NATIONAL ACADEMY OF SCIENCES

GEORGE ABRAM MILLER

1863—1951

A Biographical Memoir by

H. R. BRAHANA

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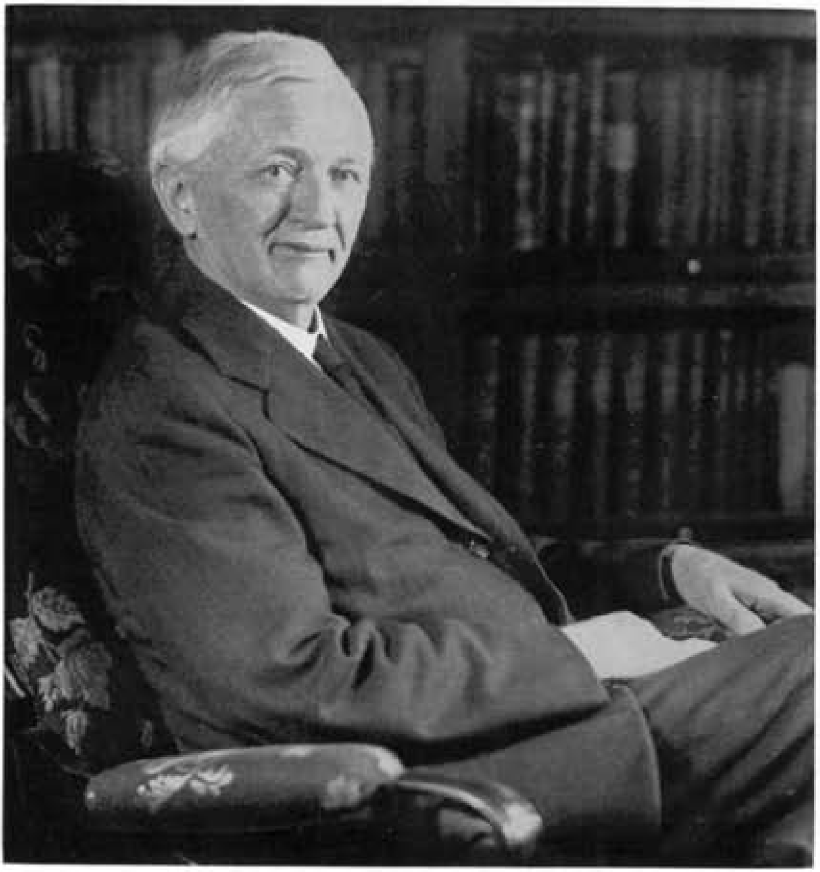

G. A. Miller

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George Abram Miller was born near Lynville, Pennsylvania, on July 31, 1863, the year the National Academy of Sciences was incorporated. He was elected to membership in the Academy in 1921. He published two papers in the first volume of the Proceedings of the Academy in 1915; he contributed five or six papers to each volume of the Proceedings for many years around the time of his retirement in 1931; and his last two mathematical papers appeared in Volume 32 of the Proceedings in 1946. He died at Urbana, Illinois, on February 10, 1951.

Miller came into prominence in the mathematical world abruptly in 1894-1895 when he completed the determination of the substitution groups of degrees eight and nine. The groups of degree eight had been listed in 1891 by Cayley, who at that time was referred to as the greatest English mathematician since Newton. Cayley's list had been corrected by F. N. Cole, and Miller started his study of groups after his association with Cole. Miller redetermined the groups and brought the number to 200 including two groups that his predecessors had missed. Camille Jordan, one of the foremost French mathematicians and a specialist in the theory of groups, had published in 1872 a list of the primitive groups of degree nine and Cole had given a list of all the groups of degree nine in 1893; Miller determined the 258 groups of this degree, adding one group to Jordan's list and two to Cole's. In the following year he published his own list of 994 intransitive groups of degree ten. These lists have stood since that time. In 1900 the Academy of Sciences of Cracow awarded him a prize for his work on the groups of degree ten, a prize which had been standing since 1886. This was the first award to an American for work in pure mathematics.

The mathematical world which Miller entered in 1894 was largely a European world, and mathematics was just getting a foothold in America. Although mathematics is an old subject and advanced vigorously during the two preceding centuries, there had been no participation by Americans in the advancement. The first notable American contribution was a memoir on "Linear Associative Algebras" by Benjamin Peirce of Harvard. This was read before the National Academy of Sciences in Washington in 1870, although at that time there was no organized mathematical group to which to present it and no American mathematical periodical in which it could be published. It was published later in the *American Journal of Mathematics.* The Johns Hopkins University, established in 1876, is considered to have started the first school of mathematics in this country, headed by the English mathematician, J. J. Sylvester. Harvard, Yale, and Princeton were offering the degree of Doctor of Philosophy in mathematics, but Harvard Fellows were going to Germany to study mathematics. Some returned to Harvard for their degrees while others took their degrees abroad. Cole got his degree at Harvard in 1886 after two years in Germany; Bôcher got his degree in 1891 at Göttingen after some years there as a Harvard Fellow. Clark University, established in 1889, had Bolza, a German, on its staff. The University of Chicago, established in 1893, had E. H. Moore and two Europeans, Bolza and Maschke, in its mathematics department; Moore had his degree from Yale and had spent a year in Europe. The University of Michigan was offering the doctor's degree and had Cole and Ziwet on its staff. Ziwet also was trained in Europe. Most of the colleges were offering mathematics as far as the calculus; those that were going further depended largely on men who had been trained in Europe. The *American Journal of Mathematics,* established by Johns Hopkins in 1878 with Sylvester as editor, depended for the first few years on contributions by Europeans. The *Annals of Mathematics* (1884) was edited by Ormond Stone at the University of Virginia. The *Bulletin of the New York Mathematical Society* (1891) was edited by Fiske, Jacoby, and Ziwet. The New York Mathematical Society had 23 members in 1890; Miller was elected to membership in 1891. The Society changed its name to the American Mathematical Society in 1894. Except in a few institutions, mathematics libraries were non-existent; for example, the mathematics books at the University of Illinois occupied one fifteen-foot shelf in 1893.

Miller had no considerable contact with mathematicians or with their works before he went as Instructor to the University of Michigan in 1893, and he had behind him no family tradition of erudition or scholarship to spur him on. He began teaching school at seventeen to earn money to permit him to continue his education. He attended Franklin and Marshall Academy, a subdivision of the College at Lancaster, in 1882-1883. He attended Muhlenburg College from January, 1884, to 1887, receiving the degree of Bachelor of Arts with honorable mention, and ranked third in his class of twelve. Muhlenburg granted him the degree of Master of Arts in 1890. The catalogue of Muhlenburg states that the recipient of the degree should be of good moral character, should have been a Bachelor of Arts for three years, and should have been engaged in liberal and professional pursuits. During the year 1887-1888 Miller was Principal in the schools of Greeley, Kansas. From 1888 to 1893 he was Professor of Mathematics at Eureka College in Eureka, Illinois. In 1892 he received the degree of Doctor of Philosophy from Cumberland University in Lebanon, Tennessee. It is not known that he ever went to Lebanon, although it is likely that he wrote some examinations there. He was registered as a graduate student at Cumberland during the year 1891-1892, but graduate work could be taken by correspondence and Miller was teaching at Eureka during the year. A thesis was a requirement for the degree, but examinations in the advanced courses could be substituted for the thesis. He was offering the same courses and the doctor's degree at Eureka and was using the same texts. He had a graduate student taking these courses for two years but the student did not receive the doctor's degree, although he did succeed Miller as Professor of Mathematics at Eureka. In the summers of 1889 and 1890 he went to Johns Hopkins and the University of Michigan, although neither university was in session during these summers. If he had contact with the mathematicians at these places there is no trace of it to be found. When he went to the University of Michigan as Instructor in 1893 he lived in Cole's home, and he credited Cole with starting him on his career in the theory of groups. But if he met Cole in the summer of 1890 no spark was struck, for if there had been Eureka would have had a course in the theory of groups.

He spent the years 1893-1895 as Instructor at Michigan. During the years 1895-1897 he was in Europe attending the lectures of Sophus Lie at Leipzig and Camille Jordan in Paris, and wrote prodigiously on the theory of groups. He is, therefore, numbered among the group of mathematicians with European training. He credited Lie with starting him on a systematic study of commutators and commutator subgroups, but otherwise Lie's influence on him was small. Neither Lie nor Miller was greatly interested in the other's groups. Jordan was interested in questions of primitivity and imprimitivity and Miller kept returning to these questions for the remainder of his life. It is doubtful, however, that Jordan influenced him greatly, for Miller worked on his own problems with his own methods and he never engaged in the explanation of Jordan's works or Jordan's methods. From 1897 to 1901 Miller was Assistant Professor at Cornell; during 1901 to 1906 he was Assistant Professor and Associate Professor at Leland Stanford; from 1906 he was at the University of Illinois serving as Associate Professor, Professor, and from 1931 Professor Emeritus.

