George W. Mackey

1916–2006

BIOGRAPHICAL *Memoirs*

A Biographical Memoir by
Calvin C. Moore

NATIONAL ACADEMY OF SCIENCES

GEORGE WHITELAW **MACKEY**

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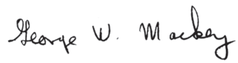


By Calvin C. Moore

George W. Mackey made fundamental contributions to ergodic theory and to the theory of unitary representations of locally compact groups. In the classical special case of finite groups, representation theory is the study of how such groups can be represented as consisting of matrices, and as such the theory is a key tool for examining these groups' structure and classification. For general locally compact groups, representation theory concerns how such groups can be realized as groups of unitary operators on a Hilbert space where the correspondence from the group element to the unitary operator is continuous in an appropriate sense.

While many aspects of the representation theory for compact and abelian groups had been developed previously, and some examples of representations of non-compact, non-abelian groups had been published, Mackey laid out for the first time a general theory of unitary representations, a way of constructing representations, and a way of determining all the representations of many groups, and he provided an overall vision of the subject. These achievements laid the groundwork for the numerous investigators who have followed in his footsteps.

It is of interest that Mackey's work on representation theory highlights the intimate relationship of representation theory and ergodic theory—that is, the theory of groups acting as transformation groups on a measure space. Such actions give rise naturally to unitary representations, and in the course of analysis of unitary representations of a group, ergodic actions arise in natural way.

In his theory of virtual groups, which Mackey developed in the 1960s, he shed new light on the relationship between representation theory and ergodic theory. The impact of these virtual groups, now more commonly identified as equivalence classes of ergodic measured groupoids, has been substantial. Finally, Mackey had a lifelong interest in the mathematical foundations of quantum mechanics and of theoretical physics generally, and he wrote extensively on these topics. A key connection here is that any group of symmetries of a quantum system will give rise to a unitary (projective) representation of that group as unitary operators on the Hilbert space associated with the system.

A natural autodidact, there were times during George's childhood when he did not attend school at all.

Mackey was born in St. Louis, Missouri, to William Sturges Mackey—at the time a bond salesman working for local firms—and Dorothy Frances Allison Mackey. George became the eldest of three children, with a sister Madge born in 1917 and a brother Bill born in 1921. In 1925, Mackey, Sr., had an attractive job offer in Hollywood, Florida, and he moved the family there. When this business opportunity did not pan out as expected, and after the family survived the 1926 hurricane that wreaked considerable damage on Hollywood and other Florida communities, Mr. Mackey took advantage of another business opportunity, this time in Texas, and moved the family permanently to Houston. He worked in various lines of business there but primarily as a petroleum broker. The family moved frequently in Houston as the fortunes of the family ebbed and flowed, and George and his siblings were educated in the Houston public schools.

George showed an early fascination with mathematics and science. At age six, he contemplated a rectangular garden from an overlooking hotel room, and noted that the length of the diagonal was less than the sum of the two adjoining sides. He became interested in ornithology, which led to his amassing of bird pictures. He described himself as an abstract bird watcher, by which he meant looking at these pictures rather than at live birds. George was an avid reader, especially of books about science, so much so that his parents worried that his extensive reading might harm his eyes.

A natural autodidact, there were times during George's childhood when he did not attend school at all. For instance, when the family arrived in Hollywood, Florida, it was several weeks after the school term had begun, and the parents decided to keep the children out of school for the year, in part to protect George's eyes. He spent much of that year reading from *The Book of Knowledge* at a relative's house. In addition, on at least one or two later occasions, George suffered an extensive period of illness when he could not do much more than lie in bed and think. His education evidently did not suffer from these intervals away from school, and they even may have helped him develop a taste for deep contemplation.

Mackey as a teenager in his home chemistry lab.

Although not at all athletic and little interested in sports as an adult, George was for a time an avid baseball fan of his home team, the St. Louis Cardinals. His father, eager for him to be more of an athlete, tried to bribe him to take up golf as a boy, but to no avail. George later commented that he had much preferred the satisfaction of making money by selling magazines, as opposed to practicing putting. He also developed a significant interest in magic—an interest that extended into adulthood; he often performed magic tricks at his daughter's birthday parties.

