a scholar and teacher, he was one of the most valuable members of the faculty at the University of Minnesota.

The Department of Mathematics at Minnesota, under its able chairman Bussey (in 1919) was proud of the excellence of its teaching. Jackson enhanced this reputation both by his teaching and by his indirect influence on the whole department. He had a strong conviction, illustrated by his own activities, that excellent teaching at both the undergraduate and graduate levels is a prime duty, and should yield great satisfaction for any member of a university department of mathematics. In his contacts with younger members of the Department, and with graduate students, he implicitly emphasized ability to communicate and scholarship as companion characteristics which should be cultivated by a member or prospective member of a college faculty. His manner in teaching was informal and friendly, but strictly businesslike. He gave dignity to undergraduate instruction by teaching courses at low levels, particularly

[10] During Jackson's lifetime at Minnesota, the chairmen were William H. Bussey, William L. Hart (twice), and Raymond W. Brink (twice), for the longest period.

elementary calculus, whenever possible. In fact, he gladly would have taught this course every year. At a time when I was chairman of the department, I urged him to teach the course only every third year. I recall his suggestion that he would be glad to teach the course as an extra load, in addition to his popular graduate courses, but I convinced him that this would be a poor policy. His success in the teaching of both undergraduates and graduates was public knowledge on the campus. He took great pleasure in personal contacts with young people, particularly undergraduates, and enjoyed having the opportunity to influence them scientifically at an early age. It was invariably the case that his students thought of him as a sincere friend as well as a skillful and inspiring teacher. It is not surprising then to find that an unusually large percentage of undergraduates who had Jackson as a teacher in elementary courses went on to graduate work in mathematics, the physical sciences, or statistics.

In regard to advanced courses, Jackson had the conviction that, in cooperating with other fields such as the physical sciences, it was the duty of the mathematician to transmit as much knowledge as possible on the basis of minimum prerequisites. Hence, it was his custom to develop courses where the prerequisite was merely a first course in calculus, but where the content as presented went deeply into fields of great usefulness. His most outstanding course of this kind for students of the physical sciences was named "Topics in Analysis," and covered essentially the content in his Carus Mathematical Monograph [67]. In its preface he remarks in connection with the low prerequisite, elementary calculus, for appreciating most of the book: "Under the circumstances, rigor in the sense of literal completeness of statement has been out of the question. It is hoped, however, that the reader who is familiar with the methods of rigorous analysis will be able to read between the lines the requisite supplementary specifications, and will find that what has actually been said is entirely accurate in the light of such interpretations." Thus Jackson aimed to tell the truth mathematically but not to make the reader unhappy by giving rigorous details beyond his comprehen-

sion. A further aim was to give the reader of low background a feeling of rigor, and confidence in the soundness of the reasoning. The content of this book for years formed a very popular course in advanced mathematics for students in pure mathematics as well as in the physical sciences. I quote, essentially, a statement made to me by an internationally famous physicist who took the course from Jackson: "It was the most valuable course which I ever had in mathematics. The content came relatively painlessly. It seemed that, with little apparent effort on the part of the students, large amounts of very important mathematics were absorbed. He seemed to make all of it appear easy." This reaction was a common one among students of Jackson. In lecturing, he never seemed to say a useless word. Without any indication of hurry on his part, it suddenly would be realized by the students that he had covered a wide expanse of territory with complete clarity.

One of Jackson's outstanding contributions at Minnesota was his role in the development of statistics throughout the University. In his research on orthogonal polynomials, their use in connection with expansions of statistical frequency functions brought him in touch with certain phases of statistical theory. Also, at the time, he was receiving numerous requests for mathematical aid from faculty members in various fields of applied statistics on the campus. Hence it was natural for him to develop interest in statistics. His publications in this field commenced in 1923 and extended over a period of fourteen years. His papers show evidence that he was giving the foundations of statistics close scrutiny, and was endeavoring to clarify various points which at the time were somewhat obscure mathematically. Hence, his publications on statistical topics [24, 28, 30, 32, 34, 37, 38, 45 (Chapter VI), and 56] involve only a few definitely new results, and are relatively of expository type. His outstanding exposition, "Mathematical Principles in the Theory of Small Samples" [56], consisted of the content of a lecture presented by him at a joint session of the A.M.S., the American Statistical Association, the Econometric Society, and Sections A and K of the A.A.A.S. At the time,