He was married in 1909 to Cassandra Boggs of Urbana, Illinois. They had no children and Mrs. Miller died in 1949.

The catalogues of Eureka College for the years 1887 to 1895 reveal the G. A. Miller that was known to the writer by daily association during his last thirty years. Under the heading of the Department of Mathematics in the Eureka catalogue of 1889-1890 appears the following:

". . . The study of mathematics—

a. Develops the power of reasoning.
b. Cultivates precise methods of reasoning.
c. Trains the mind in abstract thinking.
d. Lifts from servile imitation to original thinking.
e. Gives a steady and dignified bearing to the mind.
f. Enables the student to understand all the other sciences better.
g. Cultivates the habit of persistent and well-directed exertion, and
h. Cultivates the habit of distinguishing clearly between the known and the unknown.

Authority for all these statements may be found in the works of the greatest philosophers. . . ."

One can see the young, meagerly trained Professor of Mathematics called upon by the President for copy for the catalogue; he is not content to offer the customary list of requirements and laudatory but vague assertions about the value of mathematics; he thinks out and states in detail his philosophy of education and the part that mathematics has in it. Sixty years later he would have been more precise in referring to the philosophers, but the goals he set up in 1889 were the ones toward which his whole life was directed.

The statements under Mathematics in the catalogues change from year to year. In 1890 Miller offered the doctor's degree. New advanced courses were offered. The description of one of these in 1892-1893 reads: "The Higher Algebra and Determinants have been prepared expressly for the students of this institution. The treatments of the following subjects are more comprehensive than those given by any other American text-book on Higher Algebra, Logarithms, Choice and Chance, Apparent Paradoxes, Determinants, and Theory of Numbers. The other subjects commonly treated in works of this kind are introduced and discussed according to the most approved methods." The book to which this refers was published under the title *Determinants* in 1892 in the Van Nostrand Science Series. It could have been said of him in 1893 that here is a man of power who appreciates his own worth, who claims his place in the sun, and who, while vigorous and astute in seeking that place, offers solid achievement as the basis of the claim.

The publication of lists of all the substitution groups of low degrees was started in 1850 when a French mathematician, Serret, listed all the 19 possible groups of degrees not greater than five. Miller came upon the scene in 1893 to complete the lists for degrees eight and nine. The principles underlying the determination of all the groups of a given degree were then fairly clear. Miller put them to use and rapidly went on to determine the 994 intransitive groups of degree ten; these brought to 1,039 the total number of groups, transitive and intransitive, of degree ten. Jordan had determined the eight transitive groups of degree eleven in 1872. Miller and a student, a G. H. Ling, determined the 1,492 intransitive groups of degree eleven in 1901. Miller did the transitive groups of degree twelve in 1896; the transitive groups of degrees thirteen and fourteen and the primitive groups of degree fifteen in 1897; the primitive groups of degree sixteen in 1898; and the transitive groups of degree seventeen in 1899. The subject of the determination of substitution groups, which had interested many mathematicians besides Cayley and Jordan, was virtually taken over by Miller in 1893, and he finished it in the sense that nobody has tried to go beyond him and probably nobody has followed him so far. His work on substitution groups could be carried on but probably will not, unless someone should see how to put a high-speed computer to work on it. Any obvious attack with a computer seems sure to lead directly to insuperable difficulties.

Miller's belief in persistent and well-directed exertion would have taken him beyond the transitive groups of degree seventeen if he had not early become interested in abstract groups. Every substitution group is a representation of one and only one abstract group; an abstract group may have many representations as a substitution group on a given number of letters. In determining all the substitution groups of a given degree, it is necessary to know much about the abstract groups that can be represented on smaller numbers of letters. Every abstract group can be represented in one and only one way as a regular substitution group, that is, a group which is transitive and has its degree equal to its order. Miller's first work on abstract groups was done in terms of regular substitution groups, and throughout his life he returned to regular substitution groups to study certain properties of abstract groups. There is an element of the fantastic in his first publication on abstract groups. In 1884 Felix Klein used a certain method to prove that a group of order 60 was simple. In a paper dated November 5, 1894, E. H. Moore used the same method to prove that a group of order 168 was simple, and asked if anybody knew of a simple group on which that method would not work. On December 28 Miller gave him an example, the alternating group on 68 letters. The non-mathematical reader may be interested in knowing that the alternating group on 68 letters has order one half of 68!, that 68! is the product of all the integers from 1 to 68, and that this is a number which if written out would require about two lines on this page. One man displays an inclination to generalize from his experience with the two smallest simple groups, and the other, though a novice, answers the question even if he has to go beyond astronomical figures to do it.

Miller's first considerable contribution to abstract groups, also done in terms of regular substitution groups, was his list in 1896 of all the abstract groups of orders less than 48. This paper contained the first determination of the 15 groups of order 24 and the 51 groups of order 32. The enumeration of all the abstract groups of low order had been begun by Cayley in 1854 when he proved that there are just two groups of each of the orders 4 and 6; in 1859 Cayley determined the 5 groups of order 8, and in 1889 the 5 groups of order 12. The 14 groups of order 16 were determined in 1893 by two men independently, J. W. A. Young, an American student under Bolza at Clark, and Hölder in Germany. Shortly before Miller's paper in 1896 a French mathematician, Le Vavasseur, had announced in the *Comptes Rendus* that he had found 75 groups of order 32 and that he had not yet reached the end. Two years later an Italian, Bagnera, stated in the *Annali di Matematiche* that Miller had made a mistake and there were 50 groups of order 32, but the following year he agreed that the number is 51. The orders for which there are the largest numbers of distinct abstract groups are orders which are divisible by a high power of a prime number; the smallest of these are thus the powers of 2: 2, 4, 8, 16, 32, 64, 128. In 1930 Miller determined the 294 groups of order 64; he estimated at that time that there are more than a thousand groups of order 128. He had determined the 52 groups of order 48 and the 15 groups of order 54 in 1898; he did those of order 54 by showing that there are just 15 groups of order $2p^3$ for every odd prime p. He determined the 57 groups of order 168 in 1902, the groups of order 72 in 1929, and those of order 96 in 1930. In the 34 years between the publication of his lists of the groups of orders 32 and 64 only one man, Pôtron, had offered a list of the groups of order 64 and that was not a success. Miller thus took over the enumeration of abstract groups at order 24 and carried it to order 100. The enumeration has since been carried to order 160, omitting order 128.

The two most fundamental questions about groups are: What groups exist? How can one group be distinguished from another? Information sufficient to determine a particular group will be enough to determine all of its subgroups, and sufficient information about subgroups and their relations will be enough to determine the group. Thus these lists of substitution groups of low degrees and of abstract groups of low orders not only answer the first question as far as they go, but they provide tools for the investigation of the groups beyond.

The second question above may be restated: What questions should one ask of a group so that the answers will enable one to identify it? Miller's life was directed toward that question. When the group is abelian the answer is simple and was known before Miller's time. At the beginning of his career it was hoped that the answer for non-abelian groups, if not simple, would still be discoverable. As it became clear that the answer was not easy, it seemed apparent that the only place to look for clues was in the groups that were known and others that could be determined. Miller's attack was so direct that he never discovered that among the difficulties he encountered were difficulties other mathematicians had met in other fields which had forced abandonment of their attempts, at least temporarily.

The simple groups of composite order are few and far between. There are but 53 known simple groups of orders less than a million and they are of particular importance in the study of finite groups. The first six have orders 60, 168, 360, 504, 660, 1,092. That these are the only ones up to this order was proved by Hölder, Cole, and Burnside. Miller, with G. H. Ling, proved in 1900 that there is no other with order 2,000 or less. In 1922 he proved there is only one simple group of order 2,520. All simple groups of order not greater than 6,232 are now known.

Two things that Miller did with simple groups would have kept memory of him fresh if he had done no more. In 1900 he proved that the Matthieu fivefold transitive group of degree 24 is simple, as are its maximal subgroups of degrees 23, 22, and 21. The group was first published in 1873, but Matthieu had known it in 1861. The Matthieu groups present a challenge to mathematicians since no other fourfold or fivefold transitive group is known and nobody can prove no others exist. Moreover, these are the only known simple groups that do not belong to any of the known infinite systems of simple groups. Miller's first words on this group, whose order is just under 245 million, were contained in a paper purporting to prove that the group does not exist. This was one of the few mistakes he made. The second thing Miller did with simple groups was remarkable in its time and has not been superseded yet. He proved in 1900 that a simple group of odd composite order, if any such exists, cannot be represented as a substitution group on fewer than 51 letters. This will be forgotten if ever a simple group of odd composite order is found, or if it can be proved that none exists, but so far nothing more definitive can be said.

