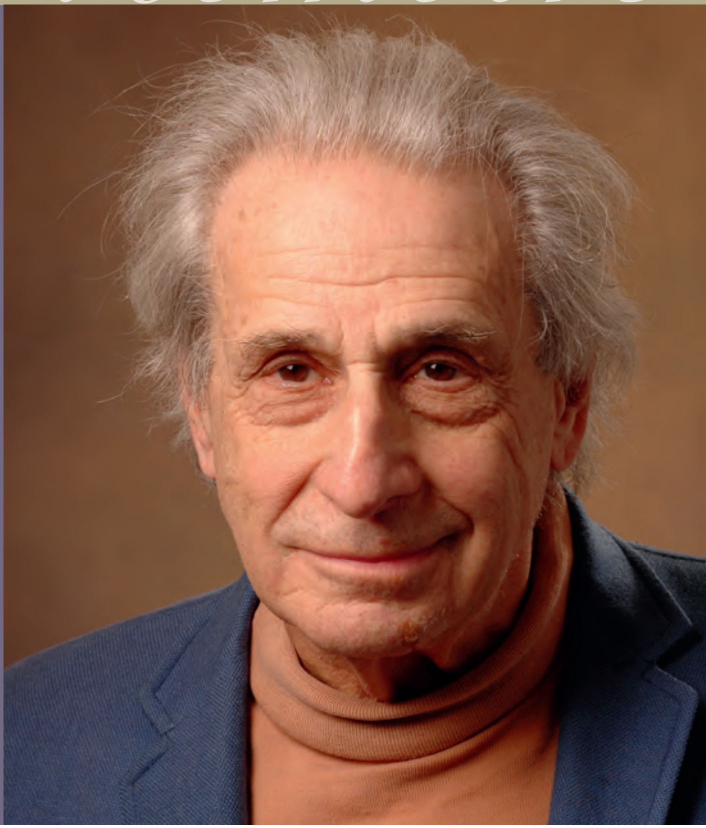
Martin D. Kruskal

1925–2006

BIOGRAPHICAL *Memoirs*



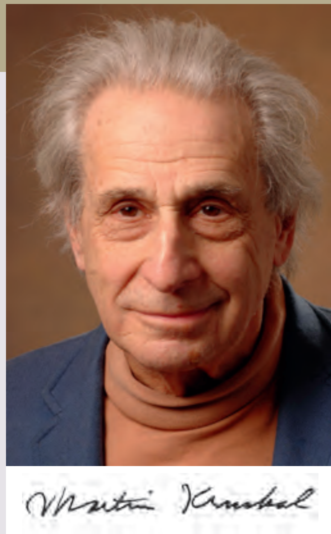
A Biographical Memoir by
Percy Deift

NATIONAL ACADEMY OF SCIENCES

MARTIN DAVID **KRUSKAL**

September 28, 1925–December 26, 2006

Elected to the NAS, 1980



Photograph courtesy of Rutgers University.

By Percy Deift

Martin David Kruskal,* an American mathematician and physicist, made fundamental contributions in wide-ranging areas of mathematics (such as nonlinear analysis and asymptotic analysis) and science (including plasma physics and general relativity). His single most celebrated contribution was the discovery, together with Norman Zabusky, of solitons and their remarkable properties.

Kruskal was a student at the University of Chicago and at New York University, where he completed his Ph.D. in mathematics under Richard Courant and Bernard Friedman in 1952. The dissertation was titled "The Bridge Theorem For Minimal Surfaces."

Kruskal spent much of his career at Princeton University, first as a research scientist at the Plasma Physics Laboratory (starting in 1951) and then as a professor of astronomy (1961), founder and chair of the Program in Applied and Computational Mathematics (1968), and professor of mathematics (1979). He retired from Princeton University in 1989 and joined the mathematics department of Rutgers University, holding the David Hilbert Chair of Mathematics.

Apart from his research, Kruskal was known as a mentor of younger scientists. He worked tirelessly and always aimed not just to prove a result but also to understand it thoroughly. And he was notable for his playfulness. He invented the Kruskal Count,[1] a magical effect that has been known to perplex professional magicians because, as he liked to point out, it was based not on sleight of hand but on a mathematical phenomenon.

