Nathan Jacobson

1910–1999

BIOGRAPHICAL *Memoirs*

A Biographical Memoir by
Georgia Benkart

NATIONAL ACADEMY OF SCIENCES

NATHAN JACOBSON

October 5, 1910–December 5, 1999

Elected to the NAS, 1954

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By Georgia Benkart

Nathan Jacobson was one of the most influential algebraists of the twentieth century. For his fundamental contributions to the theories of Lie and Jordan algebras and to the study of associative algebras satisfying polynomial identities, he was elected a member of the National Academy of Sciences in 1954. A postdoctoral position as research assistant to Hermann Weyl at the Institute for Advanced Study drew him to study Lie algebras, which became a lifelong focus of much of his research. The 1934 *Annals of Mathematics* paper "An Algebraic Generalization of the Quantum Mechanical Formalism" by Jordan, von Neumann, and Wigner introduced him to what later became known as Jordan algebras, another important topic of his work. In the 1960s and 1970s, Jacobson and Kevin McCrimmon developed the foundations of the theory of quadratic Jordan algebras. Essential to understanding one of the families of quadratic Jordan algebras was Jacobson's oft-cited 1958 paper revealing properties of the nonassociative algebra of octonions.

Nathan Jacobson began life as Nachman Arbiser in the Jewish ghetto of Warsaw, Poland, on October 5, 1910 (although official immigration records listed it as September 8).[1] His father emigrated to the United States when Nachman was five, took the name Charles Jacobson, and opened a grocery store in Nashville, Tennessee. When he had established himself well enough, he sent for his wife and their children, who in the United States would become known as Pauline Rosenberg Jacobson and Solomon (Sol) and Nathan (later universally known as Jake). The family's journey amid World War I was facilitated by the Hebrew Immigrant Aid Society and required traveling through Germany to their departure port in the Netherlands in a sealed freight car. The family moved to Alabama and then Mississippi, and Nathan received most of his elementary and secondary education in these states. He then enrolled in the University of Alabama in 1926. There, to his enduring gratitude, Professors William P. Ott and Fred Lewis encouraged him to switch from pre-law courses to mathematics. He earned a bachelor's degree from Alabama in 1930.

With additional encouragement, Jacobson applied to graduate schools and chose Princeton University because he was offered financial support. Jacobson noted that he found Princeton "a very exciting place in the thirties." The twenty or so graduate students, who all lived in the Graduate College, donned their academic gowns to dine at Procter Hall, where they talked mathematics "almost exclusively." Eminent mathematician Joseph Henry Maclagan Wedderburn directed Jacobson's dissertation, "Non-commutative Polynomials and Cyclic Algebras." He earned his doctoral degree in mathematics in 1934. The stipends and teaching salary he earned as a graduate student enabled him to put aside enough money to visit Europe in 1935 and reunite with close relatives in Warsaw, who subsequently perished in the Holocaust.

Research Inspirations and Interests

Jacobson was inspired by a course given by Wedderburn, particularly the part devoted to his structure theory of finite-dimensional associative algebras that reduced their study to one of division algebras. He remained interested in division algebras throughout his career and returned to them in his final book, published in 1996.[2]

At the recently opened Institute for Advanced Study in Princeton, Jacobson, who was hired in 1934 as Weyl's assistant pending Richard Brauer's arrival, wrote up Weyl's lecture notes on continuous groups. Sophus Lie's foundational work had reduced properties of analytic groups, or what are now called Lie groups, to corresponding properties of the tangent space at the identity element. The tangent space is a nonassociative algebra, which at that time was referred to as the "infinitesimal group." Weyl proposed an independent study of such algebras, and in the lecture notes offered the name Lie algebras for them.

Weyl believed that it would be of interest to study Lie algebras over arbitrary fields without recourse to the group or to the algebraic closure of the field. Jacobson, who was well versed in Wedderburn's similar investigations on associative algebras, readily took to the task. His first paper on the subject,[3] which appeared in 1935, acknowledged Weyl's profound influence. It re-derived the well-known theorems of Lie and Engel on solvable and nilpotent Lie algebras by using methods from elementary linear algebra, and it set the stage for "rationalizing" other parts of the theory.