There are many questions that present themselves early to the student of the theory of groups that Miller was the first to answer. One had been current for a long time. In 1856 W. R. Hamilton had studied the so-called polyhedral groups, the rotation groups of the regular polyhedra. He had shown that these groups can be defined as the groups generated by two elements s and t subject only to the conditions that s be of order 3, t of order 2, and their product st of order 3, 4, or 5: 3 for the tetrahedron, 4 for the octahedron and cube, 5 for the dodecahedron and the icosahedron. If s and t are both of order 2, the group depends only on the order of their product, and these groups were well known. It was not known whether any other set of numbers for the orders of s, t, and st were sufficient to determine a particular group. Miller proved in 1902 that for any other set of numbers, s and t may be selected to generate any one of an infinite set of groups. In such case at least one other condition must be imposed on s and t if a particular group is to be obtained. He worked off and on with problems of this kind as late as 1920, trying conditions on commutators, conditions that would ensure certain quotient groups, conditions on groups of p-th powers. One interesting set of groups near Hamilton's groups is the set obtained when s and t satisfy the conditions $s^3 = t^2 = (st)^6 = 1$; these are the groups of deformations of a regularly subdivided anchor ring into itself.

A question that was widely current around 1900 was answered by Fite in 1902, in a dissertation written under Miller at Cornell, when he exhibited a commutator subgroup containing an element which was not a commutator. Miller shortly gave an infinite system of groups having this property, i.e., the elements of the commutator subgroup are not all commutators.

Just before 1900 there was current the question whether two distinct abstract groups could have the same numbers of elements of each other. That they can is shown by the groups of order 16. Miller designated this relation by naming the groups *conformal*. Two distinct abelian groups are not conformal. In 1902 Miller determined the abelian groups that are conformal to non-abelian groups, and showed that there is no upper limit to the number of non-abelian groups conformal to each other and to the same abelian group. It is hard to believe now that this was ever a serious question, but in the 1890's a student at Chicago proposed to prove that the only groups with elements all of order an odd prime p are the well-known abelian ones, which is true when p is 2.

A similar question, whether two distinct groups can have the same group of isomorphisms, was recently asked seriously by an able and experienced student of groups. Miller answered this in 1900 when he proved that there are five groups whose groups of isomorphisms are the same group of order 24. In the same paper he proved the very useful fact that an abelian group must be cyclic if its group of isomorphisms is abelian.

A group of order n can be represented as a regular substitution group on n letters. It is thus a subgroup of the symmetric group of degree n. This symmetric group is well defined for any n and hence its subgroups are all defined and theoretically obtainable. The work with substitution groups shows that this attack on the groups of order n is not feasible. A group of order n has a composition series, which means that the group has a series of subgroups each invariant in the preceding one and such that the quotient group of one with respect to the next is a simple group. If none of these simple groups is of composite order, the original group is solvable. The solvable groups of order n can be attacked from another direction. The last group in the composition series is the identity and the one preceding that is cyclic if the group is solvable; the group which precedes that contains the cyclic group as an invariant subgroup of prime index. Miller gave in 1901 a method by which can be found all the groups of a given order which contain a given group G as a subgroup of prime index. In this paper he dealt with G of very special restricted type. He came back to this again and again, the last time in 1928 when no restrictions were placed on G. He thus offered a step-by-step method to determine all solvable groups. The method is practical only to a limited extent, as he recognized, because different series of steps may lead to the same group. It was useful to him in determining the groups of order 64.

The first two people who determined the groups of order 16 did not observe that each of the fourteen contains an abelian subgroup of order 8. Miller observed this and proved that every group of order p^m contains an invariant abelian subgroup of order p^a if m is as large as $a(a-1)/2$. This is an important fact in the application of the method of the preceding paragraph since it eliminates the first a steps.

A group of order n contains a subgroup of order p^m if p^m divides n and p^{m+1} does not. The prime power groups are therefore important. A prime power group is solvable. The term *class* of a prime power group was first used in Fite's dissertation to which reference was made earlier. A prime power group has a central which is not identity; the corresponding quotient group is a prime power group which accordingly has a central not identity. This second central determines a characteristic subgroup of the original group. This process may be continued until one arrives at the stage where the characteristic subgroup obtained is the whole of the original group. The number of steps in this process is the class of the group. The abelian groups are of class 1; Fite called groups of class 2 *metabelian*. These names have remained in the literature. Miller has said that we are inclined to give undue credit to men whose names have become prominent. To have written this paragraph, cautious though it is, during Miller's lifetime would have been to invite some uncomfortable moments pondering the treachery of circumstantial evidence.

The transitive groups of degree $p = 2q + 1$, where p and q are primes, are interesting because the Matthieu groups are closely connected with the cases $p = 11$ and $p = 23$. Jordan and de Seguier proved that for $p = 47$ and $p = 59$ there are no transitive groups except six that are easily exhibited for any such p. Miller devised a method by which the question can be answered for any such p. He verified the work of Jordan and de Sequier and used the method for $p = 83$, stating the result that there are only six transitive groups of degree 83. The method shows how to get 42 different elements of order 2, each on 82 letters. Denote one of these elements of order 2 by t, and denote the element of order 83 by s. The group $[s, t]$ is then examined. For each of the 42 t's Miller says the group $[s, t]$ is the alternating group of degree 83, of order $83!/2$. Probably nobody has ever checked his conclusion. This seems to be something a high-speed computer could do, since the computer could be told how to identify the alternating group without keeping the alternating group in its memory. Sooner or later this theorem will be checked, for mathematicians will not permanently accept defeat on the problem of fourfold transitive groups, and if more such groups are found they will still want to know about groups of degree 83.

In the more than four hundred technical papers that Miller contributed to the research journals many other subjects are treated that would merit mention here. His work on commutator subgroups, Φ-subgroups, groups of isomorphisms, and characteristic subgroups was of fundamental importance and of permanent value. The problems he could not solve usually led him to results that were of permanent value. He pursued these problems further than many others have been willing to go. He never pursued them to the point of getting results so complicated to state that other people could not understand them. He was never interested in translating a problem into a new problem unless he could thereby make progress with the original one. He advanced the science by the positive contributions he made, and also by exploring paths that look promising but that lead to difficulties not readily foreseeable.

Early in his career Miller started explaining the theory of groups and mathematics in general to people who are not expert in the respective subjects. In volumes 2 and 3 in 1895-1896 of the *American Mathematical Monthly* he published in fourteen installments the most complete treatment of the construction of substitution groups to be found in the literature. He continued all his life to promote the theory of groups, by explaining groups to non-specialists, by pointing out desirable uses of groups in teaching elementary mathematics, by publishing collections of quotations from eminent mathematicians who spoke well of groups.

Miller's most important publication on a general subject was probably "Some Thoughts on Modern Mathematical Research." This lecture was given before the University of Illinois chapter of Sigma Xi in 1912. It was printed in *Science* and was reprinted in the *Annual Report of the Smithsonian Institution* as one of the most important memoirs published in America during the year. "Some may be tempted to say that the useful parts of mathematics are very elementary and have little contact with modern research. In answer we may observe that it is very questionable whether the ratio of the developed mathematics to that which is finding direct application to things which relate to material advantages is greater now than it was at the time of the ancient Greeks." These sentences from the concluding paragraph suggest the tone; they are arresting today.

Published also in *Science* (1917) was "The Function of Mathematics in Scientific Research," a paper delivered before the Science Club of the University of Wisconsin. This is likewise profound, thought-provoking, and is still timely.

In his papers directed toward teachers of elementary mathematics he was for some years after 1900 urging the introduction of the study of groups. He wrote a "New Chapter in Trigonometry." He attempted to stimulate teachers of mathematics to study mathematics. If he felt that his own teachers had been ignorant of mathematics he never made mention of it. It is a fact that in this country a very small proportion of the teachers of secondary mathematics have ever got near mathematics. The thing Miller was trying to do is still to be done. He did more than the ordinary mathematician does to try to bring some of the ideas of higher mathematics to their attention and within their reach.

After theory of groups Miller's next interest was history of mathematics, and he studied history longer than he studied groups. He introduced a course in History of Mathematics at Eureka in 1890; he published his book *Historical Introduction to Mathematical Literature* in 1916; his last publication was "An Eleventh Lesson in the History of Mathematics" in the *National Mathematics Magazine* in 1947.

He was at his best as an historian when he was writing about the history of his own subject, the theory of groups and in particular finite groups. In 1899 he published his "Report on Recent Progress in the Theory of Groups of a Finite Order." This was history reported as it was being made. This and the second Report four years later are of extreme importance to any later historian. Abstract group theory was just acquiring autonomous status; it had many followers, especially in America; and its devotees were very active. These Reports are valuable for what they contain but are valuable also for things they do not contain. For example, in 1898 Miller had not yet met the question about commutator subgroups containing elements which are not commutators, the question which Fite and Miller answered in 1902. In the Report of 1899 Miller says of the commutators that they "form" the commutator subgroup. "Form" in this context is not precise: if it means *generate* his statement is correct; if it means *constitute* the statement is incorrect, and the latter is the more reasonable meaning. If he had met the question the statement would have been precise and would have been correct.