Kruskal was born to a Jewish family in New York City and grew up in the suburb of New Rochelle. He was generally known as Martin to the world and David to his family. His father, Joseph B. Kruskal, Sr., was a successful fur wholesaler. His mother, Lillian

* This memoir is adapted from the biography of Martin David Kruskal in *Wikipedia Online* at: *https://en.wikipedia.org/w/index.php?title=Martin %20David%20Kruskal&oldid=712924644* (accessed 2016).

Rose Vorhaus Kruskal Oppenheimer, became a noted promoter of the art of origami during the early era of television and founded New York's Origami Center of America, which later became OrigamiUSA. Martin was one of five children. His two brothers, both of whom became eminent mathematicians, were Joseph Kruskal (1928–2010), who devised multidimensional scaling, the Kruskal tree theorem, and Kruskal's algorithm; and William Kruskal (1919–2005), who created the Kruskal–Wallis test.

Martin Kruskal was married to Laura Kruskal for 56 years. Laura, who (at this writing, in July 2016) survives him, is well known as a lecturer and writer about origami and originator of many new models. Martin, who had a great love of games, puzzles, and word play of all kinds, also invented several quite unusual origami models, including an envelope for sending secret messages (anyone who unfolded the envelope to read

Martin and Laura.
(Photo courtesy of the Kruskal family.)

the message would have great difficulty refolding it to conceal the deed).

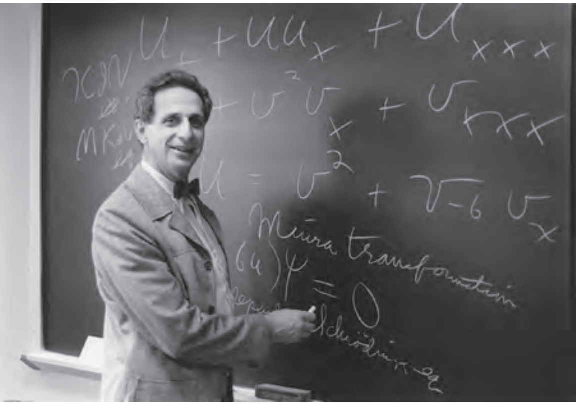
Martin on a day hike.
(Photo courtesy of the Kruskal family.)

Martin and Laura traveled extensively to scientific meetings and to visit Martin's many scientific collaborators. Laura used to call Martin "my ticket to the world." Wherever they went, Martin would be hard at work and Laura would often keep busy by teaching origami workshops in schools and other institutions, usually for the elderly and people with disabilities. Martin and Laura also had a great love of traveling during their own leisure time, and of hiking.

Their three children were Karen, Kerry, and Clyde, who respectively became an attorney, an author of children's books, and a computer scientist.

Martin Kruskal's scientific interests covered a broad range of topics in pure mathematics and in the applications of mathematics to the sciences. He had lifelong interests in many topics in partial differential equations and nonlinear analysis, and he developed fundamental ideas about asymptotic expansions, adiabatic invariants, and numerous related topics.

In the 1950s and early '60s, Kruskal worked largely on plasma physics, developing many ideas that are now fundamental to the field. His theory of adiabatic invariants was, for example, important in fusion research. Other key concepts of plasma physics he contributed, which now bear his name, include the Kruskal-Shafranov instability and the Bernstein-Greene-Kruskal (BGK) modes. And with I. B. Bernstein, E. A. Frieman, and R. M. Kulsrud, he developed the Magnetohydrodynamic (MHD) Energy Principle.[2] In plasma physics, in which Kruskal's interests extended to plasma astrophysics as well as laboratory plasmas, his work is considered by some to have been his most outstanding.



Martin lecturing on KdV and MKdV
(Photo courtesy of Princeton University.)

In 1960, Kruskal discovered the full classical space-time structure of the simplest type of black hole in general relativity.