this expository article was unique in clarifying a most important segment in the mathematical foundations of the field. His continued major activity in approximation theory apparently prevented him from performing more fundamental research in statistics. However, his publications in the field, and the nature of a course developed by him at Minnesota, make it evident that he was well along on the highway of modern mathematical statistics. Shortly before his illness at the age of fifty-two, he remarked to me that he hoped to give major attention to developments in statistics which he had in mind, as soon as his well defined program in his main field neared completion. His illness prevented such action. Hence, as his lasting products in statistics, we must point mainly to a course which he introduced at Minnesota, the outstanding record of alumni of that course, and his general influence on statistical development on the campus.

In the school year 1924–25, Jackson introduced a course entitled "The Mathematical Theory of Statistics," three hours per week for a year, at the advanced undergraduate or elementary graduate level, at a time when few courses of this type were available in American universities. He specified the prerequisite for the course as merely elementary calculus, with his usual desire for minimum prerequisites. He always presented the course purely by lectures. It was not modern by present-day standards but, I believe, had more of the spirit of modern statistics than any text of the proper level which was available at the time. Jackson last taught the course (only for part of the year) in 1944–45. Up to this time, the course had been given occasionally by two other members of the Department of Mathematics besides Jackson. The impact of this course on graduate training at Minnesota can be judged in part by the following facts about students who took the course (all except two were taught by Jackson) early in its history. Ten of them [11] have been or are heads of the instruction in bio-

[11] It is interesting to note that one of the leading modern textbooks on mathematical statistics, as presented on the basis of elementary calculus as a prerequisite, was written by one of these students: Paul G. Hoel, *Introduction to Mathematical Statistics* (1947, 1954), John Wiley and Sons, Inc. Hoel received his Ph.D. at Minnesota with mathematics as his major, and Jackson as the adviser for the doctoral dissertation (see items for 1933 in the list of dissertations directed by Jackson).

statistics or in statistics for a department of mathematics, or for a college of business administration, in major American universities; one is a world-famous statistician in the field of sociology; another is a noted statistician in the field of psychology. Approximately a dozen other students from the course went on to the Ph.D. degree in some field of applied statistics. Also, when Jackson started the course, practically all of the prominent teachers in fields of applied statistics at Minnesota took the opportunity to audit the course, or otherwise make contact with its content.

The course which we have just discussed arose very opportunely for the University of Minnesota. At approximately the same time, and with added vigor due to the existence of Jackson's course and his availability as a statistical adviser, several significant developments in applied statistics took form in the University. A strong group in biostatistics arose under the influence of the eminent botanist and biometrician, James Arthur Harris (deceased), head of the Department of Botany. He was instrumental in bringing into being a Division of Biostatistics in the Department of Botany; this Division was transferred later to its present location in the Medical School, where now the Division is very large. The first head of the Division was Alan E. Treloar, who was well-informed as to the content of Jackson's course. From the beginning, due to Jackson, and continuing down to the present date, close collaboration has existed between the Division of Biostatistics and the Department of Mathematics. In the School of Business Administration, an exceptionally strong and at that time unique undergraduate major in statistics was created by the economist Bruce D. Mudgett. This major involved the requirements of elementary calculus and Jackson's course in mathematical statistics. Substantial courses in statistics were initiated in the graduate offerings of the Department of Educational Psychology in the College of Education, under the direction of the educational psychologist and statistician Palmer O. Johnson. Statistical work on a high plane was fostered at advanced levels in the College of Agriculture by Forest R. Immer (deceased), the noted agricultural statistician,

who audited Jackson's course, and by the economist Holbrook Working.

In the Department of Mathematics, the wide utility of Jackson's course in statistics (at first given biennially) finally led to its being offered annually, with ever-increasing registration. This in turn has fostered the development of more advanced courses in statistics and in the field of probability, where the Department now exhibits particular strength.

An analysis of the research of Jackson naturally involves simultaneous scrutiny of the research directed by him. Hence, following the appended chronological list of his publications, I have listed the doctoral dissertations which he directed. He was the adviser for two students obtaining the Ph.D. degree at Harvard and nineteen at Minnesota. It will be observed that the topics of the theses are closely related to Jackson's research at corresponding periods.