An example of the rationalizing process involves Jacobson's notion of a weakly closed subset S in a finite-dimensional associative algebra A.[4] "Weakly closed" means that for each ordered pair of elements a,b in S, there is a scalar s(a,b) so ab + s(a,b)ba is in S. If every element a of S is nilpotent ($a^k = 0$ for some k), then the associative subalgebra <S> of A generated by S is nilpotent (<S>m = 0 for some m). Jacobson perfectly phrased this result so it can be invoked for Lie and Jordan algebras and Lie superalgebras. It is note-worthy as one of the few general results that apply over any field, even those of prime characteristic.

An associative algebra A under the product a!b = ½(ab + ba) is a Jordan algebra, which by custom is denoted A^+. The study of Jordan algebras was begun in 1934 by Jordan, von Neumann, and Wigner in an attempt to formulate the foundations of quantum mechanics in terms of the Jordan product a!b rather than the associative product ab.[5] Observables, which are represented as Hermitian operators on a Hilbert space, are closed under the composition a!b but not under the product ab. Their work greatly sparked Jacobson's interest in Jordan algebras, which became a major theme of his research. The subsequent algebraic development of the theory of Jordan algebras by Albert, Jacobson, and McCrimmon has had important connections to Lie groups and Lie algebras, geometry, and real and complex analysis.

Jacobson became acquainted with Emmy Noether and her groundbreaking research when she came from Bryn Mawr to lecture at the Institute for Advanced Study each week. He was asked to take over Noether's courses at Bryn Mawr for a year after she died unexpectedly in April 1935. Noether's work inspired Jacobson's study of division algebras and central simple algebras in many ways. In his expository contribution to a 1983 symposium in honor of Noether's 100th birthday, Jacobson provided a new derivation of Brauer factor sets and explained their connection with Noether factor sets.[6] He then explicated the properties of the factor sets when the associated algebra has an involution of orthogonal type. In this paper Jacobson was revisiting a subject he had treated in a different way in a paper from 1944.[7]

For any kind of algebra (associative, Lie, Jordan, etc.), the linear transformations D that satisfy the "derivative property" D(ab) = D(a)b + a D(b) are said to be derivations. Derivations are very natural objects to study especially in Lie theory, because the multi-plication operator, or adjoint mapping, of x, adx(y) = [x,y], of a Lie algebra is always a derivation. In general, the composition DD' of two derivations need not be a derivation; however, the set of all derivations is a Lie algebra under the commutator product

[D, D'] = D D' – D' D. If the underlying field has characteristic $p > 0$, then it is a consequence of Leibniz's formula that Dp is a derivation. It was Jacobson's great insight that the property of being closed under p-powers conveys important structural information. This idea led him to introduce the notion of restricted Lie algebras[8] in the mid-1930s and to further explore their properties in the 1940s.[9,10,11] For such Lie algebras L, if $x = 0$ is the only element of L with $adx = 0$, the mapping (adx)p is a derivation, and it equals the adjoint mapping of some element x[p] of L.

The Lie algebras associated to algebraic groups (the analogues of Lie groups over arbitrary fields) are always restricted, as are the characteristic p versions of the classical simple Lie algebras sl(n), so(n), sp(2n). They are simple too, except when p divides n, and it is necessary to factor out the scalar multiples of the identity matrix in sl(n). However, they and the analogues of the five exceptional Lie algebras are not the only finite-dimensional simple Lie algebras over algebraically closed fields of prime characteristic. The Witt algebra, which is the derivation algebra of the truncated polynomial algebra, provides an example, as do the Jacobson-Witt algebras, which are the derivations of the multivariable polynomials, where the pth power of the variables is 0. The latter algebras were discovered and investigated by Jacobson in the early 1940s as part of his efforts to develop a Galois theory for purely inseparable field extensions using derivations rather than automorphisms.[12] His work set the stage for Kostrikin and Šafarevič,[13] who identified four unifying families of simple Lie algebras living in the Jacobson-Witt algebras. Those four are called the Cartan type Lie algebras because they correspond to Cartan's four infinite families (Witt, special, Hamiltonian, contact) of infinite-dimensional complex Lie algebras. They played a crucial role in the classification of the finite simple restricted Lie algebras in characteristic $p > 7$,[14] and ultimately in the classification all finite-dimensional simple Lie algebras in prime characteristic, notably in the work of Helmut Strade and Alexander Premet.