He wrote many short papers on the early history of groups. Most of these, but not all, were the result of his study of the writings of the earlier workers in the subject. The culmination of his work on the history of finite groups is the set of articles he wrote for volumes I, II, and III of his *Collected Works*. In volume I are two historical notes on the determination of substitution groups of a given degree and of abstract groups of a given order, and a history of groups to 1900. In each of volumes II and III is a history of groups during the period in which the papers in the volume were first published, thus bringing the history to 1915.

His besetting sin as an historian was that he did not generally feel called upon to acquaint himself at firsthand with the original sources. This was true to a lesser extent in the history of groups, although sometimes true there. Too much of his history on subjects other than his specialty consisted in confronting historian A with historian B or of confronting historian A with contradictory statements of A's own. He did this in a suitably objective manner, but it was generally believed that Miller was too much interested in discrediting historian A. Quite different from this are his arguments about the Greeks and the quadratic equation. Many people are willing to say the Greeks solved the quadratic equation since they used the method we use today, yet such a statement does not fit well with the fact that they had no concept of imaginary numbers and were not comfortable with negative numbers.

Although Miller's friends deplored much of his history, they could not quarrel with his aims. He believed that the student, and certainly the teacher, should know something of the history of mathematics. He believed that the history that is taught should be real history and not plausible answers to historical questions, that owning ignorance where ignorance exists is preferable to offering entertaining myths.

He expended much energy and alienated many well-wishers by coming back again and again to point out errors in a history of mathematics that has been widely used. In fact no history in English has been free from repeated attacks. *Encyclopaedia Britannica* and *Webster's Dictionary* were combed for errors about groups, mathematics, and their histories. He held the dictionaries and encyclopedias to as strict account in their statements about mathematical concepts as he held the research mathematician writing about his specialty. He knew that a book which contains heroes and villains will be more widely read than one about mortals who sometimes succeed and sometimes do not, but the writer who tried to put this fact to use in furthering a knowledge of the history of mathematics was called to account for taking liberties in the most inconsequential matters. He stated in his *Historical Introduction:*

"The caliber of a mathematician can probably be judged just as accurately from the errors to which he pays attention as from the new results which he announces. In both cases, he can devote himself to trivialities or to big things." There is no doubt he saw the application to historians as well. It will always be a thankless task to root out the errors that have crept into the history of mathematics and we all have a duty in that regard. When we impute excessive zeal to Miller we must assume an awkward defense of tolerance of error.

His relentlessness in the pursuit of error is shown in the case of the French encyclopedia where it had no ill effect so far as is known. The *Encyclopédie des Sciences Mathématiques* started publication in 1904; it asked for corrections and engaged to print them in a special section called *Tribune Publique.* At the end of the first volume was a section of about eighty pages on finite groups. Miller supplied the *Tribune Publique* with nearly twenty pages of corrections, historical and technical.

He recognized that an historian who disapproves of all the available histories of mathematics is under some obligation to write a history of his own. His *Historical Introduction* was one offering and it is relatively free of the things to which he objected. Also he wrote a *History of Elementary Mathematics.* This was offered for publication in the early 1930's, which was not an opportune time. It remains unpublished; but it is planned that it will appear in his *Collected Works.*

While Miller was in California he assisted in the organization of the San Francisco Section of the American Mathematical Society, and he was its Secretary until 1906. He was a member of the Council of the Society from 1901 to 1904. He was Chairman of the Chicago Section from 1907 to 1909; he was Vice-President of the Society in 1908. He was Secretary of section A of the American Association for the Advancement of Science from 1907 to 1912. He was one of the group that founded the Mathematical Association of America in 1915; he was one of the two Vice-Presidents in 1916; he was President in 1921. He was one of the editors of the *American Mathematical Monthly* from 1909 to 1915. He was a member of the London Mathematical Society, of the Deutsche Mathematiker Vereinigung, a corresponding member of the Sociedad Matematica Española, an honorary life member of the Indian Mathematical Society. He was a Fellow of the American Academy of Arts and Sciences. He was made an honorary life member of the Mathematical Association of America in 1937. The honorary degree of Doctor of Letters was conferred on him by Muhlenburg in 1936.

On his retirement in 1931 the University of Illinois undertook the collection and republication of his technical papers. Volumes I to IV of the *Collected Works of George Abram Miller* have so far appeared. There will be one or two more volumes, the last one of which will contain his unpublished *History of Elementary Mathematics* to which reference has been made.

During the nineteen years after his retirement he followed the regime he had been following before; he went to his office in the Mathematics Building morning, afternoon, and evening. On more than one Sunday morning, when the University doors are locked, Mrs. Miller had to enlist the aid of someone with a key in order to communicate with him. He did not relax his devotion to the habit of persistent and well-directed exertion, but stopped writing in 1947 when his hand could no longer control his pen. He kept a watchful eye on the additions to the Mathematics Library, and in the margins of the books he made comments and inserted corrections of their historical blunders. He took daily walks around the campus and beyond, but he never went far from town in the nineteen years. Occasionally in the evening he played bridge and was a keenly competitive player, but he never attained to the seriousness of purpose that is common among mathematicians who play bridge. He wrote in one of his papers for the edification and guidance of the young that a man who must go afield for his fun and recreation is not likely to become a scholar.

Probably long before Eureka he was engaged in giving a steady and dignified bearing to the mind. He was easy and responsive in conversation but there was always a push away from the immediate to things more enduring and of general import. One friend might hear of a pertinent incident from his childhood on the farm; another might hear of a personal experience at Eureka or at Michigan; another might hear of his planting a rosebush in his friend's yard so he would not be forgotten when he left Cornell. Very few heard much. Yet nobody who knew him thought him secretive, even after he had completely astounded all who had known him for years.

Some weeks after his death the newspapers and the radio announced that Miller had left to the University of Illinois an estate valued at just under a million dollars. In the week before he died, when he had given in to the urging of friends and was going to the hospital, he had asked who was going to pay for this. Friends of recent date who had seen him leading the life of a professor on retirement pay were worried about the expense; those who had known him longer recognized his intention to make well-meaning but officious people squirm a little. A man with a brief case, obviously not a college professor and certainly not an insurance salesman, who had been coming to Miller's office off and on for years, had prepared many of his associates to hear that Miller had been interested in the investment market. But many a man has spent a lifetime rushing about and still the million dollars has eluded him.

Miller had never been preoccupied, or rather he had been preoccupied by a statement which he misread to mean that a Russian mathematician had proved there is no simple group of odd composite order, or by a purported dialogue between Euclid and a student which appeared in a book about mathematics written by a reputable mathematician.

Miller had not been preparing a joke on his associates. If he had thought the million dollars was important it would have appeared in the journals. The million dollars is something that attached itself to Miller as he went steadily on his way, not denying or despising the world but certainly not giving it dominion over him.

KEY TO ABBREVIATIONS

Amer. J = American Journal of Mathematics
Annalen = Mathematische Annalen
Annali = Annali di Matematiche
Annals = Annals of Mathematics
Archiv = Archiv der Mathematik und Physik
Bibliotheca = Bibliotheca Mathematica
Bull. = Bulletin of the American Mathematical Society
Bull. France = Bulletin de la Societé Mathématique de France
Bull. N. Y. = Bulletin of the New York Mathematical Society
Collected Works = The Collected Works of George Abram Miller
Comptes Rendus = Comptes Rendus de l'Académie des Sciences
Educ. Review = Educational Review
Giornale = Giornale di Matematiche
Jahresbericht = Jahresbericht der deutschen Mathematiker-Vereinigung
J. Indian = Journal of the Indian Mathematical Society
J. Math. = Journal für reine und angewandte Mathematik
Le mat. = Le matematiche pure ed applicate
L'enseignement = L'enseignement mathématique
L'Intermediare = L'Intermediare des Mathématiciens
Math. Gaz. = Mathematics Gazette
Math. Mag. = National Mathematics Magazine
Math. Student = The Mathematics Student
Math. Teacher = Mathematics Teacher
Messenger = Messenger of Mathematics
Monographs = Monographs on topics of modern mathematics, New York, 1911
Monthly = American Mathematical Monthly
Phil. Mag. = Philosophical Magazine
Pop. Sci. = Popular Science Monthly
Prace mat. = Prace matematyczno-fizyczne
Proc. AAAS = Proceedings of the American Association for the Advancement of Science
Proc. Am. Phil. = Proceedings of the American Philosophical Society
Proc. London = Proceedings of the London Mathematical Society
Proc. NAS = Proceedings of the National Academy of Sciences
Proc. Toronto = Proceedings of the Toronto Congress
Quart. J. = Quarterly Journal of Mathematics
Revista = Revista, Sociedad Matematica Española
Revista Mat. = Revista Matematica Hispano-Americana
School Sci. = School Science and Mathematics

Sci. Monthly = Scientific Monthly
Sci. Prog. = Science Progress
Scripta = Scripta Mathematica
Sigma Xi Quart. = Sigma Xi Quarterly
Smithsonian Inst. = Smithsonian Institution, Annual Report
Tôhoku J. = Tôhoku Mathematical Journal
Trans. = Transactions of the American Mathematical Society
Trib. Pub. = Tribune Publique of the Encyclopédie des Sciences Mathématiques
Vierteljahrsschrift = Vierteljahrsschrift der naturforschenden Gesellschaft in Zurich
Year-Book = American Year-Book

BIBLIOGRAPHY

Determinants. iii-110 pp. 1892 Van Nostrand Science Series.

