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TJALLING CHARLES KOOPMANS

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A Biographical Memoir by
HERBERT E. SCARF

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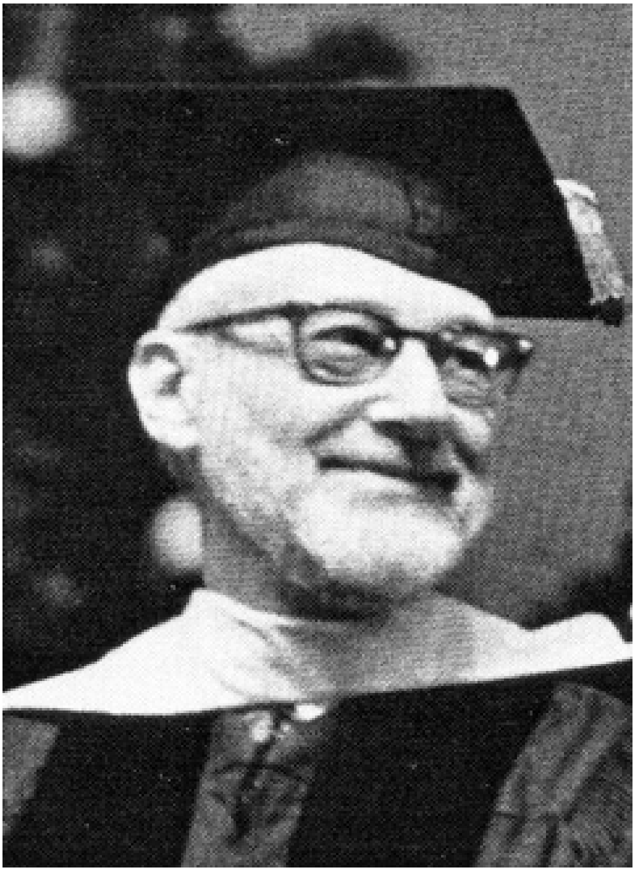

Tjalling C. Koopmans

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August 28, 1910–February 26, 1985

BY HERBERT E. SCARF

TJALLING CHARLES KOOPMANS, one of the central figures in modern economic science, played seminal roles in the modern theory of the allocation of scarce resources and in the development of statistical methods for the analysis of economic data. In both of these areas Koopmans creatively mobilized and developed the methods of other quantitative disciplines for the purposes of economics: mathematical statistics became econometrics, and linear programming became the activity analysis model of production. Koopmans was also one of the major scholars concerned with the study of economic growth and the economic consequences of the depletion of nonrenewable resources. He was a remarkably inspired and inspiring leader of research who combined his considerable mathematical power with a deep concern for the ultimate practical applications of his work.

Koopmans was born in the village of 's Graveland, near the town of Hilversum, in the Netherlands, on August 28, 1910; he was the third son of Sjoerd Koopmans and Wijtske van der Zee. Both his mother and father were born in Frisia, a province in northeastern Holland. Sjoerd's father was the owner of a small shop in the rural area of Toppenhuizen; Wijtske's father was a painter of fancy carriages and also an artist who painted many landscapes and portraits, now owned by his great-grandchildren. The family in which Sjoerd grew up was severe and Calvinistic, in contrast to Wijtske's family, which was more relaxed and liberal about religious matters. At the age of sixteen Sjoerd became the schoolteacher of a small school in Toppenhuizen and was entrusted with the education (including bible instruction) of the neighborhood children. He was said to have been very stern in the classroom, perhaps as a consequence of the many responsibilities he assumed at so early an age. Wijtske was also trained as a schoolteacher and, after their marriage, the couple left Frisia and eventually settled in 's Graveland, where Koopmans's father became the principal of a much larger "school with the bible."

The family house, as Koopmans described it in an autobiographical sketch written when he received the Nobel Prize in economic sciences in 1975,

> . . . was squeezed between two sections of that school. The row of these three buildings was, as [were] almost all houses in the village, sandwiched between one long street and a parallel straight and narrow canal marking one of the village's boundaries. Across the street were large wooded estates each with meadows and a large mansion. The occupants of the mansions kept aloof from the life of the village except for the employment of coachmen, gardeners, servants and contractors.
>
> Every weekday morning at nine, our living quarters and the narrow strip of garden at the back were engulfed by the sound of three different hymns sung dutifully, simultaneously, but independently in true Charles Ives fashion, by the schoolchildren on both sides.

Despite frequent illnesses Koopmans had a happy childhood in this rural environment, with its many meadows and canals. His formal education began at his father's school, with its heavy emphasis on biblical studies, and was followed by five years at the Christian High School at Hilversum, some ten miles away. At the high school Tjalling studied

Latin, Greek, mathematics, physics, chemistry, and three modern languages. He was instructed in the theory of evolution by a teacher who remarked at the end of the course, “the Bible says otherwise.”