The mathematical publications [12] of Jackson may be discussed in three categories: (*a*) articles on statistical topics; (*b*) publications in his major field of approximation theory and orthogonal functions; (*c*) other publications. Ample remarks have been made about category (*a*). In category (*c*), I have already mentioned a few publications produced while he was at Harvard or in Europe. In addition, his papers [21, 23, and 31], should be included in (*c*); their titles are sufficiently explanatory. His most significant research occurs in category (*b*), and can be grouped roughly as follows:

1. *His doctoral dissertation, already discussed, and closely related outgrowths of this work,* involving part of his research from 1911 to 1921. In the later stages of this work, his emphasis was on convergence properties of trigonometric and polynomial approximations to a given function $f(x)$. Synthesized general results of this nature are found in a paper [20].

2. *Research during* 1921–25 *on approximation in the sense of least*

[12] For a more detailed analysis of the publications, see the biographical sketch "Dunham Jackson, 1888–1946," by William L. Hart, *Bulletin* of the A.M.S., 54(1948): 847–60.

*m*th *powers,* $0<m$, *with a non-negative weight function,* ρ, introduced to give a measure of approximation $h_{n,m}$ to a given function $f(x)$, say on an interval $0\leqq x\leqq 2\pi$, where

$$h_{n,m}=\int_0^{2\pi}\rho(x)|f(x)-T_{n,m}(x)|^m dx,$$

and $T_{n,m}$ is either a trigonometric sum of order n or a polynomial of degree n. His dissertation had considered the case $m=2$ with $\rho(x)\equiv 1$, and later work by another mathematician had treated the case $m=2$ with $\rho(x)$ merely taken as nonnegative. First, with $\rho(x)\equiv 1$ [18, 19, 22, and 25], Jackson established the existence of a best $T_{n,m}$ for all n, and the convergence of $T_{n,m}$ to f as n becomes infinite, under various assumptions: $m\geqq 1$, $T_{n,m}$ unique; $0<m<1$, $T_{n,m}$ possibly ambiguous. He then widened the scope of the results by allowing a somewhat general ρ [25 and 29]. Related research is found in several papers [31, 32, and 33]. This research systematically extended approximation theory related to $T_{n,m}$ from the case of the classical least square measure of goodness of fit ($m=2$), to the general case where merely $m>0$, and was a significant contribution to mathematical literature.

3. *Comprehensive research reports presented to the* A.M.S. *by invitation.* In 1921, Jackson was the principal speaker at a symposium of the A.M.S., where he presented a synoptic account of his own main research through 1921 and related literature. This lecture was published [20]. Again, he was honored by being selected as the lecturer at the Colloquium of the A.M.S. for 1925, which involved giving a series of lectures at the summer meeting of the A.M.S. His subject was "The Theory of Approximation." He included complete presentations of essential parts of his own main research to date, associated new results, refinements of existing literature, and collateral classical material to give a rounded view of the field. His lectures were exceptional for their clarity. As usual, the A.M.S. issued a published form of the lectures as a volume in its Colloquium Series [45]. The preparation for the delivery of a series of colloquium lec-

tures, and their later publication in book form, is known to demand at least a few years of major effort from any mathematician thus honored by the A.M.S.

4. *Significant expository publications.* In the period from 1926 to 1946, Jackson devoted considerable time to expositions of fundamental content, written at the level of readers satisfying only minimum prerequisites, in order to disseminate the content more widely among mathematicians and their students, and among workers in fields of application. His principal publication of this type was his Carus Monograph, 1941, *Fourier Series and Orthogonal Polynomials* [67]. His other outstanding expositions are the three related papers [49, 52, and 53] on Fourier series. On account of these, in 1935 he was awarded the Chauvenet prize of the M.A.A. for the period 1932–34. This prize, for each three-year period, is designed to recognize the member of the M.A.A. who has published in English the most excellent exposition of valuable content. In mentioning Jackson's outstanding publications of expository type, it is appropriate here to refer again to his exceptional exposition on "Mathematical Principles in the Theory of Small Samples" [56]. In addition, his miscellaneous papers on statistics published from 1923 to 1935 in the *American Mathematical Monthly* were gems of exposition, while not pretending to be research, except to a minor extent.

