



Bertram Kostant

1928–2017

BIOGRAPHICAL

Memoirs

*A Biographical Memoir by
Nolan Wallach*

©2020 National Academy of Sciences.
Any opinions expressed in this memoir are
those of the author and do not
necessarily reflect the views of the
National Academy of Sciences.



NATIONAL ACADEMY OF SCIENCES

BERTRAM KOSTANT

May 24, 1928–February 2, 2017

Elected to the NAS, 1978

Shing Shen Chern gave a framed piece of his calligraphic art to Bertram Kostant for his sixtieth birthday. The Chinese characters translate to “Bertram Kostant the master of Lie groups.” Kostant’s research spanned the second half of the twentieth century and continued into the second decade of the twenty-first. This work was characterized by its elegance and ingenuity. His most important work involved Lie algebra cohomology, the orbit structure of Lie group actions and their application to representation theory, topology and physics. He received the Wigner Prize in June 2016 “for his fundamental contributions to representation theory that led to new branches of mathematics and physics.”



photo courtesy of Ann Kostant

Bertram Kostant

By Nolan Wallach

Bertram Kostant’s earliest work was on the holonomy of homogeneous spaces. This led him to a deep understanding of the structure of semi-simple Lie algebras and led to his important work on a theorem of Bott (the so-called Bott, Borel, and Weil Theorem) by explicitly writing down a basis for the pertinent cohomology spaces. This paper has played an important role in many of the future applications of Lie algebra cohomology to representation theory and to automorphic forms. He went on to study the principal three-dimensional simple Lie subalgebras of a simple Lie algebra over the complexes. He showed that up to conjugacy there is exactly one. He then used these algebras to calculate the cohomology of the simple Lie groups and the structure of the nilpotent cone in a semi-simple Lie algebra. These papers have been critical to most of the developments in representation theory over the past 50 years. His next major work involved symplectic geometry. He introduced the notion of prequantization for symplectic manifolds with integral symplectic forms. This theory allowed for a philosophical generalization of the work of Kirillov on the representation theory of simply connected nilpotent Lie groups. As a byproduct, he showed that homogeneous symplectic manifolds are coadjoint orbits

under mild conditions on the symmetry group. His “big picture” involved a search for quantizations of his prequantizations in order to construct irreducible unitary representations of reductive Lie groups. It is still unfinished, although it has been a guiding principle for many mathematicians (for example David Vogan and Michelle Vergne). For the split real forms of D_4 and E_8 , he (and Ranee Brylinski in the case of E_8) showed how to quantize the minimal nilpotent orbits to construct the unitary representation of minimal Gelfand-Kirillov dimension. He used representation theory to give explicit solutions to his generalized Toda Lattices (which include the so-called non-periodic Toda lattice). His quantization of his Toda lattices yielded Whittaker vectors for the principal series of quasi-split semi simple Lie groups. In addition, he and Stephen Rallis developed a complete generalization of Kostant’s work on the nilpotent cone in a simple Lie algebra to symmetric pairs. Kostant used this work in his 1975 paper on the spherical principal series for which he was awarded the Steele Prize in 1990.

Kostant made a beautiful conjecture about the existence of certain groups of the form $PSL(2)$ over a finite field with q elements in the adjoint group of a simple Lie group, G , over C . He used the notation $L(2,q)$ for these groups. He conjectured that if h is the Coxeter number of G and if $q = 2h+1$ is prime then $L(2,q)$ imbeds as a subgroup of the adjoint group of G . In the case of E_8 the Coxeter number is 30 and he was therefore asking about $L(2,61)$. This case was first proved by computer calculations by Greiss and coworkers and by Serre without computers. E_8 , the largest of the exceptional simple Lie groups, was his favorite among the Lie groups. He was a walking encyclopedia on it and found amazing connections between it and truly diverse phenomena (e.g. the Buckeyball and perhaps HIV the AIDS virus). In addition, anyone who did research with Kostant knows that he remembered every piece of work that he had done, down to giving page numbers for specific theorems.

Bertram Kostant was born on May 24, 1928 in Borough Park, Brooklyn. He graduated from Stuyvesant High School in 1945 and did his undergraduate work at Purdue University, graduating with distinction in mathematics in 1950. Professor Arthur Rosenthal, a German emigre, was a major influence on Kostant at Purdue and was one of the reasons he switched his major from Chemical Engineering to Mathematics. He did his graduate work at the University of Chicago, partially supported by an Atomic Energy Commission Fellowship for graduate studies. His thesis advisor was Irving Segal, and he received his Ph.D. under Segal’s direction in 1954. At that time, the University of Chicago was one of the most important centers of mathematics in the world; the “Nancago” (short for Nancy and Chicago) University involved a constant flow of

Bourbaki luminaries to Chicago. Also on the faculty were Marshall Stone, Adrian Albert, Shing Shen Chern, Paul Halmos, Irving Kaplansky, Irving Segal, and as a member of Nancago, Andre Weil.

Kostant was a member of the Institute for Advanced Study in Princeton from 1953–1956. In his talk at the banquet for his 65th birthday conference, he mentioned that he had met Einstein during that period. He had a conversation with Einstein and then drove him home. He was one of the last people to see Einstein alive. During the years, 1956–1962 Kostant was a professor at the University of California, Berkeley. This was the time when the “center of gravity” of American mathematics moved to Berkeley with Chern, Smale, Spanier, and others moving from Chicago to Berkeley. In 1962, Kostant accepted a professorship at MIT, where he stayed until he became Professor Emeritus in 1993. His was a very active “retirement,” with many invitations to lecture at universities throughout the world and, of course, continuing his brilliant research in Lie Theory and its applications. A perusal of MathSciNet shows that he published at least 46 peer-reviewed papers after his retirement.