The Koopmans family was very musical, and sang together regularly. Sjoerd played the harmonium, and Tjalling was taught the violin as a child. He was not entirely satisfied with his skill on this instrument, and in his later life he replaced the violin with the piano. Both secular and sacred learning were highly valued in the Koopmans household. There were prayers before every meal and Bible reading in the evening, with the servants called from the kitchen to collect around the dinner table and participate in religious instruction. Tjalling’s father was the dominant influence, and the atmosphere in the home and the school was a stern and disciplined one.

Tjalling left home for the University of Utrecht at the age of seventeen. At Utrecht, boarding was arranged with the minister to the city prisons, whose surname was Couvée. This was an experience very different for Koopmans from living at home; there were many young children, some close to Tjalling in age, and much lively social activity. Due to his post the father of the family had seen a good deal of the raw life of the city, and, while religious, he was not strict nor dogmatic. The mother was French, and Tjalling became quite comfortable with the language. He stayed with the family for two years.

It was customary for a young man to take formal religious vows at the age of seventeen or eighteen. Koopmans wrestled with the issue for a considerable period of time, and, in what was a difficult experience both for himself and his parents, he formally renounced his ties to the Protestant faith while at the university. But the moral and educational values of his early home remained with him and were probably the central source of the great personal integrity and strong sense of purpose that he displayed throughout his lifetime.

Koopmans's academic abilities must have been apparent quite early, for he was awarded a generous stipend by a private foundation—the St. Geertruidsleen—at the age of fourteen. This scholarship supported his studies until his twenty-sixth birthday and relieved his family of the financial burden of his education. At the university Koopmans commenced with the study of mathematics—in particular, analysis and geometry. He had a vivid geometrical intuition, and, in many of his subsequent publications, elaborate analytical arguments are frequently simplified by the use of insightful geometrical figures. He read widely in other subjects, ranging from physics to history, psychology, and psychiatry. For a while he contemplated entering the profession of psychiatry, but, in a somewhat less dramatic change of field, he moved (in 1930) from pure mathematics to theoretical physics. This shift in subjects, a first step toward his eventual decision to take up economics, was "a compromise between my desire for a subject matter closer to real life and the obvious argument in favor of a field in which my mathematical training could be put to use."

Koopmans's professor at Utrecht was Hans Kramers, the leading theoretical physicist in Holland at the time. He admired Kramers enormously and described him as "a humane and inspiring person with a gentle wit." In 1933 Koopmans wrote an important paper on quantum mechanics, which is still frequently cited by physicists many years after its publication. But, of course, these were the years of the Great Depression, and theoretical physics must have seemed remote from the distress of daily economic life. As Koopmans later said, "It dawned on me that the economic world order was unreliable, unstable, and most of all, iniquitous." He began, at the suggestion of fellow students, to read the works of Karl Marx; this was his first exposure to abstract economic reasoning. While he was not persuaded by Marxian economic analysis, he felt deeply moved by Marx's description of the plight of workers during the Industrial Revolution.

It was at this point that Koopmans was introduced to Jan Tinbergen, who was seven years older and already one of the leaders in the new field of mathematical economics. Tinbergen, who was to share the first Nobel Prize in economic science with Ragnar Frisch in 1969, had been trained in mathematical physics as a student of Ehrenfest. He had been a conscientious objector to military service at the age of eighteen and, as an alternative obligation, was required to spend some time at the Statistical Office in the Hague, where he became acquainted with and concerned about social and economic issues. Despite his change in interest Tinbergen continued to work with Ehrenfest; his Ph.D. thesis, written in 1929 at Leiden, was on the topic of minimization problems in both physics and economic theory. After receiving his degree Tinbergen began to develop the elements of a mathematical theory of business cycles and to construct a formal mathematical model of the Dutch economy.

Koopmans decided to affiliate himself with Tinbergen. He moved from Utrecht to Amsterdam in January of 1934 and joined a group of Tinbergen's young disciples, among them Truus Wanningen, whom Koopmans was to court and, finally, marry in October 1936.

Tinbergen offered a weekly lecture in economics, which Koopmans attended. As he later said in his Nobel biographical sketch,

In the first half of that year [1934], I had the privilege of almost weekly private tutoring from him over lunch after his lecture. I have been deeply impressed by his selflessness, his abiding concern for economic well-being and greater equality among all of mankind, his unerring priority at any time for problems then most crucial to these concerns, his ingenuity in economic modeling and his sense of realism and wide empirical knowledge of economic behavior relations.

Tinbergen instructed Koopmans in many aspects of mathematical economics and econometrics. He suggested that Koopmans read the works of the theorists Cassel and Wicksell and that he become familiar with the field of statistics and its applications to economic problems.