A spherically symmetric black hole can be described by the Schwarzschild solution, which was derived in the early days of general relativity. However, in its original form, this solution only describes the region exterior to the horizon of the black hole. Kruskal (in parallel with George Szekeres) developed the maximal analytic continuation of the Schwarzschild solution, which he exhibited elegantly by using what are now called Kruskal-Szekeres coordinates.

This led Kruskal to the astonishing discovery that the interior of the black hole looks like a “wormhole” connecting two identical and asymptotically flat universes. His finding provided the first real example of a wormhole solution in general relativity, wherein the wormhole collapses to a singularity before any observer or signal can travel from one universe to the other. This is now believed to be the general fate of wormholes in general relativity. In the 1970s, when the thermal nature of black-hole physics was discovered, the wormhole property of the Schwarzschild solution turned out to be an important ingredient. Nowadays, it is considered a fundamental clue in attempts to understand quantum gravity.

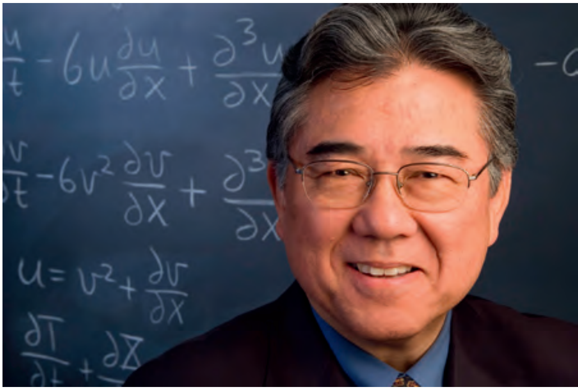
Kruskal’s most widely known work was his discovery in the 1960s of the integrability of certain nonlinear partial differential equations that involve functions of one spatial variable as well as of time. This advance began with a pioneering computer simulation, which Kruskal performed with Norman Zabusky and Gary Deem, of a nonlinear equation known as the Korteweg–de Vries equation (KdV)—an asymptotic model of the propagation of nonlinear dispersive waves. But the researchers made the startling discovery of a “solitary wave” solution of the KdV equation that propagates nondispersively and even regains its shape after a collision with other such waves. Because of the particle-like properties of such a wave, they named it a “soliton,” a term that caught on almost immediately.

This work was partly motivated by the near-recurrence paradox that had been observed in a very early computer simulation of a nonlinear lattice by Enrico Fermi, John Pasta, and Stanislaw Ulam, together with Mary Tsingou, at Los Alamos in 1955.[3] Those authors had observed long-time nearly recurrent behavior for a one-dimensional chain of anharmonic oscillators, in contrast to the rapid thermalization that had been expected. The KdV equation is a continuum limit of that lattice, and the discovery by Kruskal et al. of solitonic behavior, which is the opposite of thermalization, turned out to be the heart of the phenomenon.

Norman Zabusky and Martin Kruskal.
(Photo courtesy of Norman Zabusky.)

Solitary wave phenomena had been a 19th-century mystery dating back to work by John Scott Russell, who in 1834 had observed what we now call a soliton propagating in a canal, and he chased it on horseback.[4] Scott Russell subsequently observed solitons in many, various wave - tank experiments, but his experimental observations, presented in his Report on Waves to the British Association for the Advancement of Science in 1844, were viewed with skepticism by George Airy and George Stokes because their linear water-wave theories were unable to explain them. Some years later Joseph Boussinesq (1871) and Lord Rayleigh (1876) published mathematical theories justifying Scott Russell's observations, and in 1895, Diederik Korteweg and Gustav de Vries formulated the KdV equation to describe shallow-water waves and showed that it had a solution—the 1-soliton solution—which describes Russell's solitary wave. However, the essential properties of this equation were not understood until the work of Kruskal and his collaborators in the 1960s.

Solitonic behavior suggested that the KdV equation must obey conservation laws beyond the obvious conservation laws of mass, energy, and momentum. A fourth conservation law was discovered by Gerald Whitham and a fifth one by Kruskal and Zabusky. Several other new conservation laws were discovered by hand by Robert Miura, who also showed that many conservation laws existed for a related equation known as the modified Korteweg–de Vries (MKdV) equation.[5] Imbedded in Miura's calculation, there was a remarkable connection (now called the Miura transformation) between solutions of the KdV and MKdV equations.