In the depths of the Great Depression there were few faculty openings, and even fewer for young Jewish academics. Following a year (1936-37) as a National Research Council fellow working with Adrian A. Albert at Chicago, Jacobson considered himself extremely fortunate to be hired for one of two research professorships created by the University of North Carolina through an extraordinary collaboration between its president, Frank Graham, and Department of Mathematics Chair Archibald Henderson. The U.S. Navy opened a pre-flight school on the North Carolina campus after the United States entered World War II, and Jacobson was one of the professors it sent to Chicago for "teacher training." There, he reconnected with Florence "Florie" Dorfman, one of Albert's doctoral students, and they were married in the summer of 1942. Because of changes in the Navy program, the Jacobsons moved to Baltimore in 1943, where Johns Hopkins University, which had a similar program associated with the U.S. Army, also wanted an algebraist for its faculty. Son Michael was born there in 1944. Daughter Pauline Ida (Polly) followed in 1947, and the couple's third joint collaboration, a paper showing that the universal algebras of semisimple Jordan algebras of characteristic zero are semisimple, establishing the complete reduciblity of the representations of such algebras.[15]

"At the termination of World War II," Jacobson recalled, "discrimination against Jews at the major American universities began to wane." As a result, he "became the first Jew to hold a tenured position in the mathematics department" at Yale University. En route to New Haven, the Jacobsons took what he called "a detour to Chicago," where he taught summer courses on Galois theory while working with Irving Kaplansky. Besides lessening discrimination against Jewish academics, the postwar world offered opportunities for travel and international research collaboration unavailable during the decades of the Depression, fascism, and war.

In the context of associative rings, in 1945, Jacobson investigated the ideal J of elements of a ring that annihilate all left (or equivalently all right) simple R modules.[16] J is now called the Jacobson radical. Said differently, we define an ideal M to be primitive if it is the annihilator of a simple left R module. J is, then, the intersection of all primitive ideals. If M is a primitive ideal, then Jacobson and Chevalley showed that R/M (which is called a primitive ring) is a dense subring of the endomorphism ring of a vector space over a division ring. Then for any ring R, R/J is a so called subdirect sum of such dense subrings. This gives a powerful structure theory for arbitrary noncommutative rings, generalizing the Artin-Wedderburn theorem for Artinian rings. One early consequence was a remarkable commutativity theorem of Jacobson. Namely, if R is a ring (perhaps without unit) and for every $x \in R$ there was a natural number $n(x)$ such that $x^{n(x)} = x$, then R is commutative. .

Jacobson's 1952 paper[17] showed that for any finite-dimensional Lie algebra L of characteristic p and for any element a of its universal enveloping algebra U(L), there is a polynomial f(x) such that f(a) lies in the center of U(L). Although rather easy to state and prove, this result had deep consequences: It provided a proof that every finite-dimensional Lie algebra of characteristic p has a faithful representation; it gave a proof of Chevalley's conjecture that every such Lie algebra has a representation that is not faithful; and it showed that U(L) can be embedded in a division algebra. Because the polynomial f(a) must act as a scalar when the field is algebraically closed, Jacobson's result also led to the notion of a reduced enveloping algebra and central characters, important concepts for modular representation theory.[18]

From the 1950s through the 1980s, Jacobson held visiting professorships or lectureships in at least nine different countries outside North America and gave research talks in numerous others. In addition to participating in meetings of the International Congress of Mathematicians (ICM) and the International Mathematical Union (IMU) held throughout the world, Jacobson spoke at a congress of mathematicians of the Soviet Union (USSR) in Leningrad in 1961. At the 1962 ICM in Stockholm, he observed that none of the Jewish mathematicians from the USSR were present to give their invited addresses. By the 1970s, Soviet anti-Semitism would be a serious concern for Western mathematicians in general as well. In the meantime, American academia faced urgent domestic issues, especially over U.S. involvement in the war in Vietnam.