Tinbergen had a profound influence on Koopmans's professional career, and it may be useful to make a brief digression about Tinbergen's work on business cycles and macroeconomic models. In order to place this work in perspective, let me describe a fundamental distinction between two attitudes toward dynamic models in economic theory. We are all familiar with the basic idea that prices are determined so as to equate the supply and demand for goods and services. In its most elementary form, the demand for a particular commodity may be thought of as a function of its price (and perhaps the prices of other competing commodities) and demand declines as the price rises. Similarly, the supply brought forth by producers of a particular commodity may be viewed as a function of the price at which the commodity may be sold (and the prices of the factors of production required to manufacture the commodity); typically, the supply of a commodity rises as its price increases. The static equilibrium price is at the intersection of these two curves.

Suppose that we wish to examine a dynamic variant in which the commodity is produced and consumed at a sequence of consecutive points of time. On the one hand, we can imagine that the production and consumption decisions are made in the presence of perfect futures markets and with the full knowledge of the prices that are expected to prevail over time. Making use of this information, producers purchase factors of production and consumers purchase outputs at times when they are inexpensive and store them for future use, seeking to smooth their production and consumption plans over time. On the other hand, we can imagine that the imperfections of financial institutions require that such choices be made in a myopic fashion, attending only to those prices and values of other significant economic variables that prevail today.

In the first version, prices would clear both spot and futures markets instantaneously; the model would describe an economic situation of full dynamic equilibrium with no underemployment of resources. In the latter variant, markets would respond sluggishly to previous signals and the evolution of the economy might best be described by a mathematical system in which the future values of major economic variables are an extrapolation of their past values.

Clearly, the depression years of the early 1930s could not be accurately described by a classical model in which all economic resources are fully employed. Tinbergen was drawn to the alternative formulation, which had played an important role in the analysis of business cycles and which was ultimately to lead to the Keynesian model. For example, Tinbergen published a paper in 1931 in which cycles in shipbuilding are analyzed by means of a simple difference-differential equation stating that the increase in available shipping tonnage at a particular time is related linearly to the stock of tonnage with a fixed time delay. There is no explicit consideration of freight rates or the costs of constructing new shipping. Freight rates are examined in subsequent papers but not in the neoclassical manner as those prices that equilibrate the demand for shipping services with its supply. Instead, Tinbergen engaged in skillful curve fitting; he fitted a regression of freight rates to a pair of indices purporting to measure the demand and supply of shipping services and the cost of coal.

A number of themes that appear in these early works of Tinbergen became major influences in Koopmans's later research agenda. Tinbergen's concerns with the shipping industry were to stimulate Koopmans's subsequent interest in formal mathematical models of transportation. Tinbergen's use of statistical analysis opened up a series of questions that were to preoccupy Koopmans and other scholars for many years, and Koopmans's fundamental research in economic growth theory very probably had its roots in the early dynamic models of Tinbergen.

Koopmans's Ph.D. dissertation, titled "Linear Regression Analysis of Economic Time Series," was supervised jointly by Tinbergen and Kramers; the degree was granted in November 1936. In retrospect, this thesis can be seen as an important step in the development of modern econometric methodology. By the 1930s economists had already been exposed to the use of regression analysis and other statistical techniques in analyzing the relationship between the demand for a particular good and its price and in the study of business cycles. The parameters in Tinbergen's model of the Dutch economy had been estimated using multiple correlation analysis with a degree of care and detail not seen in previous economic reports, and Frisch had developed his own ingenious statistical methods. But the new paradigm for statistics offered by R. A. Fisher had not yet found its way into econometric analysis prior to Koopmans's thesis.

The major innovation suggested by Fisher was an assessment of the merits of various statistical methods based on a formal probabilistic model. To take an important example, consider a set of observations $(y_i, x_i)_{i=1,\ldots,T}$ of a dependent variable y and an independent variable x. A linear relationship, $y = \alpha x + \beta$, between these two variables can be obtained by a least squares regression of y on x. But such a regression is essentially an exercise in curve fitting, and the parameters could equally well be found by other contending methods, such as one that minimizes the sum of the absolute values of the deviations, rather than the sum of their squares. In order to justify the use of one particular method, Fisher introduced an underlying probabilistic model that is assumed to generate the observed data. For example, assume that the observations y_i are independently drawn from normal distributions with means $ax_i + b$, and with a common standard deviation σ. Given the parameters a, b, and σ and the sequence of values of the independent variable $x = (x_1,\ldots,x_T)$, the probability of observing the sequence $y = (y_1,\ldots,y_T)$ can be expressed as a function $F(\mathrm{y}|a,b,\sigma;x)$. For the observed sequence (y, x), Fisher suggests that the parameters a, b, and σ be selected so as to maximize this likelihood function, that is, to select those parameters that give the highest probability to the sequence of observed data.