When George was about 15, his father started chasing a new business opportunity, which involved extracting iodine from seaweed, and he tried to secure George's help by suggesting that he study a little chemistry. George quickly developed an intense interest in the subject, even before taking any high-school science courses, and he pursued this interest—practically pushing all other subjects aside—through independent reading; he also set up his own home chemistry laboratory. In 1934, when he entered the Rice Institute (later renamed Rice University), George's stated aim was to study chemical engineering—which he saw as a compromise between his love for pure science and his father's wish for him to become a businessman. But as his interest in mathematics further developed and deepened during his undergraduate years, George liberated himself from family career expectations and switched his major to physics, also taking many courses in mathematics. Although the practice did not formally exist at the time, he effectively completed a double major—in physics and mathematics—and in fact, with his independent reading and study in chemistry, he might even be described as having had a triple major.

Although he was not a mathematics major, the mathematics faculty at Rice were well aware of Mackey's talent in their field; and they encouraged him to take the William Lowell Putnam Examination, a mathematics competition for undergraduates in the United States and Canada, the first time it was offered (during the 1937–38 academic year). The mathematics faculty's judgment of Mackey's ability was validated when he achieved one of the top five scores among all those in the two countries who had taken the test.

As Mackey looked to his future beyond Rice, he decided to become a theoretical physicist who would probe the connections between physics and mathematics. To achieve this goal, he conceived the strategy of first obtaining a Ph.D. in mathematics and then proceeding to attack fundamental problems in physics. Toward that end, the Rice mathematics faculty encouraged him to apply to Harvard University for graduate study in mathematics, pointing out that John H. Van Vleck, an eminent theoretical physicist who held a joint appointment at Harvard in physics and mathematics, was someone who might be relevant to Mackey's goals. Thus Mackey applied to Harvard—also to the University of California, Berkeley—for graduate study in mathematics, and both institutions invited him to enroll.

Harvard initially did not offer financial aid, while Berkeley came through with a teaching assistantship, which Mackey quickly accepted. But when Harvard learned of his Putnam score, it added significant financial aid, including full tuition. This state of affairs placed Mackey in a difficult position, and he wrote to Griffith Evans, the mathematics chair at Berkeley (who coincidentally had been long-time mathematics chair at Rice before leaving for Berkeley in 1934). Mackey asked Evans if he could be released from his acceptance at Berkeley, and Evans complied. As an added note, the Harvard mathematics department began that year its long tradition of reviewing the top five Putnam finishers and selecting one of them for a full scholarship. Mackey was not selected for this honor, but rather it went that year to Irving Kaplansky.

So in the fall of 1938, Mackey entered Harvard for doctoral study in mathematics. In his first year he took a course from Marshall Stone and did very well in it. This was his first exposure to Stone, but the decisive interaction came when Mackey discovered his teacher's magisterial *Linear Transformations in Hilbert Space and their Applications to*

Analysis (Stone 1932), which convinced him that he must study under the writer of this magnificent book. In 1939, after Mackey's first year of graduate work, he asked Stone rather than Van Vleck to be his dissertation supervisor, and Stone accepted.

In 1942 Mackey completed his dissertation, The Subspaces of the Conjugate of an Abstract Linear Space. In this work he explored the different locally convex topologies that an infinite-dimensional vector space can carry. The most significant result to emerge can be stated as follows:

Consider two (say, real) vector spaces V and W, which are in perfect duality by a pairing