5. *Detached publications in Jackson's main field from 1923 to 1946.* His research after 1925 involved several excursions, already mentioned, into the theory of statistics. He also published miscellaneous results on approximation theory apart from a coordinated major program which will be mentioned later. Thus he considered approximation to a real valued function f, whose domain is a finite real interval, by functions of particular types, not always linear families of functions [27, 38, 42, 48, and 50]. Also, he considered approximation to complex valued functions of complex variables [41, 44, 47, and 52].

6. *A relatively coordinated major program of research from 1933 to 1946,* including related theses of his doctoral candidates. First he

developed general methods with some results based on theorems of Bernstein and Markoff, for establishing an upper bound for the measure of error $h_{n,m}$ discussed in (2), and obtained corresponding convergence theorems for the sequence $\{T_{n,m}\}$ related to a function f [49, 51, 52, 55, and 57]. Of these, [51] produced useful general methods. For the remainder of his life, except for the preparation of his Carus Monograph [67], he devoted almost all of his research activity to the problem of approximating a function f of two or of three variables, where the domain of f was assumed to be of various types. In preparation for other work, he discussed [58 and 63] fundamental properties of orthogonal polynomials in two and in three variables. He obtained [61] analogs of the theorems of Bernstein and Markoff for the two-dimensional case, and then proved results for approximation, by use of polynomials $P_n(x, y)$, to a function f whose domain is a two-dimensional region R in the xy-plane. His methods here were similar to those which he had used in the one-dimensional case, with application of his generalizations of the theorems of Bernstein and Markoff.

The titles of the papers relating to orthogonal polynomials and approximation [59–61 and 63–75], and the topics of associated doctoral dissertations by his students, show that he was carrying out a systematic program. These publications attack the problems of approximation to $f(x, y)$ or to $f(x, y, z)$ or various types of point sets, mainly defined algebraically, in two or in three dimensions which can be dealt with conveniently. In a few of the publications, he synthesized general methods, and proved related theorems, apparently in an effort to indicate that the situations considered in many of his papers, and in dissertations of his students, were just particular cases of a large class of similar problems which could be proposed and solved by corresponding devices. His last publication [75], which was finished while he was a heart invalid restricted to his room at home or in a hospital, emphasized the general viewpoint mentioned in the preceding sentence. When he handed this paper to me with the request that I should arrange for its publication, he stated that it was

the last in a sequence of investigations which he had outlined in his mind, many years before, as pertinent in the theory of approximation by polynomials (and sometimes by trigonometric sums) in the two- and three-dimensional cases. Then he remarked significantly, "I am glad that I have been able to finish the job."

Jackson's enforced early retirement from active participation in national mathematical affairs at the age of fifty-two, and his death on November 6, 1946, after a long period of invalidism, were the cause of deep grief throughout the University of Minnesota, and among his former students and host of friends. His past associates recall him not only because of his scholarly attainments and his ability as a counsellor and teacher, but also because of his friendly, sympathetic personality, lively sense of humor, his high ideals, and his unswerving loyalty to his friends and his convictions. Feelings of regret at his departure are tempered by the realization that Dunham Jackson lived a very full life, working successfully in a field which he enjoyed to the hilt. In spite of his innate modesty, he could not have helped realizing that he was continually receiving the most precious of all rewards, the satisfaction of knowing that his actions and attainments in his profession and daily life were well appreciated by all of his associates, and that his affectionate feelings for them were matched by their reactions. It appears particularly fitting that I should close by quoting the final sentence in a letter which I received from a prominent mathematician in answer to a request for certain information about Jackson for this biography: "I need hardly remark in connection with your project that all those who worked for and with Jackson . . . almost worshipped him." He was that sort of man, as well as a mathematician, teacher, and educator of exceptionally high quality.

KEY TO ABBREVIATIONS

Am. J. Math. = American Journal of Mathematics
A.M.S. Coll. Pub. = American Mathematical Society Colloquium Publications
Am. Math. Month. = American Mathematical Monthly
Ann. Math. = Annals of Mathematics
Bull. A.M.S. = Bulletin of the American Mathematical Society
Carus Math. Monog. = Carus Mathematical Monographs of the Mathematical Association of America
Duke Math. J. = Duke Mathematical Journal
J. Reine Angew. Math. = Journal für die Reine und Angewandte Mathematik
Proc. Am. Acad. Arts Sci. = Proceedings of the American Academy of Arts and Sciences
Proc. Nat. Acad. Sci. = Proceedings of the National Academy of Sciences
Rend. Circ. Math. Palermo = Rendiconti del Circolo Mathematico di Palermo
Trans. A.M.S. = Transactions of the American Mathematical Society

BIBLIOGRAPHY

PUBLICATIONS BY DUNHAM JACKSON

1909

1. Resolution into Involutory Substitutions of the Transformations of a Nonsingular Bilinear Form into Itself. Trans. A. M. S., 10:479–84.