Economic data are distinctly different in at least two very significant ways from those arising in the agricultural experiments that motivated Fisher's analysis. Economic data are similar to astronomical observations in the sense that they are natural observations that do not arise in experimental laboratories. The independent variables x, which might represent temperature and other experimental parameters in Fisher's controlled experiments could, in an econometric study, become the prices at which a sequence of commodity demands were observed. But even if prices were thought of as being independent variables in the sense that the price of food would cause a certain level of demand for food to arise, these prices could not be set by the experimenter and would, themselves, be measured with error.

After an exposition of Fisher's program, Koopmans's thesis contains a lucid set of proposals for accommodating the particular econometric problem that all of the relevant variables might be measured with error. He does not, at this point, address a second major problem, that is, the fact that causal connections are far from obvious in economics and the values of many economic variables might very well be considered to be simultaneously determined. This is a point that will arise again.

In the period 1936-38 Tinbergen was called to the League of Nations at Geneva to find out, with the aid of statistics, which theory of the business cycle was closest to reality. At Geneva Tinbergen also prepared a business-cycle model of the United States. Koopmans took over the teaching of his class in mathematical economics at the Netherlands School of Economics in Rotterdam. During this time Koopmans embarked on a lengthy study of the relationship between freight rates and the construction of oil tankers. The study was not based on a formal mathematical model, but it did display a sure grasp of economic theory and a detailed knowledge of the tanker industry that was remarkable for a young scholar recently preoccupied with mathematical physics. The work was published as a monograph titled *Tanker Freight Rates and Tankship Building* by the Netherlands Economic Institute in 1939. There is a clear foreshadowing in the monograph of Koopmans's subsequent interest in the construction of optimal transportation routes.

In 1938 Tinbergen and Koopmans exchanged places. Tinbergen returned to Rotterdam and Dr. and Mrs. Koopmans moved to Geneva, where Koopmans was assigned the task of constructing a mathematical model of the United

Kingdom's economy. In early 1939 he attended a conference on Tinbergen's work at Oxford University. At the conference Koopmans met a number of economists, including Jacob Marschak, with whom he was to have a long and significant relationship. Later in the year the Koopmans went on a leisurely vacation, traveling through the French Alps by bus. As Mrs. Koopmans later related to me, "We had a good time and I became pregnant." Their first child, Anne, was born prematurely in April of 1940.

It was, of course, a time when the signs of war were everywhere; the invasion of Poland took place during the Koopmans's vacation. In April 1940 the Germans invaded Norway, and the Koopmans family decided to leave Europe for the United States. As Mrs. Koopmans described it to me:

> Not a stitch of work was being done because everybody foreign to Switzerland was struggling desperately to get away. We ourselves were scrambling for a visa—to the U.S., Canada, Cuba, even to Martinique. We were lucky; we had an invitation to come to Princeton, arranged for us by Professor Samuel Wilks, with whom we had become very friendly the year before, and we had gotten a visitors' visa. Furthermore, because Tjalling's term at the League of Nations was coming to an end, we had already arranged for passage on a Dutch ship for Genoa to the U.S. Somehow that passage on the Dutch ship was converted into passage on an American ship almost on the spot. I believe that happened in Bordeaux.
>
> The chance to get away came up suddenly, so I had hurriedly packed a small trunk with necessities and clothes, and a suitcase with diapers and milk powder for our 6-weeks-old baby. Then we got word that the U.S. ship (the *Washington*) was ordered to Bordeaux instead of to Genoa after Italy entered the war. We heard that at 9 a.m. on June 4; at 12:00 noon, we were on the train to Bordeaux. The Polak family had given us a travel basket for the baby; others supplied us with sleeping bags; Tjalling carried his briefcase, the luggage and gas masks; I carried the baby. We never saw our trunk again. Because we had a baby, we were given a small cabin to ourselves while the rest of the ship slept dormitory style. The vessel was only half full in Bordeaux—the day after we left Switzerland France closed all its

borders—and many Americans who had been booked to sail were stranded in Italy and Spain. But while we were en route, the ship was ordered to Lisbon to pick up many people there, so that then the ship was filled to its capacity of 1,000 passengers. After that, we went to pick up more Americans in Galway, Ireland. Our adventure was not over for on the way to Ireland we were halted by German submarines and ordered into the lifeboats. Fortunately, it got across to the Germans that the ship was an American one, and America had not entered the war yet, so after some 4 hours of terror in the water, we were on our way again. In Galway, we took aboard another 1,000 persons. The rest of the trip was uneventful. We learned of the fall of Paris while at sea and we arrived in New York with only the clothes on our back, the child in her basket and some borrowed money. We had nothing else whatsoever.

The next several years were to be peripatetic. The departure from Europe was sudden, and long-term employment could not be arranged before arriving in this country. In 1940-41 Koopmans was engaged as a research assistant at Princeton and, simultaneously, taught a course in statistics at NYU. During this time, Koopmans worked on a celebrated problem of mathematical statistics in the tradition of earlier work by R. A. Fisher: the exact distribution of the serial correlation coefficient in normal samples. Koopmans derived a representation for this distribution by means of a contour integral and illustrated the use of an ingenious smoothing approximation that facilitated numerical computations. His paper, titled "Serial Correlation and Quadratic Forms in Normal Variables," was published in the *Annals of Mathematical Statistics*. It remains a permanent contribution to a problem that was never fully solved analytically yet absorbed the interest of many of the world's leading mathematical statisticians throughout the 1940s.