1894

Note on Substitution Groups of Eight Letters. Bull. N. Y., 3:168-169.

Note on Substitution Groups of Eight and Nine Letters. Bull. N. Y., 3:242-245.

On the Non-primitive Substitution Groups. Bull., 1:67-72.

An Instance Where a Well-known Test to Prove the Simplicity of a Simple Group is Insufficient. Bull., 1:124-125.

Note on Transitive Substitution Groups of Degree Twelve. Bull., 1:255-258.

Intransitive Substitution Groups of Ten Letters. Quart. J., 27:99-118.

1895

A Simple Proof of the Fundamental Theorem of Substitution Groups, and Some Applications of the Theorem. Bull., 2:75-77.

Remarks on Substitution Groups. Monthly, 2:142-144, 179-180.

1896

Introduction to Substitution Groups. Monthly, 2:211-213, 257-260, 267-268, 304-309, 351-354; 3:7-13, 36-38, 69-73, 104-108, 133-136, 171-174.

Applications of Substitution Groups. Monthly, 2:197-202.

Lie's Views on Several Important Points in Modern Mathematics. Monthly, 2:295-296.

On the Lists of all the Substitution Groups That Can Be Formed with a Given Number of Elements. Bull., 2:138-145.

The Substitution Groups Whose Orders Are the Products of Two Unequal Prime Numbers. Bull., 2:332-336.

On Several Theorems of Operation Groups. Bull., 3:111-116.

List of Transitive Substitution Groups of Degree Twelve. Quart. J., 28:193-231.

The Regular Substitution Groups Whose Orders Are Less than 48. Quart. J., 28:232-284.

The Substitution Groups Whose Order Is Four. Phil. Mag., 41:431-437.

The Operation Groups of Order 8*p*, *p* Being Any Prime Number. Phil. Mag., 41:195-200.

The Non-regular Transitive Substitution Groups Whose Order Is the Cube of Any Prime Number. Annals, 10:156-158.

Sur les groupes des substitutions. Comptes Rendus, 122:370-372, 123:91-92.

1897

On the Transitive Substitution Groups Whose Orders Are the Products of Three Prime Numbers. Bull., 3:213-222.

The Non-regular Transitive Substitution Groups Whose Orders Are the Products of Three Unequal Prime Numbers. Vierteljahrsschrift, 42:68-73.

On the Transitive Substitution Groups of Degrees Thirteen and Fourteen. Quart. J., 29:224-249.

On the Primitive Substitution Groups of Degree Fifteen. Proc. London, 28:533-544.

Sur l'énumération des groupes primitifs dont le degré est inférieur à 17. Comptes Rendus, 124:1505-1508.

The Transitive Substitution Groups of Order 8*p*, *p* Being Any Prime Number. Phil. Mag., 43:117-125.

On the Transitive Substitution Groups That Are Simply Isomorphic to the Symmetric and Alternating Groups of Degree Six. Proc. AM. Phil., 36:208-215.

1898

Note on the Practical Application of a Substitution Group in Spherical Trigonometry. Monthly, 5:102-103.

On Several Points in the Theory of Groups of a Finite Order. Monthly, 5:196-197.

On Groups Which Are Determined by a Given Group. Monthly, 5:221-224.

On a Method to Construct Intransitive Substitution Groups. Monthly, 5:288-290.

On the Commutator Groups. Bull., 4:135-139.
On the Limit of Transitivity of the Multiply Transitive Groups That Do Not Contain the Alternating Group. Bull., 4:140-143.
On an Extension of Sylow's Theorem. Bull., 4:323-327.
On the Hamiltonian Groups. Bull., 4:510-515.
On the Primitive Substitution Groups of Degree Sixteen. Amer J., 20:229-241.
On the Perfect Groups. Amer. J., 20:277-282.
On the Operation Groups Whose Orders Are Less than 64 and Those Whose Order Is $2p^3$, p Being Any Prime Number. Quart. J., 30:243-263.
Sur les groupes hamiltoniens. Comptes Rendus, 126:1406-1408.
On an Important Theorem with Respect to the Operation Groups of Order p^α, p Being Any Prime Number. Messenger, 27:119-121.
On the Supposed Fivefold Transitive Function of 24 Elements and 19!/48 Values. Messenger, 27:187-190.
On the Simple Isomorphisms of a Substitution Group to Itself. Phil. Mag., 45:234-242.
On the Quaternion Group. Proc. AM. Phil., 37:312-319.

1899

On the Total Number of Transitive Substitution Groups That Have a 1, α Isomorphism to a Given Group. Messenger, 28:84-87.
On the Groups That Can Be Represented as Multiply Transitive Substitution Groups. Messenger, 28:104-107.
Report on Recent Progress in the Theory of Groups of a Finite Order. Bull., 5:227-249.
Note on Burnside's *Theory of Groups*. Bull., 5:249-251.
On the Simple Isomorphisms of a Hamiltonian Group to Itself. Bull., 5:292-296.
Note on Simply Transitive Primitive Groups. Bull., 6:103-104.
On the Commutators of a Given Group. Bull. 6:105-109.
On the Representation of a Group of Finite Order as a Substitution Group. Messenger, 28:149-150.
Sur les groupes d'opérations. Comptes Rendus, 128:227-229.
Memoir on the Substitution Groups Whose Degrees Do Not Exceed Eight. Amer. J., 21:287-338.
On the Transitive Substitution Groups of Degree Seventeen. Quart. J., 31:49-57.
On the Primitive Substitution Groups of Degree Ten. Quart. J., 31:228-233.

On Several Classes of Simple Groups. Proc. London, 31:148-150.
On the Simple Groups Which Can Be Represented as Substitution Groups That Contain Cylical Substitutions of a Prime Degree. Monthly, 6:102-103.
Some Reminiscences with Regard to Sophus Lie. Monthly, 6:191-193.
Some Elements of Substitution Groups. Monthly, 6:255-257.

1900

Report on the Groups of an Infinite Order. Bull., 7:121-130.
Note on Netto's Theory of Substitutions. Annals, 2:1, 71-73
On the Product of Two Substitutions. Amer. J., 22:185-190.
On the Groups Which Have the Same Group of Isomorphisms. Trans., 1:395-401.
Note on the Group of Isomorphisms. Bull., 6:337-339.
Burnside's Theory of Groups. Bull., 6:390-398.
On the Transitive Substitution Groups Which Are Isomorphic to a Given Group. Giornale, 38:63-71.
On the Groups Which Are the Direct Products of Two Subgroups. Trans., 1:66-71.
On the Holomorph of the Cyclical Group and Some of Its Subgroups. Quart. J., 31:382-384.
Sur plusieurs groupes simples. Bull. France, 28:266-267.
Proof That There Is No Simple Group Whose Order Lies between 1092 and 2001. Amer. J., 22:13-26.
Some Methods of Constructing Substitution Groups. Monthly, 7:277-281.
In a Simple Group of an Odd Composite Order Every System of Conjugate Operators or Subgroups Includes More than Fifty. Proc. London, 33:6-10.
An Index of Some New Fields of Thought in Mathematics. Bull., 7:121-130.
Sur les groupes des isomorphismes. Comptes Rendus, 130:316-317.
Examples of a Few Elementary Groups. Monthly, 7:9-13.

1901

Sur les groupes d'opérations. Comptes Rendus, 132:912-914.
Determination of All the Groups of Order p^m Which Contain the Abelian Group of Type $(m-2, 1)$, p Being Any Prime. Trans. 2:259-272.
Sur les groupes de substitutions. Comptes Rendus, 133:624-625.
On the Groups Generated by Two Operators of Order Three Whose Product Is Also of Order Three. Annals, 3:40-43.