Although Jacobson devoted most of his research to associative and Jordan algebras after the mid-1950s, he wrote two books, *Lie Algebras*[19] and *Exceptional Lie Algebras*[20] and supervised a number of graduate students in Lie theory. *Lie Algebras* transformed the earlier work of Cartan and Killing into a highly understandable text. Its status as a "classic" having been confirmed by its 1979 re-publication in the Dover series, *Lie Algebras* remains the best basic reference for restricted Lie algebras and for an exposition of the famous embedding result known as the Jacobson-Morosov theorem. *Exceptional Lie Algebras* details the construction of the five exceptional simple Lie algebras, and it is a valuable resource for anyone wanting to know about their structure. These books, like Jacobson's papers, have a timeless quality, and one marvels at just how readable his works are even now, more than seventy years after many of them were written. Jacobson's book *Structure and Representations of Jordan Algebras*[21] and his *Lectures on Quadratic Jordan Algebras*[22] made Jordan algebras accessible to a wide audience. The only previous book on the subject, *Jordan-Algebren*[23] by Hel Braun and Max Koecher, was motivated by questions in analysis despite being essentially algebraic in nature.

Critical to understanding one of the families of quadratic Jordan algebras was Jacobson's influential 1958 paper[24] establishing properties of the octonions and algebras that admit a norm form with composition. Jacobson had realized much earlier that these algebras were responsible for exceptional phenomena,[25] in particular the 27-dimensional simple Jordan algebra that he had studied earlier with Albert and which cannot be realized as an algebra A^+.[26] He showed that it is an indispensable ingredient in the construction of the five exceptional simple Lie algebras E_6, E_7, E_8, F_4, G_2,[27] and that the group of automorphisms of this exceptional 27-dimensional simple Jordan algebra provides valuable data on forms of exceptional Lie algebras and on the principle of triality.[28-31] The definition of Jordan algebras was unsuitable for fields of characteristic 2 and for Jordan algebras over arbitrary rings. In the 1960s and 1970s, Jacobson and his doctoral student Kevin McCrimmon set about to remedy this situation in a series of articles by introducing the notion of a quadratic Jordan algebra and developing results on the structure and representations of quadratic Jordan algebras.[32-36]

In 1964 Jacobson gave a talk (published 1966) entitled "Forms of Algebras"[37]. In this work Jacobson unified known results that classified algebras, Lie Algebras, and Jordan algebras which become canonical forms of these objects over the algebraic closure. In this work there is an early version of the now standard result relating forms of algebraic objects and the nonabelian first Galois cohomology group.

In 1965, Jacobson became chair of the Yale Mathematics Department, which further developed his skills and reputation beyond the realm of mathematical research. In close cooperation with Yale's high-profile president, Kingman Brewster, Jacobson led the department's creation of the Gibbs Instructorships for early-career mathematicians and negotiated senior faculty appointments for Abraham Robinson and Robert Langlands. At the same time, he was moving through successively higher offices in the American Mathematical Society (AMS), of which he became president in January 1971, after his chairmanship at Yale but during the war in Vietnam. The previous president had tried to keep the society distanced from such a "political issue," and this mindset now had ramifications for the way the AMS operated. After a resolution by Anatole Beck calling for prompt termination of American participation in the War in Southeast Asia was defeated at the AMS annual meeting in January 1971, the summer meeting in August of that year was forced to deal with AMS policies and procedures. Beck and Mary Gray received special permission to address the AMS Council, and Gray's focus on the "undemocratic" nature of the nominating process resulted in the appointment of an AMS Committee for Women chaired by Cathleen Morawetz, as well as in the founding of the Association for Women in Mathematics in 1971, of which Gray served as the first president.