After a year the jobs at Princeton and NYU were terminated, and Koopmans took a position as an economist at the Penn Mutual Life Insurance Company in Philadelphia. A paper, "The Risk of Interest Fluctuations in Life Insurance Operations," which does not seem to have been published, was written at this time.

In 1942 the family left Philadelphia for Washington, where Koopmans was to be employed for two years as a statistician for the British Merchant Shipping Mission. The work was interesting though routine, and Koopmans found the time to initiate a line of inquiry about the economics of cargo routing. This was eventually to be of great significance in the development of linear programming and in the study of the activity analysis model of production.

Koopmans's problem can be described in the following way. Given a list of ports, the flows of a homogeneous shipborne cargo can be described by a graph, whose vertices are the ports and whose edges are marked by the tonnage shipped between that pair of ports. Given also a fixed set of supplies at some ports and demands at others, an increase in the amount shipped from one particular port to another will cause compensating changes in the matrix of flows between other pairs of ports. In the paper, "Exchange Ratios Between Cargos in Various Routes," written in 1942, Koopmans showed how to calculate these compensating changes and their consequences for the total cost expressed in ton-miles.

The problem of determining the shipping plan that minimizes total cost, given a preassigned pattern of availabilities of supplies and demands, is known as the transportation problem. It is one of the most elementary examples of a linear programming problem, that is, the maximization of a linear function of several variables, subject to a series of linear inequality constraints. But in 1942 the concept of linear programming had not yet been proposed in the West, and Koopmans was unable to see his work as an instance of this more general problem.

In 1939 Jacob Marschak, whom Koopmans had previously met in Oxford, left Europe to become a professor at the New School for Social Research. There he organized a seminar in mathematical economics and econometrics, and the relationship between the two scholars was renewed when Koopmans attended the seminar on a regular basis in 1940 and 1941. In 1943 Marschak was appointed director of research at the Cowles Commission for Research in Economics at Chicago, and in 1944 Koopmans wrote to Marschak about his desire to leave Washington. Soon after, Koopmans accepted Marschak's invitation to join the staff of the Cowles Commission, and thus began a long association—both with Marschak and the commission—that was to prove extraordinarily productive.

The Cowles Commission for Research in Economics was founded in 1932 by Alfred Cowles, the president of Cowles and Company, an investment counseling firm with offices in Colorado Springs, Colorado. Mr. Cowles's initial motivation in establishing the commission was to assemble a group of mathematicians, statisticians, and economists whose combined efforts might provide a rational basis for investment choices. The formal charter of the organization, however, allowed for a broader mandate and contained the phrase, "The particular purpose and business for which said corporation is formed is to educate and benefit its members and mankind, and to advance the scientific study and development . . . of economic theory in its relation to mathematics and statistics." It was this broader mandate that was ultimately adopted by the commission, which, during its long history, was to become a primary vehicle for the elaboration and dissemination of quantitative methods in economics. During the last half-century, the subject of economics has been transformed by the introduction of quantitative techniques, and the Cowles Commission has played a major role in this process. I know of no other example in the history of science in which a research institution, founded and nourished by a private patron, has had so profound an impact on an intellectual discipline.

Initially the organization was located in Colorado Springs, with a small research staff headed by Charles A. Roos, who became the commission's first director of research in 1934. Starting in 1935, summer conferences were held regularly, with an ever-widening research agenda and group of participants from the United States and abroad. As pleasant as the location was for summer conferences, however, Mr. Cowles found it difficult to attract permanent staff to Colorado Springs, and he arranged for the commission to move to Chicago, where it became affiliated with the University of Chicago in 1939. Theodore Yntema, the first director of research at Chicago, was succeeded by Jacob Marschak in 1943.

Marschak was a scholar of great intellectual force, curiosity, and initiative. As director he continued the program of summer conferences, but now there was a dramatic increase in the number of visitors and the size of the resident staff. Marschak organized a series of weekly seminars, as well, and initiated the practice of disseminating research results as discussion papers and reprints. Leonid Hurwicz had been recruited by Yntema, and in the next several years Trygve Haavelmo, Koopmans, Herman Rubin, Lawrence Klein, Theodore Anderson, Kenneth J. Arrow, Herman Chernoff, Herbert Simon, and other distinguished statisticians and economists were to be associated with the commission in one way or another. The early research agenda, set by Marschak, was primarily concerned with the particular statistical problems arising in the estimation of parameters in a set of simultaneous equations.