1911

2. Über die Genauigkeit der Annäherung stetiger Funktionen durch ganze rationale Funktionen gegebenen Grades und trigonometrische Summen gegebener Ordnung. Gekrönte Preisschrift und Inaugural-Dissertation, Göttingen University. 98 pp.

1912

3. On the Degree of Convergence of the Development of a Continuous Function according to Legendre Polynomials. Trans. A. M. S., 13: 305–18.
4. On Approximation by Trigonometric Sums and Polynomials. Trans. A. M. S., 13:491–515.

1913

5. On the Approximate Representation of an Indefinite Integral and the Degree of Convergence of Related Fourier Series. Trans. A. M. S., 14:343–64.

6. On the Accuracy of Trigonometric Interpolation. Trans. A. M. S., 14:453–61.

1914

7. A Formula for Trigonometric Interpolation. Rend. Circ. Math. Palermo, 37:371–75.
8. On the Degree of Convergence of Sturm-Liouville Series. Trans. A. M. S., 15:439–66.

1915

9. Expansion Problems with Irregular Boundary Conditions. Proc. Am. Acad. Arts Sci., 51:381–417.

1916

10. Note on Rational Functions of Several Complex Variables. J. Reine Angew. Math., 146:185–88.
11. Nonessential Singularities of Functions of Several Complex Variables. Ann. Math., 17(2):172–79.
12. Algebraic Properties of Self-adjoint Systems. Trans. A. M. S., 17: 418–24.

1917

13. Note on Representations of the Partial Sums of a Fourier Series. Ann. Math., 18(2):139–46.
14. Roots and Singular Points of Semi-analytic Functions. Ann. Math., 19(2):142–51.

1919

15. The Method of Numerical Integration in Exterior Ballistics. Ordnance Textbook (prepared in the Office of the Chief of Ordnance), Washington, U. S. Govt. Print. Off. 43 pp.

1920

16. On the Order of Magnitude of Coefficients in Trigonometric Interpolation. Trans. A. M. S., 21:321–32.
17. Note on a Method of Proof in the Theory of Fourier Series. Bull. A. M. S., 27:108–10.

1921

18. On Functions of Closest Approximation. Trans. A. M. S., 22:117–28.
19. On the Convergence of Certain Trigonometric and Polynomial Approximations. Trans. A. M. S., 22:158–66.

20. The General Theory of Approximation by Polynomials and Trigonometric Sums. Bull. A. M. S., 27:415–31.
21. Note on the Picard Method of Successive Approximations. Ann. Math., 23(2):75–77.
22. Note on a Class of Polynomials of Approximation. Trans. A. M. S., 22:320–26.

1922

23. Note on an Irregular Expansion Problem. Bull. A. M. S., 28:37–41.

1923

24. Note on Quartiles and Allied Measures. Bull. A. M. S., 29:17–20.
25. Note on Convergence of Weighted Trigonometric Series. Bull. A. M. S., 29:259–63.
26. Note on an Ambiguous Case of Approximation. Trans. A. M. S., 25: 333–37.
27. An Approximation by Functions of Given Continuity. Trans. A. M. S., 25:449–58.
28. The Method of Moments. Am. Math. Month., 30:307–11.

1924

29. A General Problem in Weighted Approximation. Trans. A. M. S., 26:133–54.
30. A Symmetric Coefficient of Correlation for Several Variables. Bull. A. M. S., 30:536–42.
31. On the Method of Least *m*th Powers for a Set of Simultaneous Equations. Ann. Math., 25(2):185–92.
32. On the Trigonometric Representation of an Ill-defined Function. Ann. Math., 26(2):8–20.
33. A General Class of Problems in Approximation. Am. J. Math., 46: 215–34.
34. The Algebra of Correlation. Am. Math. Month., 31:110–21.
35. The Trigonometry of Correlation. Am. Math. Month., 31:275–80.
36. The Elementary Geometry of Function Space. Am. Math. Month., 31:461–71.