On the Groups Generated by Two Operators of Orders Two and Three Respectively Whose Product Is of Order Six. Quart. J., 33:76-79.
On a Special Class of Abelian Groups. Annals, 3:77-80.
On the Transitive Substitution Groups Whose Order Is a Power of a Prime Number. Amer. J., 23:173-178.
On the Groups Generated by Two Operators. Bull., 7:424-426.
On Holomorphisms and Primitive Roots. Bull., 7:350-354.
O pewnem twierdzeniu elementarnem w teoryi grup podstawien. Prace mat., 12:136-138.
On the Product of Two Commutative Operators. Monthly, 8:213-216.
List of the Intransitive Substitution Groups of Degree Eleven. Quart. J., 32:342-368.
On the Abelian Groups Which Are Conformal with Non-Abelian Groups. Bull., 8:154-156.
On the History of Several Fundamental Theorems in the Theory of Groups of Finite Order. Monthly, 8:213-216.
On the Concepts of Number and Group. Monthly, 8:137-139.
Sui gruppi generati da due operatori. Le mat., 1:231-234.

1902

Second Report on Recent Progress in the Theory of Groups of Finite Order. Bull., 9:106-123.
Determination of All the Groups of Order p^m, p Being Any Prime, Which Contain the Abelian Group of Order p^{m-1} and of Type (1, 1, 1, . . .). Bull., 8:391-394.
On the Groups of Order p^m Which Contain Operators of Order p^{m-2}. Trans., 3:383-387.
Groups Defined by the Orders of Two Generators and the Order of Their Product. Amer. J., 24:96-100.
On a Method of Constructing All the Groups of Order p^m. Amer. J., 24:395-398.
Note on the Group of Isomorphisms of a Group of Order p^m. Annals, 3:180-184.
On an Infinite System of Conformal Groups. Messenger, 31:148-150.
On the Group of Isomorphisms of an Abelian Group. Prace mat., 13:155-158.
Gruppi d'ordine p^m (p primo) non conformi con gruppi abeliani. Le mat., 2:19-21.
Determination of All the Groups of Order 168. Monthly, 8:1-5.
On the Primitive Groups of Class Four. Monthly, 8:63-66.

Linear Groups with an Exposition of the Galois Field Theory. Science, 15:113-114.
On Ball's *History of Mathematics*. Monthly, 9:280-283.
Kronecker's *Lectures on the Theory of Numbers*. Bull., 8:303-307.

1903

Non-Abelian Groups in Which Every Subgroup Is Abelian. Trans., 4:398-404.
On the Holomorph of a Cyclic Group. Trans., 4:153-160.
Sur les groupes de substitutions. Comptes Rendus, 136:294-295.
On the Mathieu System of Triply Transitive Groups. Quart. J., 34:232-234.
A Fundamental Theorem with Respect to Transitive Substitution Groups. Bull., 9:543-544.
Note on Abelian Groups. Giornale, 41:336.
A New Proof of the Generalized Wilson's Theorem. Annals, 4:188-190.
On the Definition of an Infinite Number. Monthly, 10:154-155.
An Elementary Example of Modular Systems. Monthly, 10:27-30.
On the Groups of the Figures of Elementary Geometry. Monthly, 10:215-218.
Appreciative Remarks on the Theory of Groups. Monthly, 10:87-89.
Some Fundamental Discoveries in Mathematics. Science, 17:496-499.
On the Generalization and Extension of Sylow's Theorem. Proc. AAAS, 53:373.

1904

Sur les groupes d'opérations. Comptes Rendus, 138:888-890.
On the Subgroups of an Abelian Group. Annals, 6:5-6.
On the Roots of the Operators of a Group. Quart. J., 36:51-55.
Note on the Groups Whose Orders Are Powers of an Odd Prime Number. Messenger, 33:164-165.
Extension of a Fundamental Theorem in Group Theory. Messenger, 34:96.
Addition to a Theorem Due to Frobenius. Bull., 11:6-7.
Note on Sylow's Theorem. Annals, 5:187.
An Extension of Sylow's Theorem. Proc. London, 2:2, 142-143.
On the Generalization and Extention of Sylow's Theorem. Monthly, 11:29-32.
On the Number of Sets of Conjugate Subgroups. Prace mat., 15:87-89.
Two Infinite Systems of Groups Generated by Two Operators of Order Four. Monthly, 11:184-185.

On the Totitives of Different Orders. Monthly, 11:129-130.
What Is Group Theory? Pop. Sci., 64:369-374.
The Subtraction Groups. Monthly, 11:199-202.
Groups of Elementary Trigonometry. Monthly, 11:225-226.
On the Definition of an Infinite Number. Monist, 14:469-472.

1905

Extension of a Theorem Due to Sylow. Bull., 11:367-369.
Determination of All Characteristic Subgroups of Any Abelian Group. Amer. J., 27:15-24.
The Groups of Order 2^m Which Contain an Invariant Cyclic Subgroup of Order 2^{m-2}. Bull., 11:494-499.
On the Possible Numbers of Operators of Order 2 in a Group of Order 2^m. Bull., 12:74-77.
Determination of All the Groups of Order 2^m Which Contain an Odd Number of Cyclic Subgroups of Composite Order. Trans., 6:58-62.
Generalization of the Hamiltonian Groups. Annalen, 60:597-606.
On the Invariant Subgroups of Prime Index. Trans., 6:326-331.
Sur les sous-groupes invariants d'indice p^2. Comptes Rendus, 140:32-33.
Groupes contenant plusieurs opérations de l'orde deuxième. Comptes Rendus, 141:591-592.
Groups of the Fundamental Operations of Arithmetic. Annals, 6:41-48.
Groups of Subtraction and Division. Quart. J., 37:80-87.
Theorems Relating to Quotient Groups. Prace mat., 16:109-112.
Some Relations between Number Theory and Group Theory. Amer. J., 27:315-322.
Application of Several Theorems in Number Theory to Group Theory. Monthly, 12:81-84.
The Groups Generated by Two Operators Which Have a Common Square. Archiv, 9:6-7.
Note on the Totient of a Number. Monthly, 12:41-43.
Groups Containing the Largest Possible Number of Operators of Order Two. Monthly, 12:228-232.
Mathematics in Japan. Science, 22:215-216.
The Mathematical Handbook of Ahmes. School Sci., 5:567-574.
A Minor Reform. School Sci., 5:394-395.

1906

On the Number of Abelian Subgroups Whose Order Is a Power of a Prime. Messenger, 36:79-80.

Groups Containing Only Three Operators Which Are Squares. Trans., 7:94-98.

The Groups of Order p^m Which Contain Exactly p Cyclic Subgroups of Order p^a. Trans., 7:228-232.

Note on the Possible Number of Operators of Order 2 in a Group of Order 2^m. Annals, 7:55-60.

The Groups Containing Thirteen Operators of Order Two. Bull., 12:289-302.

Groups in Which All the Operators Are Contained in a Series of Subgroups Such That Any Two Have Only Identity in Common. Bull., 12:446-449.

Groups Generated by Operators Which Transform Each Other into Their Powers. Quart. J., 37:286-288.

On the Commutators of a Group of Order p^m. Quart. J., 37:349-352.

A New Chapter in Trigonometry. Quart. J., 226-234.

Groups Generated by Two Operators Which Transform Each Other into the Same Power. Prace mat., 17:119-122.

Several Fundamental Theorems in Group Theory. Monthly, 13:10-11.

The Groups Which Contain Less than Twenty Operators of Order Three. Monthly, 13:27-29.

Some Useful Groups in the Teaching of Elementary Trigonometry. Math. Gaz., 3:353-357.

Group Theory for Teachers of Elementary Mathematics. School Sci., 6:752-756.

Reform in Mathematical Instruction. Science, 24:493-496.

On a Fundamental Theorem in Trigonometry. Monthly, 13:101-103.

Note on the Addition Theorem in Trigonometry. Monthly, 13:226-227.

Lodge's *Easy Mathematics*. Science, 24:114-115.

The Study of History of Mathematics. School Sci., 6:294-296.

Recent Tendencies in Mathematical Instruction. Pop. Sci., 68:161-165.

1907

The Groups in Which Every Subgroup of Composite Order Is Invariant. Archiv, 11:76-79.

Third Report on Recent Progress in the Theory of Groups of Finite Order. Bull., 14:78-91, 124-133.

Groups of Order p^m Containing Exactly $p+1$ Abelian Subgroups of Order p^{m-1}. Bull., 13:171-177.

On the Minimum Number of Operators Whose Orders Exceed Two in Any Finite Group. Bull., 13:235-239.