The idea that the relationships among economic variables are best described by a set of simultaneous equations is a time-honored concept of economic theory. The price of a given commodity and the quantity purchased may be depicted by the intersection of a demand curve and a supply curve—the first relating the demand for the commodity to its price (given the incomes of consumers), and the second relating the supply of the commodity to its price (given the prices of the factors used in its production). Each of these equations will involve various parameters whose estimation is required if the system is to be used for the prediction of future values of price and quantity. The naive approach is to estimate the parameters in each equation separately using ordinary least square regressions. The question was: How good are the naive methods?

In several extremely important publications, Trygve Haavelmo, previously a student of Frisch, laid the groundwork for answering this question. Using the probabilistic methods of R. A. Fisher, Haavelmo assumed that the observed series of economic variables satisfied a system of, say, linear equations with stochastic errors governed by specific probability distributions with unknown parameters. Given the parameters of the error terms and of the equations themselves, any particular set of possible values will have a well-defined probability. The maximum likelihood estimates of the unknown parameters are those that give the highest probability to the values of the economic variables actually observed. As Haavelmo had shown, these maximum likelihood estimates could differ substantially from ordinary least squares estimates.

At an even more basic level, the structure of the system of equations may make estimation of the unknown parameters impossible. If, for example, prices and quantities are derived from the intersection of demand and supply curves, there may not be enough information to ascertain the separate slopes of each of these curves. It was the study of these statistical problems that Koopmans took up as his major area of concern soon after arriving at the Cowles Commission. A first paper concerned the bias arising from an ordinary least squares regression of the parameters of a single equation, if the equation is, in reality, part of a larger system. A second paper, written with the assistance of Herman Rubin and Roy Leipnik, provided a complete solution to the problem of "identification," that is, a description of the necessary and sufficient conditions that permit the structural parameters of a linear system to be determined uniquely from the probability distributions of the data and hence amenable to statistical estimation. This latter paper also developed systems of maximum likelihood estimators and derived their large sample statistical properties. The theoretical advances in this paper proved to be of lasting significance. Its results are still the core of the theory of simultaneous equations and endure in every textbook treatment of the subject.

In addition to his research on these and other aspects of econometrics, Koopmans organized a Cowles Commission Conference (in early 1945) devoted to the statistical problems arising from a system of simultaneous equations. He also edited the report of the conference, published as Cowles Commission Monograph No. 10, in 1950. This volume eventually became a classic in the field, and its themes have been fundamental in both the teaching of econometrics and subsequent research.

Koopmans became the acknowledged leader of that school of econometrics, focusing on the problem of simultaneity and insisting on a complete probabilistic model of the data to be analyzed. In 1947 he took the battle to the profession as a whole in his review of the volume, *Measuring Business Cycles*, authored by Arthur F. Burns and Wesley C. Mitchell. Koopmans found this work, written by two senior economists associated with the National Bureau of Economic Research, deficient in several respects. First of all, it was a detailed analysis of a great volume of data relating to business cycles, but its categories were not based on an underlying theoretical model incorporating maximizing behavior of the individual agents in the economy. Second, the statistical approach was eclectic, with no formal probabilistic model to account for the data and to justify the use of the author's statistical techniques. The methodology used by Burns and Mitchell was descriptive, Koopmans maintained, rather than flowing from the logical and analytical stance toward economic data that was at the heart of the Cowles program.

A passionate rebuttal to Koopmans's review was offered by Rutledge Vining, who stressed the merits of a synthetic approach capable of suggesting tentative hypotheses in an important area of economic discourse lacking a formal model. There was much jockeying about on the issue of whether economics was currently in the Tycho Brahé phase—simply codifying and mastering unstructured masses of data—or in the Keplerian and Newtonian phase in which a parsimonious and robust paradigm was available for explanation and illumination. Both the review and the rebuttal were written with such lucidity, scholarship, and care for these eternal economic concerns as to commend them to the general reader some four decades later.

At the Cowles Commission, Koopmans continued his study of the transportation problem that he had initiated in 1942. By the end of 1946 he realized that his earlier problem of transporting a homogeneous commodity from a set of origins to a set of destinations so as to minimize the total cost of transportation could be formulated as a problem of minimizing a linear function of a number of variables, subject to a set of linear inequalities constraining the values assumed by these variables. He also proposed a method of solution based on an economic idea that was to become of central importance in his subsequent research.

A particular instance of the transportation problem is specified by the supply at each origin, the demand at each destination, and a matrix of unit costs for shipping from each origin to each destination. Koopmans observed that a vector of prices, one for each location, could be associated with the optimal shipping plan. The prices would meet the condition that each route in use would make a profit of zero, in the sense that the price at the destination would equal the price at the origin plus the unit cost of shipping along that route. The routes not in use would, moreover, have a profit less than or equal to zero. He also demonstrated that if such a system of prices could be associated with an arbitrary feasible solution to the constraints of the transportation problem, the feasible solution would indeed be the optimal solution. The arguments made use of the theory of convex sets, which were to become of great importance in the study of the general linear programming problem.