$$V \times W \to \mathbb{R}$$

so that each may be viewed as linear functionals on the other. It was obvious that there is a weakest (smallest) locally convex topology on V (or W) such that the linear functionals coming from W (or V) are exactly the continuous ones, called the weak topology. What Mackey proved was the non-obvious fact that there is also a unique strongest (largest) locally convex topology such that the linear functionals coming from W are the continuous ones. As Mackey showed, this is the topology of convergence of elements of V, now viewed as linear functionals on weakly compact convex subsets of W. All locally convex topologies on V, for which the linear functionals from W are exactly the continuous ones, lie between these two (weakest and strongest) topologies (Mackey 1943 and 1946). This topology of uniform convergence on weakly compact convex sets has become universally known as the Mackey topology. Stone arranged for Mackey to be awarded a Sheldon traveling fellowship during 1941–42, his last year of graduate study. This fellowship is ordinarily granted for foreign travel, but owing to the war, its application to domestic travel was allowed. Mackey used it to spend the first semester at the California Institute of Technology and the second at the Institute for Advanced Study in Princeton, New Jersey, where he met many mathematicians and forged lifelong friendships. Mackey's first postdoctoral position was as an instructor at the Illinois Institute of Technology for the 1942–43 academic year. He then was engaged in war-related research—for what was later to become the U.S. Air Force—at Columbia University and in High Wycombe, England. Following this wartime service, he returned to Harvard as an assistant professor (in 1946). Mackey was subsequently promoted up the ladder and was appointed full professor in 1956; in 1969, he became the Landon T. Clay Professor of Mathematics and Theoretical Science.

As noted above, Mackey's original intention was to become a theoretical physicist, but as he delved further into mathematics research he dropped that idea, having become, as he put it, seduced by mathematics. Of course, he retained a lifelong interest in theoretical physics, and soon after his initial work he turned his attention to the theorem of Stone (Stone 1930) and John von Neumann (von Neumann 1931). This theorem asserts that a family of $2n$ self-adjoint operators $p(i)$ and $q(i)$ on a Hilbert space satisfying the quantum-mechanical commutation relations,

$$\left[p(k), q(j)\right] = i\delta(j,k)I$$

and with no common closed invariant subspaces, is unique. Mackey realized that it was really a theorem about a pair of continuous unitary representations—one U of an abelian locally group A and the other V of its dual group $\hat{A}$—that satisfied

$$U(s)V(t) = (s,t)V(t)U(s),$$

where (s,t) is the usual pairing of the group and its dual. He showed (Mackey 1949a) that such a pair is unique if the group and its dual jointly leave no closed subspace invariant, and in general any pair is isomorphic to a direct sum of copies of the unique irreducible pair. When G is Euclidean n-space, this result becomes the classic theorem about the quantum-mechanical commutation relations. Mackey then also saw immediately that there was a version for non-abelian locally compact groups, known as his imprimitivity theorem, which played a fundamental role in his subsequent work.

This imprimitivity theorem (Mackey 1949b) states that a unitary representation U of a group G is induced by a unitary representation V of a closed subgroup H of G if and only if there is a projection valued measure on the measure space $(G/H, \mu)$—with values projections on the same Hilbert space as U—that is covariant with respect to U in the natural sense. Here μ is a quasi-invariant measure on G/H. Moreover, U and V uniquely determine each other up to unitary equivalence. Mackey's theorem above on unicity of pairs of representations of an abelian group A and its dual group $\hat{A}$ is a special case obtained by applying the imprimitivity theorem to a generalized Heisenberg group built from A and $\hat{A}$.

At the same time, Mackey initiated a systematic study of unitary representations of general locally compact second-countable groups (note that all groups will be assumed to be second-countable without further mention), the work for which he is most famous. Von Neumann had developed a theory of direct integral decompositions of operator algebras in the 1930s as an analog of direct sum decompositions for finite-dimensional algebras. But he did not publish it until F. I. Mautner persuaded him to do so in 1948. Adapted by Mackey to representation theory, direct integral theory became an important tool that Mackey used and further developed. For representations of finite groups, induced representations—wherein one induces a representation of a subgroup H of G up to the group G—is an absolutely essential tool.

In a series of papers (Mackey 1951, 1952, and 1953), Mackey systematically studied the process of induction of a unitary representation of a closed subgroup H of a locally compact group G to form the induced representation of G. When the coset space G/H has a G-invariant measure, the definition is straightforward, but when it has only a quasi-invariant measure, some extra work is needed. Mackey developed analogs for locally compact groups of many of the main theorems about induced representations of finite groups. (The process of induction had appeared in some special cases a year or two earlier in the work of I. M. Gelfand and his collaborators on unitary representations of the classical Lie groups.)