The Groups Generated by Three Operators Each of Which Is the Product of the Other Two. Bull., 381-382.
Note on the Commutator of Two Operators. Bull., 13:497-501.
The Invariant Substitutions under a Substitution Group. Bull., 13:497-501.
The Groups Which Contain Less than Six Cyclic Subgroups of the Same Order. Annalen, 64:344-356.
The Groups of Isomorphisms of the Simple Groups Whose Degrees Are Less than Fifteen. Archiv, 12:249-251.
Generalization of the Groups of Genus Zero. Trans., 8:1-13.
The Groups in Which Every Subgroup Is Either Abelian or Hamiltonian. Trans., 8:25-29.
The Groups Which Contain Less than Fifteen Operators of Order Two. Amer. J., 29:1-12.
Groups in Which Every Subgroup Is Either Abelian or Dihedral. Amer. J., 29:289-294.
The Groups Generated by Two Operators Such That Each Is Transformed into Its Inverse by the Square of the Other. Annals, 9:48-52.
Group of Order p^6 Which Does Not Include an Abelian Subgroup of Order p^4. Messenger, 36:188-189.
Note on the Definition of a Complete Group. Messenger. 37:54-55.
The Groups Which Are Generated by Two Operators of Orders Two and Four Respectively Whose Commutator Is of Order Two. Proc. Am. Phil., 46:146-150.
Note on the Use of Group Theory in Elementary Trigonometry. Annals, 8:97-102.
On the Groups Generated by Two Operators of Order Three Whose Product Is of Order Four. Prace mat., 18:235-240.
La théorie des groupes appliquée aux mathématiques elementaires. L'enseignement, 9:192-198.
Review of *Theory of Determinants* by Thomas Muir. Bull., 13:244-246.
Several Historic Problems Which Have Not Yet Been Solved. Monthly, 14:6-8.
Dividing by Zero. Monthly, 14:49-51.
Note on the Postulate That a Part Is Equivalent to the Whole. Monthly, 14:100-101.
On Some of the Symbols of Elementary Mathematics. School Sci., 7:406-410.
Note on the Solution of $a/x + b/y = c$ for $x = y = o$. School Sci., 7:667-669.

Review: *Pierpont on Functions of a Real Variable.* Science, 27:299-300.
Mathematical Prodigies. Science, 26:628-630.
Library Aids to Mathematical Study. Monthly, 14:193-196.
Note on the Definition of a Variable. Monthly, 14:213-215.
On the Development of Geometry in the Nineteenth Century. School Sci., 7:752-755.
Origine de quelques signes d'inegalité. L'Intermediare, 17:226.

1908

Groups Defined by the Orders of Two Generators and the Order of Their Commutator. Trans., 9:67-78.
On the Holomorph of the Cyclic Group of Order p^m. Trans., 9:232-236.
On the Multiple Holomorphs of a Group. Annalen, 66:133-142.
Groups Generated by n Operators Each of Which Is the Product of the $n - 1$ Remaining Ones. Amer. J., 30:93-98.
Transitive Groups of Degree $p = 2q + 1$, p and q Being Prime Numbers. Quart. J., 39:210-216.
The Groups of Isomorphisms of the Groups Whose Degree Is Less than Eight. Phil. Mag., 15:223-232.
Groups in Which the Subgroup Which Involves All the Substitutions Omitting a Given Letter Is Regular. Prace mat., 19:17-19.
Answer to a Question Raised by Cayley as Regards a Property of Abstract Groups. Bull., 15:72-75.
On the Groups Generated by Two Operators Satisfying the Condition $s_1s_2 = s_2^a s_1^{-2}$. Bull., 15:162-165.
The Classification of Mathematics. Pop. Sci., 73:319-325.
Note on the Groups of Subtraction and Division, and on the Hyperbolic Functions. Math. Gaz., 4:381-384.
Generalization of the Positive and Negative Numbers. Monthly, 15:117-119.
Definition of the Term Mathematics. Monthly, 15:25-28.
External Encouragement for the Study of Higher Mathematics. Monthly, 15:25-28.
Some Questionable Terms and Definitions Used in Elementary Mathematics. School Sci., 8:294-299, 571-576.
Appointments in Colleges and Universities. Science, 28:561-562.

1909

The Central of a Group. Trans., 10:50-60.
Automorphisms of Order Two. Trans., 10:471-478.

On the Groups Generated by Two Operators (s_1, s_2) Satisfying the Equation $s_1s_2 = s_2\alpha s_1\beta$. Quart J., 40:197-209.

Note on the Groups Generated by Operators Transforming Each Other into Their Inverses. Quart. J., 40:366-367.

Methods to Determine the Primitive Roots of a Number. Amer. J., 31:42-44.

Finite Groups Which May Be Defined by Two Operators Satisfying Two Conditions. Amer. J., 31:167-182.

Sur les groupes engendrés par deux opérateurs dont chacun transforme le carré de l'autre en son inverse. Comptes Rendus, 149:843-846.

Groups Formed by Prime Residues with Respect to Modular Systems. Archiv, 15:115-121.

The Possible Abstract Groups of the Ten Orders 1909-1919. Monthly, 16:25-27.

The Groups Which May Be Generated by Two Operators s_1, s_2 Satisfying the Equation $(s_1s_2)\alpha = (s_2s_1)\beta$, α and β Being Relatively Prime. Bull., 16:67-69.

Note on the Groups Generated by Two Operators Whose Squares Are Invariant. Bull., 16:173-174.

Groups Generated by Two Operators Each of Which Is Transformed into a Power of Itself by the Square of the Other. Bull., 16:466-473.

Extensions of Two Theorems Due to Cauchy. Bull., 16:510-513.

The Future of Mathematics. Pop. Sci., 75:117-123.

On the Definition of an Angle. School Sci., 9:45-47.

On a Few Points in the History of Elementary Mathematics. Monthly, 16:177-179.

Proposed Publication of Euler's Works. Science, 30:10-12.

1910

Groups Generated by Two Operators Satisfying the Condition $s_1s_2 = s_2^{-1}s_1^{n}$. Prace mat., 20:193-197.

On a Method Due to Galois. Quart. J., 41:382-384.

On the Solution of a System of Linear Equations. Monthly, 16:137-139.

Note on the Equation $s_1s_2 = s^2{}_2s^2{}_1$, s_1 and s_2 Being Operators of a Finite Group. J. Math., 139:80-84.

Groups Generated by Two Operators (s_1, s_2) Satisfying the Equation $s_1s_2^2 = s_2s_1^2$. Trans., 11:341-350.

Groups Generated by Two Operators Each of Which Transforms the Square of the Other into a Power of Itself. Proc. Am. Phil., 49:238-242.

Some Relations Between Substitution Group Properties and Abstract Groups. Proc. Am. Phil., 49:307-314.
Groups Involving Only a Small Number of Sets of Conjugate Operators. Archiv, 17:199-204.
Multiply Transitive Groups of Transformations of Complete Sets of Conjugates. Jahresbericht, 19:318-320.
Generalizations of the Tetrahedral and Octahedral Groups. Amer. J., 32:65-74.
Groups of Transformations of Sylow Subgroups. Amer. J., 32:299-304.
Generalizations of the Icosahedral Group. Quart. J., 41:168-174.
The Founder of Group Theory. Monthly, 17:162-165.
The Group Generated by Two Conjoints. Prace mat., 21:61-64.
Mathematics beyond the Calculus. Monthly, 17:127-132.
Explanation of the Term Fourth Dimension. School Sci., 10:43-47.
Appreciative Remarks on the Theory of Groups. School Sci., 10:279-282.
Recent Changes of View as Regards Some Points in the History of Elementary Mathematics. Educ. Review, 39:403-406.
Historical Sketch of the Development of the Theory of Groups of Finite Order. Bibliotheca, 10:317-329.

1911

On the Use of the Co-Sets of a Group. Trans., 12:326-334.
Isomorphisms of a Group Whose Order Is a Power of a Prime. Trans., 12:387-402.
Note on the Imprimitive Substitution Groups. Jahresbericht, 20:192-196.
Abstract Definitions of All the Substitution Groups Whose Degrees Do Not Exceed Seven. Amer. J., 33:363-372.
Effect on the Product When Its Factors Are Permuted in Every Possible Manner. Quart. J., 42:177-180.
Groups Generated by Two Operators Satisfying Two Conditions. Bull., 17:333-340.
Number of Abelian Subgroups in the Possible Groups of Order 2^m. Messenger, 41:28-31.
On the Totality of the Substitutions on *n* Letters Which Are Commutative with Every Substitution of a Given Group on the Same Letters. Proc. Am. Phil., 50:139-146.
The Cyclic Group as a Basic Element in the Theory of Numbers. Monthly, 18:204-209.
Extension of a Group by Operators of Orders Two and Four. Prace mat., 22: 69-71.

Note on William R. Hamilton's Place in the History of Abstract Group Theory. Bibliotheca, 11:314-315.

Kronecker and the Galois Theory of Equations. Bibliotheca, 11:182.

The Algebraic Equation. Monographs, pp. 211-260.

Appreciative Remarks on the Theory of Groups. Math. Gaz., 6:148-151.

Tests of Symmetric Polynomials. Monthly, 18:49-52.

Reduction of the Trigonometric Functions of Any Angle to the Functions of Angles in a Small Interval. Monthly, 18:171-174.

Courses in Higher Mathematics. Science, 33:984-987.

American Mathematics. Pop. Sci., 79:459-463.