Koopmans presented these ideas at a meeting of the International Statistical Conference in Washington in September 1947. Several months earlier he had a consequential meeting with George B. Dantzig, who was the first Western scholar to study the general linear programming problem. Dantzig had initiated his work on linear programming while employed by the U.S. Department of the Air Force, and in the summer of 1947 he developed the details of the simplex method, an algorithm for their solution. The simplex method is a remarkably effective computational technique that converges to the optimal solution in a relatively small number of iterations, even for problems of substantial size. The method makes use of a system of dual variables—one for each inequality—that are used at each step of the algorithm to test whether some of those activities not currently in use should be introduced. In the special case of the transportation problem, these dual variables are precisely those prices previously employed by Koopmans.

Subsequent to his meeting with Dantzig, Koopmans extended his observations about the relationship between prices and optimality to the general activity analysis model of production. In an activity analysis model the possible techniques of production available to a firm, or to the economy as a whole, are given by a finite list of elementary activities that can be used simultaneously and at arbitrary non-negative levels. The resulting production possibility set is a polyhedral cone, approximating the smooth transformation sets of neoclassical economics to an arbitrary degree of accuracy. The activity analysis model, a generalization of the Leontief input/output model, can be used to generate a large number of distinct linear programs, depending on the objective function to be chosen and on the specific set of factor endowments.

Koopmans demonstrated that an efficient plan—a plan for which no alternative existed using less inputs and providing no less of any output—would be associated with a vector of prices with a special property. The prices, intimately related to Dantzig's dual variables, would yield a zero profit for the activities used in that plan and a profit less than or equal to zero for all the remaining activities. Conversely, a feasible production plan associated with such a vector of prices would in fact be efficient. This permitted Koopmans to make the fertile suggestion that if the correct prices were known the optimal selection of activities could be accomplished in a decentralized fashion by managers who were mindful of their private considerations of profit maximization. In this way Koopmans gave precision to the intuitive beliefs of economists, from Adam Smith onwards, that a decentralized competitive economy achieves socially optimal results "as if by an invisible hand."

In 1948 Koopmans succeeded Marschak as the director of the Cowles Commission. A conference on activity analysis was sponsored by the commission in 1949, and the results of the conference appeared in Cowles Commission Monograph No. 13 in 1951. The monograph, edited by Koopmans, contained a paper by Dantzig on linear programming as well as a lengthy exposition of the activity analysis model by the editor. In this paper and in a nontechnical essay published in *Econometrica*, Koopmans demonstrated a sharp awareness of the relationship of these ideas to the fascinating discussion of socialist economic planning in the 1930s.

His strong convictions regarding the importance of the activity analysis model for economic planning in Eastern Europe led Koopmans to make extended trips to the Soviet Union in 1965 and 1970. There he met Leonid Kantorovich, a Soviet mathematician who independently initiated the study of linear programming in 1939. Kantorovich, who was to share the Nobel Prize with Koopmans in 1975, had developed a test for optimality and an outline of an algorithm for linear programming that was similar to but more cumbersome than the simplex method. In Kantorovich's work the problem of the optimal allocation of resources was approached not only from the point of view of a pure mathematician, but also with the economist's appreciation of the fundamental role played by prices in reaching an optimal decision.

Research in econometric methodology continued at the Cowles Commission, but under Koopmans's leadership and guidance new lines of activity in economic theory were initiated. The modern study of the general equilibrium model, in which the theory of production is united with a description of consumer preferences, was inaugurated by Arrow and Gerard Debreu; Arrow's classic *Social Choice and Individual Values* was in the making. At the same time Harry Markowitz was working on portfolio analysis; Arrow, Theodore Harris, and Marschak were writing an optimal inventory policy, and formal theories of decision-making under uncertainty were proposed.

In 1955 the commission left the University of Chicago for Yale University, where it was renamed the Cowles Foundation for Research in Economics. James Tobin, whom the commission had earlier tried to lure to Chicago, assumed the directorship in New Haven. Moving along with Koopmans were Debreu, Marschak, Roy Radner, and Martin Beckmann.

The last several years at Chicago were charged with intellectual disagreements between the staff of the Cowles Commission and members of the Department of Economics. Tjalling felt under considerable pressure and began to compose music. The Koopmans and their three children, Anne, Henry, and Helen, spent two summers at Bennington, visiting with friends and attending a composers' conference in which instruction in composition was given and the members of the group had their works played and recorded. The children were small and the family—which was of great importance to Tjalling—enjoyed swimming, hiking, and other outdoor activities.