These results provided the foundation for what was to become known as the Mackey little group method—or, as some have called it, the Mackey machine—for calculating the irreducible unitary representations of a group based on information about its subgroups. But before this program could get underway, Mackey had to put in place some building blocks, or preliminaries—in particular, he set forth basic facts about a Borel structure (a set together with a sigma-field of subsets) in Mackey 1957b. He identified two kinds of very well-behaved types of Borel structures, which he called standard and analytic, based on some deep theorems in descriptive set theory of the Polish school. An equivalence relation on a Borel space leads to a quotient space with its own Borel structure. If the original space is well behaved, there is a kind of dichotomy for the quotient space; it can be very nice—one of the two well-behaved types above—or if not, it is quite pathological. If the former holds, then the equivalence relation is said to be smooth. The set of concrete irreducible unitary representations of a group G can be given a natural well-behaved Borel structure, and then the equivalence relation of unitary equivalence yields the quotient space—that is, the set of equivalence classes of irreducible unitary representations—which he termed the dual space $\hat{G}$ of G. If the equivalence relation is well behaved, $\hat{G}$ is a well-behaved space, and Mackey said then that G had a smooth dual. This was a crucial concept in the program.

Also, by adapting von Neumann's type theory for operator algebras, Mackey adapted this notion and introduced as well the notion of a type I group, by which he meant that all its representations were type I—or, equivalently, that all of its primary representations were multiples of an irreducible representation. On the basis of his work in classifying irreducible representations of a group—e.g., calculating $\hat{G}$—Mackey observed that the property of a group G having a smooth dual seemed to be correlated with the absence of non-type I representations of G. Mackey then made the bold conjecture that a locally compact group G had a smooth dual if and only if it is type I. It was not too long before James Glimm provided a proof of this conjecture (Glimm 1961).

Some facts about actions of locally compact groups on Borel spaces and measure spaces constituted another building block for the Mackey machine. A group action is ergodic with respect to a quasi-invariant measure—the concept of ergodicity played a central role in Mackey's work over decades—if the only fixed points in the measure algebra are 0 and 1. An important observation is that if the equivalence relation induced on X by the action of G is smooth, then any ergodic measure is concentrated on an orbit of G; moreover, up to null sets, the action is transitive. Concerning point realizations of actions of a group, suppose that G acts as a continuous transformation group on the measure algebra $M(X,\mu)$ of a measure space where X is a well-behaved Borel space. Then it is natural to ask if one can modify X by μ-null sets if necessary and show that this action comes from a Borel action of G, on the underlying space X, that leaves the measure quasi-invariant. In an associated paper (Mackey 1962), Mackey showed that the answer was affirmative, thereby extending an earlier result of von Neumann's for actions of the real line.

Another preliminary was to deal with what one would call projective unitary representations of a group G, which are continuous homomorphisms from G to the projective unitary group of a Hilbert space. Such projective representations not only arise naturally in the foundations of quantum mechanics; it also became clear that when one started to analyze ordinary representations, this led naturally to projective representations. By using a lifting theorem from the theory of Borel spaces, Mackey showed that a projective representation could be thought of as a Borel map U from G to the unitary group satisfying

$$U(s)U(t) = A(s,t)U(st)$$

where A is a Borel function from $G \times G$ to the circle group T that satisfies a certain cocycle identity. For finite groups, it was clear that any projective representation of a group G could be lifted to an ordinary representation of a central extension of G by a cyclic group. In the locally compact case, one would like to have the same result, but with a central extension of G by the circle group T. Mackey, by a very clever use of

Weil's theorem on the converse to Haar measure, showed how to construct this central extension. He also began an exploration of some aspects of the cohomology theory that lay in the background (Mackey 1957a).

In Mackey 1958, his little group method—the Mackey machine—starts with a group G for which one wants to compute $\hat{G}$, and it is assumed that N is a closed normal subgroup that has a smooth dual (and hence is now known to be type I). If it is also assumed that $\hat{N}$ is known, then G (or really, G/N) acts on $\hat{N}$ as a Borel transformation group via automorphisms of N. Any irreducible representation U of G yields upon restriction to N a direct integral decomposition into multiples of an irreducible representation with respect to a measure μ on $\hat{N}$, which Mackey showed was ergodic. Then if the quotient space of $\hat{N}$ by this action is smooth, any ergodic measure is transitive, and is carried on some orbit of G on $\hat{N}$. Hence the representation U has a transitive system of imprimitivity based on G/H, where H is the isotropy group of a point on the orbit. Further, U is induced by a unique irreducible representation of H, whose restriction to N is a multiple of V. These results can be classified in terms of irreducible representations or projective representations of H/N, which is called the "little group." This work of Mackey built in part on Eugene Wigner's analysis (Wigner 1939) of the special case of unitary representations of the inhomogeneous Lorentz group.