Robert Miura.
(Photo courtesy of the New Jersey Institute of Technology.)

This was the clue that enabled Kruskal, with Clifford S. Gardner, John M. Greene, and Miura, to discover a general technique for the exact solution of the KdV equation and to understand its conservation laws.[6] This discovery was the "inverse scattering method," a surprising and elegant method that demonstrates that the KdV equation admits an infinite number of Poisson-commuting conserved quantities and is completely integrable. The discovery provided the basis for understanding the soliton phenomenon. Soon after, Peter Lax famously interpreted the inverse scattering method in terms of isospectral deformations and so-called Lax pairs.[7]

The inverse scattering method has had an astonishing variety of generalizations and applications in different areas of mathematics and physics. Kruskal himself pioneered some of the generalizations, such as the existence of infinitely many conserved quantities for the sine-Gordon equation. This led to the discovery of an inverse scattering method for that equation by M. J. Ablowitz et al.[8] The sine-Gordon equation is a relativistic wave equation, in 1+1 dimensions, that also exhibits the soliton phenomenon and that became an important model of solvable relativistic field theory. In seminal work preceding Ablowitz et al., Zakharov and Shabat discovered an inverse scattering method for the nonlinear Schrödinger equation.[9]

Solitons are now known to be ubiquitous in nature, from physics to biology, in large part because of the groundbreaking work of Kruskal and Zabusky. In 1986, the two shared the Howard N. Potts Gold Medal from the Franklin Institute "for contributions to mathematical physics and early creative combinations of analysis and computation, but most especially for seminal work in the properties of solitons." In awarding the 2006 Steele Prize to Gardner, Greene, Kruskal, and Miura, the American Mathematical Society (AMS) stated that before their work, "there was no general theory for the exact solution of any important class of nonlinear differential equations." The AMS added, "In applications of mathematics, solitons and their descendants (kinks, anti-kinks, instantons, and

Martin with the National Medal of Science.
(Photo courtesy of the Kruskal family.)

breathers) have entered and changed such diverse fields as nonlinear optics, plasma physics, and ocean, atmospheric, and planetary sciences. Nonlinearity has undergone a revolution: from a nuisance to be eliminated to a new tool to be exploited."

Kruskal received the National Medal of Science in 1993 "for his influence as a leader in nonlinear science [and] for more than two decades as the principal architect of the theory of soliton solutions of nonlinear equations of evolution."

In the 1980s, Kruskal developed a strong interest in the Painlevé equations, which frequently arise as symmetry reductions of soliton equations; in particular, he was intrigued by the intimate relationship that appeared to exist between the properties characterizing these equations and completely integrable systems. Much of his subsequent research was driven by a desire to understand this relationship and to develop new direct and simple methods for studying the Painlevé equations.

The six Painlevé equations have a characteristic property called the Painlevé property: their solutions are single-valued around all isolated singularities in the complex plane. In Kruskal's opinion, given that this property defines the Painlevé equations, one could start with it and, without any additional structures, be able to work out all of the required information about the equations' solutions. The first result was an asymptotic study, with Nalini Joshi, of the Painlevé equations—unusual at the time in that the approach did not require the use of associated linear problems. Kruskal's persistent questioning of classical results led to a direct and simple method, also developed with Joshi, to prove the Painlevé property of the Painlevé equations.