Koopmans's strong desire to make the results of theoretical and mathematical analysis available to a wide audience of nonspecialists is revealed in the remarkable volume, *Three Essays on the State of Economic Science*, published in 1957. The relationship between prices and economic efficiency in both static and dynamic models of production and the role played by the assumption of convexity in welfare economics are discussed by means of simple geometric diagrams and with a lucidity rarely attained by an active research scientist. A second expository tour de force was his paper, "Selected Topics in Economics Involving Mathematical Reasoning," written jointly with Bausch, which appeared in 1959.

In the decade of the 1960s Koopmans's major research preoccupation was the theory of economic growth, in which he directly addressed questions of efficiency and optimality in dynamic models of production. He published a masterful paper, "On the Concept of Optimal Economic Growth," in which his original presentation of the calculus of variations was used to study the maximization of an objective function given by a discounted sum of utilities. In the model the input of labor is assumed to be exogenously growing. Output, which can be allocated between consumption and investment, is specified by a production function based on inputs of capital and labor. In several other publications he introduced a class of stationary utility functions that properly included the previous discounted sum of utilities, and he used this larger class to study the concept of "impatience": roughly speaking, a preference for current rather than postponed consumption. The analysis was based on a sophisticated generalization of the concept of Haar measure independently arrived at by Koopmans and his collaborator, Richard Williamson.

In the autobiographical sketch written when he received the Nobel Prize, Koopmans says, "In most of my Yale period my research, chiefly on optimal allocation over time, had more of a solitary character." But this is only in contrast to the Chicago days, when the energies of the entire Cowles team were focused on specific projects. In Chicago the commission was engaged in a methodological revolution involving the use of formal mathematics in economic theory and econometrics. By 1960 the battle had been won; the troops no longer had to be massed for assaults on exposed positions. Mathematical reasoning had become an accepted mode of exposition for economic arguments, and the members of the Cowles Foundation felt freer to pursue their own individual substantive interests.

By the early 1970s Koopmans may have felt that the mathematical revolution led by him had been too successful—that elaborate mathematical arguments were being advanced throughout the profession to the neglect of more immediate practical concerns. He began to apply the techniques of growth theory to the study of exhaustible resources and, in particular, those resources used in the provision of energy. A lengthy study of copper supplies was initiated, in collaboration with William Nordhaus, his colleague in the Department of Economics, and Robert Gordon and Brian Skinner, both geologists at Yale. He took on the chairmanship of a committee of the National Academy of Sciences devoted to the study of alternative energy systems. This was followed by a one-year visit to the International Institute for Applied Systems Analysis (IIASA), in Laxenburg, Austria, where he succeeded George Dantzig (in the second half of 1974) as the leader of the Methodology Group.

On the morning in October 1975 when his Nobel Prize was announced, I visited Tjalling and Truus Koopmans at their home. The prize was shared with Kantorovich for their independent work on the optimal allocation of resources. Much of our conversation was taken up by Tjalling's distress about the fact that George Dantzig had not shared the prize. In a characteristic gesture involving a fine blend of morality and precise computation, Tjalling told me that he had decided to devote one-third of his prize to the establishment of a fellowship in honor of Dantzig at IIASA. As we left the house for a press conference at Cowles, Tjalling said, with a certain shy amusement about what was awaiting him, "Now I have become a public man."

In 1978 Koopmans agreed to assume the presidency of the American Economics Association, after the death of his longtime friend, Marschak, who had been president-elect. His presidential address, "Economics Among the Sciences," was devoted to a discussion of the differences in outlook of economists, engineers, and natural scientists engaged in interdisciplinary collaboration. The paper, written with Tjalling's characteristic conceptual clarity and mastery of the facts, was illustrated by his work on energy modeling and other topics addressed in recent reports of the National Research Council.

Looking back, one can see a pattern in Koopmans's professional career. He would invest himself for an extended period of time in a particular area of study in which his analytical capabilities could be used to clarify a large issue of potential practical value. He would gather together a group of collaborators, scholars with diverse backgrounds, and energize them with his benignly patriarchal sense of purpose and direction. He would make personal friendships with his intellectual associates, play chess with them, listen to music with them, and take them on canoe trips and long walks. The customary anxieties of the isolated research scholar would be handed over to Tjalling, the leader of the group, whose confidence and resolve would provide comfort and quiet any doubts. But, at the same time, he himself would be engaged in an internal debate about the merits of the collaborative activity—and, if the reckoning so indicated, he could deliberately take leave of the activity and prepare himself for the next venture.

Tjalling suffered a series of cerebral strokes in the last months of 1984. In the short time between then and his death on February 26, 1985, at the age of seventy-four, he was still capable of intellectual and social interaction with his family and with the loving friends who surrounded him.

I AM VERY GRATEFUL for many conversations with Truus Koopmans and for the advice and assistance given to me by Kenneth J. Arrow, Gerard Debreu, George Dantzig, Leo Hurwicz, Alvin Klevorick, Peter Phillips, Martin Shubik, Herbert Simon, T. N. Srinivasan, Jan Tinbergen, and James Tobin.

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