1925

37. The Geometry of Frequency Functions. Bull. A. M. S., 31:63–73.

1926

38. Note on a Problem of Approximation with Auxiliary Conditions. Bull. A. M. S., 32:259–62.
39. Some Convergence Proofs in the Vector Analysis of Function Space. Ann. Math., 27(2):551–67.

1927

40. Some Notes on Trigonometric Interpolation. Am. Math. Month., 34: 401–5.

1928

41. On the Approximate Representation of Analytic Functions. Bull. A. M. S., 34:56–62.
42. Some Nonlinear Problems in Approximation. Trans. A. M. S., 30: 621–29.
43. The Human Significance of Mathematics. Am. Math. Month., 35: 406–11.

1930

44. On Certain Problems of Approximation in the Complex Domain. Bull. A. M. S., 36:851–57.
45. The Theory of Approximation. A. M. S. Coll. Pub., 11. 8+178 pp.

1931

46. Note on the Convergence of a Sequence of Approximating Polynomials. Bull. A. M. S., 37:69–72.
47. On the Application of Markoff's Theorem to Problems of Approximation in the Complex Domain. Bull. A. M. S., 37:883–90.

1933

48. Problems of Approximation with Integral Auxiliary Conditions. Am. J. Math., 55:153–66.
49. Series of Orthogonal Polynomials. Ann. Math., 34(2):527–45.
50. The Convergence of Some Nonlinear Processes of Approximation. Am. J. Math., 55:515–24.
51. Certain Problems of Closest Approximation. Bull. A. M. S., 39:889–906.
52. Orthogonal Trigonometric Sums. Ann. Math., 34(2):799–814.

1934

53. The Convergence of Fourier Series. Am. Math. Month., 41:67–84.
54. A Proof of Weierstrass's Theorem. Am. Math. Month., 41:309–12.
55. The Summation of Series of Orthogonal Polynomials. Bull. A. M. S., 40:743–52.

1935

56. Mathematical Principles in the Theory of Small Samples. Am. Math. Month., 42:344–64.

1936

57. Bernstein's Theorem and Trigonometric Approximation. Trans. A. M. S., 40:225–51.
58. Formal Properties of Orthogonal Polynomials in Two Variables. Duke Math. J., 2:423–34.

1937

59. Orthogonal Polynomials on a Plane Curve. Duke Math. J., 3:228–36.
60. Polynomial Approximation on a Curve of the Fourth Degree. Bull. A. M. S., 43:388–93.

1938

61. Problems of Closest Approximation on a Two-dimensional Region. Am. J. Math., 60:436–46.
62. Orthogonal Polynomials in Two Complex Variables. Ann. Math., 39(2):262–68.
63. Orthogonal Polynomials in Three Variables. Duke Math. J., 4:441–54.

1939

64. A Class of Orthogonal Functions on Plane Curves. Ann. Math., 40: 521–32.
65. A New Class of Orthogonal Polynomials. Am. Math. Month., 46: 493–97.

1940

66. Orthogonal Polynomials with Auxiliary Conditions. Trans. A. M. S., 48:72–81.

1941

67. Fourier Series and Orthogonal Polynomials. Carus Math. Monog., 6. 8+234 pp.

1942

68. Generalization of a Theorem of Korous on the Bounds of Orthonormal Polynomials. Bull. A. M. S., 48:602–8.
69. The Instantaneous Motion of a Rigid Body. Am. Math. Month., 49: 661–67.

1943

70. Legendre Functions of the Second Kind and Related Functions. Am. Math. Month., 50:291–302.

1944

71. The Harmonic Boundary Value Problem for an Ellipse or an Ellipsoid. Am. Math. Month., 51:555–63.
72. Boundedness of Orthonormal Polynomials on the Loci of the Second Degree. Duke Math. J., 11:351–65.

1945

73. The Boundedness of Certain Sets of Orthonormal Polynomials in One, Two and Three Variables. Trans. A. M. S., 58:167–83.