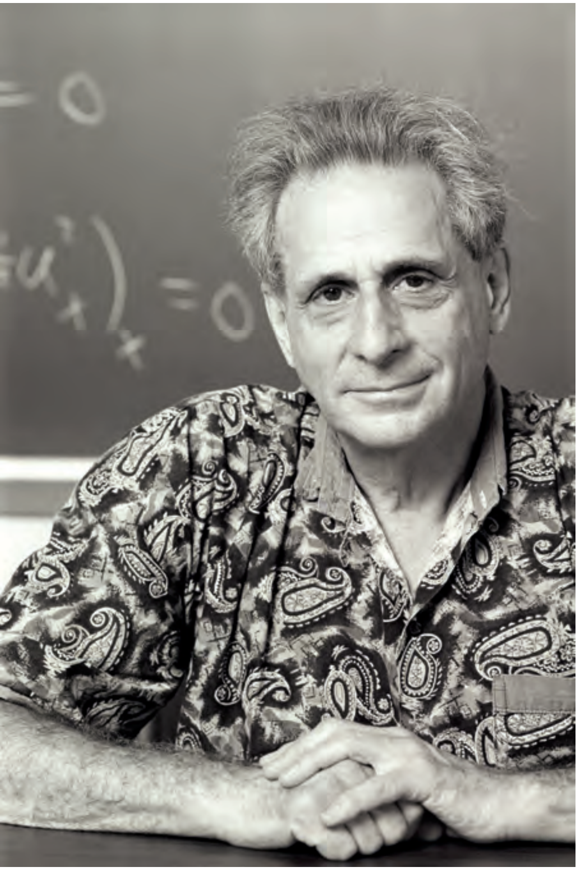
In the latter part of his career, one of Kruskal's chief interests was the theory of surreal numbers, which, when defined constructively, have all the basic properties and operations of the real numbers. Surreal numbers include the real numbers, alongside many types of infinities and infinitesimals. Kruskal contributed to the foundation of the theory, to defining surreal functions, and to analyzing their structure. Among his results, he discovered a remarkable link between surreal numbers, asymptotics, and exponential asymptotics.

From left to right: Russell Kulsrud, Martin Kruskal, Carl Oberman, and Norman Rostoker.
(Photo courtesy of Rutgers University.)

A major open question, raised by John Conway, Kruskal, and Simon Norton in the late 1970s, and investigated by Kruskal with great tenacity, is whether sufficiently well behaved surreal functions possess definite integrals. This question was answered negatively, in the full generality, by Ovidiu Costin, Harvey M. Friedman, and Philip Ehrlich in 2015. However, their analysis showed that definite integrals do exist for a sufficiently broad class of surreal functions for which Kruskal's vision of asymptotic analysis, broadly conceived, goes through. At the time of Kruskal's death, he and Costin were in the process of writing a book on surreal analysis.

Kruskal coined the term "asymptotology" to describe the "art of dealing with applied mathematical systems in limiting cases."[10] He formulated seven Principles of Asymptotology:

1. The Principle of Simplification
2. The Principle of Recursion
3. The Principle of Interpretation
4. The Principle of Wild Behavior
5. The Principle of Annihilation
6. The Principle of Maximal Balance
7. The Principle of Mathematical Nonsense.



(Photo courtesy of Rutgers University.)

But the term asymptotology is not as widely used as the term soliton. Asymptotic methods of various types have been successfully used since almost the birth of science itself. Nevertheless, Kruskal tried to show that asymptotology is a special branch of knowledge—intermediate, in a sense, between science and art. His proposal has been found to be very fruitful.[11, 12, 13]

SELECTED AWARDS AND HONORS

- Gibbs Lecturer, American Mathematical Society (1979)
- Dannie Heineman Prize, American Physical Society (1983)
- Howard N. Potts Gold Medal, Franklin Institute (1986)
- Award in Applied Mathematics and Numerical Analysis, National Academy of Sciences (1989)
- National Medal of Science (1993)
- John von Neumann Lectureship, Society for Industrial & Applied Mathematics (1994)
- Honorary D. Sc., Heriot-Watt University (2000)
- Maxwell Prize, International Council For Industrial and Applied Mathematics (2003)
- Steele Prize, American Mathematical Society (2006).

MEMBERSHIPS

- Member of the National Academy of Sciences (1980)
- Member of the American Academy of Arts and Sciences (1983)
- Foreign member of the Royal Society of London (1997)
- Foreign member of the Russian Academy of Arts and Sciences (2000)
- Foreign member of the Royal Society of Edinburgh (2001).

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