His honors include a Guggenheim Fellowship to Paris (1959–1960), a Sloan Fellowship (1961–63), election as a Fellow of the American Academy of Arts and Sciences in 1962 and to the National Academy of Sciences in 1978. In 1990, he was awarded the Steele Prize for Seminal Contribution to Research, and in 2016, he was awarded the Wigner Prize.

Kostant was a devoted husband to Ann, his wife of 49 years. Ann Kostant has been a force in mathematical publishing through her work for Springer. He and Ann were the parents of five children: Abbe Kostant Smerling, Steven Kostant, Elizabeth Loew, David Amiel, and Shoshanna Kostant. They also had nine grandchildren and two great-grandchildren.

SELECTED BIBLIOGRAPHY

- 1958 A theorem of Frobenius, a theorem of Amitsur-Levitski and cohomology theory. *J. Math. Mech.* 7(2):237-264.
- 1959 A formula for the multiplicity of a weight. *Trans. Amer. Math. Soc.* 93:53-73.
The principal three-dimensional subgroup and the Betti numbers of a complex simple Lie group. *Amer. J. Math.* 81:973-1032.
- 1961 Lie algebra cohomology and the generalized Borel-Weil theorem. *Ann. of Math.* 74(2):329-387.
- 1963 Lie algebra cohomology and generalized Schubert cells. *Ann. of Math.* 77(2):72-144.
Lie group representations on polynomial rings. *Bull. Amer. Math. Soc.* 69:518-526.
- 1966 *Groups over Z. Algebraic Groups and Discontinuous Subgroups* (Proc. Sympos. Pure Math., Boulder, Colo., 1965) Pp. 90-98. Providence, RI: Amer. Math. Soc.
- 1967 With L. Auslander. Quantization and representations of solvable Lie groups. *Bull. Amer. Math. Soc.* 73:692-695.
- 1969 On the existence and irreducibility of certain series of representations. *Bull. Amer. Math. Soc.* 75:627-642.
- 1970 *Quantization and unitary representations. I. Prequantization. Lectures in modern analysis and applications, III.* pp. 87-208. Lecture Notes in Math., Vol. 170. Berlin, Germany: Springer, Berlin.
- 1971 With L. Auslander. Polarization and unitary representations of solvable Lie groups. *Invent. Math.* 14:255-354.
- 1975 *On the existence and irreducibility of certain series of representations.* Lie groups and their representations (Proc. Summer School, Bolyai János Math. Soc., Budapest, 1971). Pp. 231-329. New York, NY: Halsted.
On the tensor product of a finite and an infinite dimensional representation. *J. Functional Analysis* 20(4):257-285.

- 1976 On the structure of certain subalgebras of a universal enveloping algebra. *Trans. Amer. Math. Soc.* 218:133–154.
- On Macdonald's eta function formula, the Laplacian and generalized exponents. *Advances in Math.* 20(2):179–212.
- 1977 Harmonic analysis on graded (or super) Lie groups. In *Lecture Notes in Physics*. Vol. 79, pp. 47–50. Berlin, Germany: Springer.
- 1978 With D. Kazhdan and S. Sternberg. Hamiltonian group actions and dynamical systems of Calogero type. *Comm. Pure Appl. Math.* 31(4):481–507.
- On Whittaker vectors and representation theory. *Invent. Math.* 48(2):101–184.
- 1979 The solution to a generalized Toda lattice and representation theory. *Adv. in Math.* 34(3):195–338.
- 1984 On finite subgroups of $SU(2)$, simple Lie algebras, and the McKay correspondence. *Proc. Nat. Acad. Sci.* 81(16):5275–5277. DOI: 10.1073/pnas.81.16.5275
- 1988 The Principle of Triality and A Distinguished Unitary Representation of $SO(4,4)$. In *Differential Geometrical Methods in Theoretical Physics*. Vol. 250. Pp. 65-108. Dordrecht, The Netherlands: Springer.
- 1992 With Ranee Brylinski. Nilpotent orbits, normality and Hamiltonian group actions. *Bull. Amer. Math. Soc.* (N.S.) 26(2):269–275.
- 1997 Quantum cohomology of the flag manifold as an algebra of rational functions on a unipotent algebraic group. *Math. Phys. Stud.* 20:157–175.
- 1999 A cubic Dirac operator and the emergence of Euler number multiplets of representations for equal rank subgroups. *Duke Math. J.* 100(3):447–501.
- 2006 With Nolan Wallach. Gelfand-Zeitlin theory from the perspective of classical mechanics. I. *Studies in Lie theory*, pp. 319–364. *Progr. Math.*, 243. Boston, MA: Birkhäuser Boston.
- 2012 The cascade of orthogonal roots and the coadjoint structure of the nilradical of a Borel subgroup of a semi simple Lie group. *Mosc. Math. J.* 12 (3):605–620, 669.

Published since 1877, *Biographical Memoirs* are brief biographies of deceased National Academy of Sciences members, written by those who knew them or their work. These biographies provide personal and scholarly views of America's most distinguished researchers and a biographical history of U.S. science. *Biographical Memoirs* are freely available online at www.nasonline.org/memoirs.