By 1973, the AMS had a Human Rights Committee chaired by Jacobson, who also was a vice-president of the IMU and chair of the Mathematics Section of the National Academy of Sciences. In those capacities, he took up the case of I. I. Pietetski-Shapiro, a top mathematician in the USSR, who had been unemployed since making an unsuccessful request for a visa to Israel. Finally allowed to emigrate in 1976, Pietetski-Shapiro divided his time between Israel and Yale.

As a member of the Consultative Committee for the ICM at Vancouver in 1974, Jacobson became more aware of both the restrictions imposed on Soviet Jewish invitees and the obstructive efforts of committee member S. V. Jablonskii, who had tried to prevent Israel Gelfand from chairing any panel at the ICM in 1970. Ultimately, none of the invited Soviet Jewish speakers appeared in Vancouver.

In 1975 Jacobson published *PI-Algebras, an Introduction*[38] in the Springer Lecture Note series. First, of course, this book was a considerable spur to research in polynomial identity rings. However, almost half of these notes are devoted to a clear development of the properties of the so called universal division algebra and an exposition of the remarkable noncrossed product result of Amitsur. These notes were extremely influential in subsequent research into division algebras. In 1978, the AMS *Notices* published "The Situation in Soviet Mathematics" by anonymous recent émigrés from the USSR. Jacobson was one of sixteen mathematicians who signed an appended statement in support of the article. A consequence was an article in the Russian Academy of Sciences journal *Uspekhi Matematicheskikh Nauk* attacking Jacobson as an "aggressive Zionist." Its author was L.S. Pontryagin, his Soviet former co-vice president of the IMU, who also boasted of having kept Jacobson from becoming president of the IMU. Jacobson published his response in *Notices* of the AMS.[39]

The celebration of Jacobson's seventieth birthday in 1980 was followed in June 1981 by a conference in honor of his retirement from Yale. More than seventy mathematicians attended, including twenty-four of his doctoral graduates. The conference proceedings were published in the AMS series Contemporary Mathematics.[40] Jacobson described retirement as a "comparatively minor discontinuity" in his career. He kept his office at Yale, continued his editorial work on two journals, and retained a travel grant. Florie Jacobson retired in 1983 from Albertus Magnus College, where she had taught since 1955 and had chaired the mathematics department since 1958. They travelled extensively after retirement, and both lectured in Taiwan on a trip to Hong Kong, the People's Republic of China, and Taiwan. In 1992, Nathan suffered a debilitating stroke, but with the aid of Florie, he was able to finish his last book.[41] Her death in 1996, following 54 years of marriage, was a crushing blow to him. Although his health declined further after that, he was still able to attend the Joint Mathematics Meetings in Baltimore in January 1998, where he was honored with the AMS Leroy P. Steele Prize for Lifetime

Achievement. The citation read:

> *To Nathan Jacobson for his many contributions to research, teaching, exposition, and the mathematical profession. Few mathematicians have been as productive over such a long career or have had as much influence on the profession as has Professor Jacobson.*

A prolific writer, Jacobson authored fourteen volumes of lecture notes and books that have been revised and reprinted numerous times over the years. Many present at the ceremony had received their first taste of abstract algebra from this well-known text series.

Jacobson was extraordinarily busy during my graduate years at Yale, a period when he was vice-president of the IMU and president of AMS. Yet he was a calm, reassuring mentor who never seemed rushed. In the summer before I graduated, he took up gardening with the same zeal he tackled so many things. When he summoned me to his office, which was rather unusual, it caused me a great deal of angst. But the purpose was to present me with a grocery bag full of zucchini. "Well," he said, "I guess I learned one thing—two people don't need thirty zucchini plants." Although that was among many things he taught me, I have since warned graduate students that when their advisor offers to support them, be sure to ask how. We, his thirty-three Ph.D. students who experienced his gentle guidance and appreciated his gracious kindness, owe him a special debt that perhaps can be repaid only in kind, by emulating his behavior with our own graduate students.

Nathan Jacobson died December 5, 1999. He and Florie were survived by their son Michael and their daughter Polly, one granddaughter and three great-grandchildren. His mathematical legacy continues to inspire future generations.

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