Mackey's little group method was an enormously effective tool for analyzing representations of many different groups. It has been used to good effect by many workers and extended in different directions. For example, after reading Andre Weil's paper on metaplectic representations (Weil 1964), Mackey wrote a lengthy (2,500-word) and insightful review of the paper (Mackey 1965); he used the review to show how Weil's construction fitted into his little group method.

In the summer of 1955, Mackey was an invited visiting professor at the University of Chicago, where he gave a course that explained the theory of group representations he was developing. J. M. G. Fell and D. B. Lowenslager's notes from the course circulated informally for years, and generations of students (including the author) learned Mackey's theory from these famous notes (Fell and Lowenslager 1955). In 1976, Mackey finally agreed to publish an edited version together with an expository article summarizing the field's progress during the intervening years (Mackey 1976).

Well into his 40s, George led a bachelor's life, living in a small and sparsely furnished apartment. Totally devoted to his mathematics, he had a regular pattern of going to his office and working. He settled early on a dress style and never varied from it. He was, for


George with his clipboard.

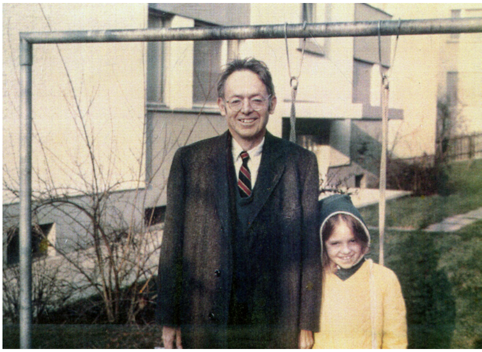
instance, reluctant to take off his jacket for any occasion—even at a picnic—compromising only to switch from tweed to a lighter-weight fabric such as seersucker when the season demanded. George was identifiable by his clipboard, which he always carried with him so that he could sit down almost anywhere to do mathematics. And he was known to keep very detailed records in notebooks of every penny he spent and what it was for.

In conversations George was always completely honest and straightforward, which some found off-putting, but he was committed to a principle of integrity and had a strong sense of ethics—though he never wished to offend and so often had to carefully consider how to phrase his comments. He would not hesitate, however, to express his opinions on such things as bureaucratic inefficiency—for example, that the federal government was wasting its money on research grants, given that mathematicians would certainly spend their summers doing mathematics whether they received the extra two months of summer salary or not. He also was concerned that such research support would over time lead to governmental control of the research agenda in universities. While not many people would agree with the former opinion, many would find wisdom in the latter. As his daughter Ann put it her eulogy and its written version in Doran and Ramsey 2007, he was "notoriously eccentric and proud of it." He also described himself as "a gregarious loner."

In December 1960, George surprised many of his friends and colleagues by getting married. The bride was Alice Willard, a native of Groton, Connecticut, and a Wellesley graduate (class of 1941). George and Alice, who worked as a buyer for the Jordan Marsh department store in Boston, had dated on and off for some 14 years. Their marriage was a happy one, with deep love and respect on both sides, and produced one daughter, Ann Sturges Mackey. George persisted in many of his bachelor habits, while also adapting them in order to become a dutiful husband and father.

For example, he carried his famous clipboard on vacations with his family and might be seen sitting on a park bench doing mathematics while Alice and Ann were off sightseeing or shopping. "I loved my father dearly," said Ann. "Although he was fond of asserting to anyone who would listen that he'd never wanted a family, it was clear to everyone that once he stumbled into marriage and fatherhood, he relished and cherished [them], even as he struggled to adapt to the compromises [they] asked of him."

In 1960 Mackey was invited to give the Colloquium Lectures at the 1961 Annual Summer Meeting of the American Mathematical Society (AMS). He used this prestigious lecture series to summarize his theory of unitary representations and his ergodic theory. In addition to the exposition, Mackey laid out for the first time his new concept of a virtual group, which he saw as a simple and elegant way to visualize ergodic theory and its connections with representation theory. The concept can also be seen as a derivative and culmination of his famous imprimitivity theorem.