1946

74. The Boundedness of Orthonormal Polynomials on Certain Curves of the Third Degree. Bull. A. M. S., 52:899–907.

1948

75. The Boundedness of Orthonormal Polynomials on Certain Curves of the Fourth Degree. Trans. A. M. S., 63:193–206.

DISSERTATIONS UNDER THE DIRECTION OF DUNHAM JACKSON

1915

Milne, William E. The Degree of Convergence of Birkhoff's Series. Harvard.

Wilder, C. E. The Degree of Approximation to Discontinuous Functions by Trigonometric Sums. Harvard.

1924

Carlson, S. Elizabeth. On the Convergence of Certain Methods of Closest Approximation. Minnesota.

Jensen, Carey M. Some Problems in the Approximate Representation of a Function by a Sturm Interpolating Formula. Minnesota.

1928

Earl, James M. Polynomials of Best Approximation on an Indefinite Interval. Minnesota.

Phipps, Cecil Glenn. Problems in Approximation by Functions of Given Continuity. Minnesota.

Risselman, William C. Approximation to a Given Function by Polynomials in Another Function. Minnesota.

1929

Michelson, Earl E. On the Approximate Representation of a Function of Two Variables. Minnesota.

1930

McEwen, William H. Problems of Closest Approximation Connected with the Solution of Linear Differential Equations. Minnesota.

1931

Scherberg, Max G. The Degree of Convergence of a Series of Bessel Functions. Minnesota.

1933

Hoel, Paul G. Certain Problems in the Theory of Closest Approximation. Minnesota.

Wilder, Marian A. Some Problems in Closest Approximation over a Discrete Set of Points. Minnesota.

1934

Oberg, Edwin N. Approximate Solution of Integral Equations. Minnesota.

1938

Peebles, Glenn H. Some Generalizations of the Theory of Orthogonal Polynomials. Minnesota.

1939

Koehler, Fulton. Systems of Orthogonal Polynomials on Certain Algebraic Curves. Minnesota.

Dimsdale, Bernard. Lacunary Orthogonal Polynomials. Minnesota.

only a few months before he died. In spite of his long and trying illness, he maintained creative ability, mathematical contacts with his advanced students, interest in the affairs of the A.M.S., and his normal relations with his family and many friends to the end of his life.

With the preceding sketch as a background, I shall now consider Jackson's life and achievements in more detail.

Dunham Jackson was born on July 24, 1888, son of William Dunham Jackson and Mary Vose Jackson in Bridgewater, Massachusetts. The family was of straight Puritan stock, descended on both sides from passengers on the "Mayflower." His father was a professor of science and mathematics in the Normal School at Bridgewater. This was an institution of high quality emphasizing experimental departures, and finally became a four-year college. Jackson's parents, as well an many relatives, were graduates of the Normal School. The father took added training, but no degree, at the Massachusetts Institute of Technology. Originally he taught botany, geology, physics, chemistry, and astronomy at the Normal School. When it became a college, he taught physics and mathematics through the level of elementary calculus.

The early life of Jackson had an intelligently religious orientation. The members of his family were Congregationalists.[3] His father was a deacon and treasurer of the church and, for many years, was superintendent of the Sunday School. The tone of the household was consistent with this background. When Jackson was in his teens, the minister of the church was a man of exceptional talents, brilliant, widely read, and alive to modern currents of religious thought. He was very stimulating to the young people of his congregation and gave them a firm, enlightened religious foundation. Jackson maintained a corresponding attitude with respect to religion throughout his life. His high ideals were in harmony with these early influences in his home and community.

[3] I am indebted to Jackson's sister, Dr. Elizabeth Jackson, Associate Professor of English at the University of Minnesota, and to Jackson's diary, for many of the details in this biography.

1944

Martin, Margaret Pearl. Some New Systems of Orthogonal Polynomials on Algebraic Curves. Minnesota.

1946

Saunders, Roy. Polynomials Approximation over Certain Infinite Intervals. Minnesota.

1947

McEwen, William R. An Upper Bound for the Number of Polynomials of any Specified Degree Associated with an Algebraic Curve in Space. Minnesota. (Codirector, Robert H. Cameron.)

Munro, William D. Orthogonal Trigonometric Sums with Auxiliary Conditions. Minnesota. (Codirector, Robert H. Cameron.)

Swanson, Leonard William. Solution of Certain Differential Equations Associated with the Theory of Orthogonal Polynomials. Minnesota. (Directed by Robert H. Cameron after death of Dunham Jackson.)