George and daughter Ann in Zürich, ca. 1971.

He began by observing that a Borel group action of a group G on a measure space (Y,μ) defines a groupoid—a set with a partially defined multiplication where inverses exist. The groupoid as a set is $G \times Y$ and is given the product Borel structure and the product measure μ on Y with Haar measure on G. If the measure μ were ergodic, then Mackey would call the construction an ergodic measured groupoid. He also noted his realization that different objects of this type needed to be grouped together under a notion he called similarity, and he defined a virtual group to be an equivalence class under similarity of ergodic measure groupoids. In the case of a groupoid coming from a transitive action of G on a coset space G/H of itself, the similarity notion makes the ergodic measured groupoid $G \times (G/H)$ similar to the group H (with Haar measure), which puts them in the same equivalence class. Hence in this case the transitive measured groupoid is literally a subgroup of G, and Mackey's point here was that it would be very productive to look at a general ergodic action of G as a kind of generalized (or virtual) subgroup of G via the language of groupoids and virtual groups.

Then Mackey began a systematic discourse on the representations of virtual groups, induced representation, and related concepts. He pointed out, for example, that the imprimitivity theorem remains true in the ergodic nontransitive case in that the irreducible representation of G is now induced by an irreducible representation of a virtual subgroup. A lengthy paper based on these AMS Colloquium Lectures appeared in Mackey 1963. He laid out his theory of virtual groups in more detail in a subsequent publication (Mackey 1966), and again in his invited lecture at the International Mathematical Congress in 1970, with the written version appearing as Mackey 1971.

One particularly rich theme has emerged from the special case when the group action is free, in which case the groupoid is simply an equivalence relation. Mackey defined what one means by a measured ergodic equivalence relation. Isomorphisms of measured equivalence relations amount to orbit equivalence of the group actions, a notion that was somewhat foreign in ergodic theory at first but that has been of overriding importance in subsequent developments. In fact, in Mackey 1966 he foreshadowed some of the advances that would spring from his work. Mackey was perhaps a bit disappointed that his elegant notion and language of virtual groups did not catch on, but he would have been pleased to see that this work laid the foundation for and inspired subsequent efforts by many on groupoids and their applications in noncommutative geometry and topology.

As noted earlier in this memoir, Mackey maintained a lively and inquiring lifelong interest in mathematical physics, and especially in the basics of quantum theory, quantum field theory, and statistical mechanics. In Mackey 1957c, he explored the abstract relationship between quantum states and quantum observables, and he raised the question of whether some very general axioms about that relationship necessarily led to the classical von Neumann formulation. This exposition inspired Andrew Gleason to prove a strengthened version of Mackey's results (Gleason 1957), which then enabled Mackey to formulate a general result that showed that the von Neumann formulation followed from a much weaker set of axioms. Also, Mackey's work on the unicity of the Heisenberg commutation relations gave an indication why the uniqueness breaks down when the number of $p(i)$'s and $q(i)$'s is infinite (quantum field theory). Following his interests in quantum mechanics from many years earlier, Mackey wrote a book based on a course he gave at Harvard (Mackey 1963b) that explained the field's mathematical foundations. Although he was clearly influenced by von Neumann and chose for his title the English translation of von Neumann's classic book (1932) of three decades earlier, he offered his own inimitable and fascinating perspective.

This was the first of a series of books and essays that Mackey wrote of an integrative or historical nature about group representations, harmonic analysis, and their applications and significance for other areas of mathematics and for the mathematical foundations of physics—especially quantum mechanics and statistical mechanics. The theme of Norbert Wiener's definition of chaos, or homogeneous chaos, was a favorite theme in Mackey's writings, as were applications of group representations to number theory. Mackey was invited to be the George Eastman Visiting Professor at Oxford University for 1966–67, and he, his wife Alice, and daughter Ann spent the year there, during which he gave a wide-ranging series of lectures, subsequently publishing these lectures' notes as Mackey 1978. Other publications of this nature include Mackey 1968, 1974, 1980, and 1992. The 1992 book, a collection of a number of his integrative essays on mathematics and theoretical physics, is a showcase of his thoughtful and insightful writing on these topics.

George in his office at Harvard, 1970s.

Mackey was accorded many honors during his lifetime, including election to the National Academy of Sciences in 1962, the American Academy of Arts and Sciences, and the American Philosophical Society. As noted, he was appointed to the George Eastman Visiting Professorship at Oxford and was a prize holder of the Alexander Humboldt Foundation. One sign of the recognition of Mackey's integrative publications was the awarding of the American Mathematical Society's Leroy Steele Prize to him for his magisterial paper on ergodic theory and statistical mechanics (Mackey 1974). In 1982, he received a Distinguished Alumnus Award from Rice University. Mackey was frequently invited to visit mathematical centers around the world to give lectures, including in: Leningrad and Moscow (1962); Les Houches, France, the Tata Institute, Bombay, and ETH, Zürich (1970–71); the Institute for Advanced Study, Princeton, Jussieu/Université de Paris VI, and Max Planck Institute, Bonn (1985–86); and China (1990).

Let me close with some personal thoughts and recollections. I first met George in 1956, when I was entering my junior year at Harvard. He became my advisor and mentor there, not only during the rest of my undergraduate days but also when I continued as a graduate student. I learned how to be a mathematician from him, and I have valued his friendship, guidance, and encouragement ever since—for over 50 years. Many of my own accomplishments can be traced back to ideas and advice coming from George, who was a uniquely gifted and inspiring individual.

Alice and George Mackey in Houston, 1991.

George visited the University of California, Berkeley (where I have long been based) on several occasions, and two incidents stick in my mind that reflect his productivity as well as his modesty and warmth. One time, probably in the 1960s or 70s, a group of us were walking to lunch and talking mathematics. In this discussion I described a certain relevant theorem (unfortunately, I cannot recall what it was), and George remarked to the effect, "That's a very nice result. Who proved it?" My response was, "You proved it." It is nice to have proved so many good theorems that you can forget a few.

The other incident—actually, a series of incidents—occurred in 1983–84, when I had arranged for George to visit at Berkeley's Mathematical Sciences Research Institute for a year. The MSRI's housing officer found him and Alice a beautiful rental house that belonged to Geoff Chew, a faculty member in physics, who was on sabbatical. The only problem was that it came with some animals—cats and a dog, which the tenants would have to take care of. But the Mackeys said this would be no problem. George actually arrived a month or two before Alice could come so that the house would not be vacant, with no one to look after the animals. George truly had his hands full as a result, and even after Alice arrived to take over housekeeping, they amassed many amusing tales to tell. When they returned to Cambridge after their year in the "wild west," Alice wrote a fascinating and hilarious article for the Wellesley alumnae magazine about their travails with the Chew menagerie.

George died on March 15, 2006, from complications of pneumonia in Belmont, Massachusetts, at the age of 90. He will be remembered and honored for his seminal contributions to group representations, ergodic theory, and mathematical physics and for his fascinating expositions on these subjects. Additional material on his life and works contributed by colleagues, friends, and students can be found in Doran and Ramsey 2007.

ACKNOWLEDGEMENTS

In 2006 I wrote an essay on George Mackey's mathematical work that was published in Doran and Ramsey (2007), a publication of the American Mathematical Society. That text of that essay, on which I retained the copyright, has been incorporated nearly verbatim into this biographical memoir. I have complemented it with other material concerning George's personal life, some of which comes from other contributions to the Doran and Ramsey collection.

Alice and Ann Mackey have kindly provided me with much useful information, and Ann in particular has shared parts of the autobiographical essay that her father prepared for her, which was especially useful to me in writing paragraphs two through eight of this memoir. In addition, George published an article on his mentor Marshall Stone (Mackey 2004), which, being partly autobiographical, I have also used in writing paragraphs two through eight. The American Mathematical Society holds the copyright to Mackey 2004 and I am grateful to the AMS for granting permission to use this material. I also thank Ann for her help in tracking down photographs and for granting permission to use them in this memoir. Finally, I thank Alice, Ann, and Robert Doran for reading the manuscript and for their helpful comments.

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