A Few Mathematical Errors in the Recent Edition of the *Encyclopedia Britannica*. Science, 34:761-762.

1912

Abstract Properties of the Finite Groups Which Can Be Represented Linearly on One Variable. Tôhoku J., 2:1-4.

New Proof of the Invariants of an Abelian Group. Tôhoku J., 2:137-139.

Some Thoughts on Modern Mathematical Research. Science, 35:877-887.

Some Properties of the Groups of Isomorphisms. Archiv, 19:295-301.

Infinite Systems of Indivisible Groups. Trans., 13:419-433.

A Third Generalization of the Groups of the Regular Polyhedrons. Annals, 13:419-433.

Note on the Maximal Cyclic Subgroups of a Group of order p^m. Bull., 18:189-191.

Note on Ratio Matrices. Quart. J., 43:217-218.

On the Sum of the Numbers Which Belong to a Fixed Exponent as Regards a Given Modulus. Monthly, 19:41-46.

A Few Theorems Relating to Sylow Subgroups. Bull., 19:63-66.

Gauss's Lemma and Some Related Group Theory. Prace mat., 23:25-29.

The Product of Two or More Groups. Bull., 19:303-310.

On the Representation Groups of Given Abstract Groups. Trans., 14: 444-452.

On the Introduction of the Word Group as a Technical Mathematical Term. Bibliotheca, 13:62-64.

Mathematical Encyclopedias. School Sci., 12:27-30.

Some Useful Mathematics Books beyond the Elementary Calculus. Monthly, 19:63-68.

Some Appreciative Remarks on the Theory of Numbers. Monthly, 19:113-115.

A New Mathematical Prize. Science, 36:374.
M. Henri Poincaré. Science, 36:425-429.

1913

Groups Containing a Given Number of Operators Whose Orders Are Powers of the Same Prime Number. Amer. J., 35:1-9.
Second Note on the Groups Generated by Operators Transforming Each Other into Their Inverses. (A correction.) Quart. J., 44:142-146.
Some Properties of the Commutators Arising from an Operator of a Given Order. Quart. J., 44:326-330.
A Non-Abelian Group Whose Group of Isomorphisms Is Abelian. Messenger, 43:124-125.
A Group of Order p^m Whose Group of Isomorphisms Is of Order p^a. Messenger, 43:126-128.
A Theorem in Number Theory Proved by Isomorphisms of Special Abelian Groups. Tôhoku J., 4:105-106.
Maximal Order of the Multiplying Group Corresponding to a p-Isomorphism of an Abelian Group of Order p^m. Jahresbericht, 22:291-294.
Groups Which Contain an Abelian Subgroup of Prime Index. Annals, 14:95-100.
Some Properties of the Group of Isomorphisms of an Abelian Group. Bull., 20:364-368.
A Group Admitting Outer Isomorphisms Which Do Not Permute Any Sets of Conjugate Operators. Bull., 20:311.
Mathematical Definitions in the New Standard Dictionary. Science, 38:772-773.
Some Thoughts on Modern Mathematical Research. J. Indian, 5:162-175.
Modern Mathematical Research. Smithsonian Inst.: pp. 187-198.
Errors in the Literature on Groups of Finite Order. Monthly, 20:14-20.
Mathematical Literature for High School Teachers. Monthly, 20:92-97.
Errors in Mathematics. School Sci., 13:313-319.
Mathematical Troubles of the Freshman. Monthly, 20:235-242.

1914

Note on the Multiply Transitive Solvable Substitution Groups. Archiv, 23:32-33.
The Group of Isomorphisms of an Abelian Group and Some of Its Abelian Subgroups. Amer. J., 36:47-52.
Examples of Normal Domains of Rationality Belonging to Elementary Groups. Annals, 15:118-124.

Independent Generators Contained in a Subgroup of an Abelian Group. Tôhoku J., 5:10-18.
Outer Isomorphisms of a Group Whose Inner Isomorphisms Form a Group Having the Square of a Prime for Its Order. Jahresbericht, 23:144-148.
Representation Groups of Well-Known Systems of Groups. Tôhoku J., 6:35-41.
Remarks on the Bearing of the Theory of Groups. Tôhoku J., 6:73-78.
Primitive Substitution Groups of Degree Nine. Quart. J., 46:13-18.
Note on Intransitive Substitution Groups. Quart. J., 46:94-96.
Groups of the Figures of Elementary Geometry. Monthly, 21:285-286.
Suggestions for the Prospective Mathematician. School Sci., 14:26-30.
Ideals Relating to Scientific Research. Science, 39:809-819.
Recent Mathematical Activities. Pop. Sci., 85:457-462.
Grupos: nomenclatura y notacion. Revista, 3:193-199.

1915

Note on the Potential and the Antipotential Group of a Given Group. Bull., 21:221-227.
A New Proof of Sylow's Theorem. Annals, 16:169-171.
Limits of the Degree of Transitivity of Substitution Groups. Bull., 22:68-71.
The Φ-Subgroup of a Group. Trans., 16:20-26.
Independent Generators of a Group of Finite Order. Trans., 16:399-404.
Groups Possessing at Least One Set of Independent Generators Composed of as Many Operators as There Are Prime Factors in the Order of the Group. Proc. NAS, 1:241-244.
Note on Several Theorems Due to A. Capelli. Giornale, 53:313-315.
Groups of Subtraction and Division with Respect to a Modulus. Monthly, 22:47-49.
Successive Transforms of an Operator with Respect to a Given Operator. Messenger, 45:92-94.
Groups Which Are Commutator Subgroups of Other Groups. Tôhoku J., 8:67-72.
The Preparation of Mathematics Teachers in the United States of America. L'enseignement, 17:325-335.
History of Mathematics. Monthly, 22:229-304.
The Training of Mathematics Teachers. School Sci., 15:1-12.
Historical Notes in Text-Books on Secondary Mathematics. School Sci., 15:806-809.

Shamelessness as Regards Mathematical Ignorance. School and Society, 1:441-444.
Quantity and Quality. Science, 42:327-330.
A Few Classic Unknowns in Mathematics. Sci. Monthly, 1:93-97.
Sur le théoréme de Sylow. Comptes Rendus, 160:97.
The Φ-Subgroup of a Group of Finite Order. Proc. NAS, 1:6-7.

1916

Finite Groups Represented by Special Matrices. Trans., 17:326-332.
Graphical Method of Finding the Possible Sets of Independent Generators of an Abelian Group. Bull., 23:14-17.
Orders of Operators of Congruence Groups Modulo 2^r3^s. Messenger, 46:101-103.
Upper Limit of the Degree of Transitivity of a Substitution Group. Proc. NAS, 2:61-62.
Automorphisms of Period Two of an Abelian Group Whose Order Is a Power of Two. Tôhoku J., 10:118-127.
Reforms in the Teaching of Mathematics. School Sci., 16:35-39.
Fundamental Conceptions of Modern Mathematics, Variable and Quantities, with a Discussion of the General Concept of Functional Relation. Book notice. Science, 44:173.
Sylvester and Cayley. Science, 44:173.
Historical Introduction to Mathematical Literature. Macmillan. xiii + 302 pp.
With H. F. Blichfeldt and L. E. Dickson. Theory and Applications of Finite Groups. Part I, 192 pp. John Wiley & Sons.

1917

Automorphisms of Period Two of an Abelian Group Whose Order Is a Power of Two. Tôhoku J., 12:13-16.
Substitution Groups and Possible Arrangements of the Players at Card Tournaments. Annals, 19:44-48.
Possible Characteristic Operators of a Group. Amer. J., 39:404-406.
Groups Generated by Two Operators of the Same Prime Order Such That the Conjugates of One under the Powers of the Other Are Commutative. Bull., 23:283-287.
Exposition of Groups of a Galois field. Tôhoku J., 11:200-204.
The Function of Mathematics in Scientific Research. Science. 45:549-558.
Mathematics in the *New International Encyclopedia*. Monthly, 24:106-109.
The Obsolete in Mathematics. Monthly, 24:453-456.

Modern Developments in Elementary and Secondary Mathematics. School Sci., 17:32.
Science and Learning in France, with a Survey of Opportunities for American Students in French Universities. Book notice. School Sci., 17:652
The Use of the Radical Symbol. Math. Teacher, 9:154.
School Mathematics and War. School and Sociey, 4:588.
History and Use of Mathematical Text-Books. School and Society, 4:918.
War Decalogue of Italian Teachers. School and Society, 6:383.
Education for Those Who Become Failures. School and Society, 6:712-713.

1918

Mathematical Encyclopedic Dictionary. Monthly, 25:383-387.
Group-Theory Proof of Two Elementary Theorems in Number Theory. Tôhoku J., 13:314-315.
Sets of Independent Generators of a Substitution Group. Trans., 19: 299-304.
Substitution Groups on the Terms of Symmetric Polynomials. Quart. J., 48:147-150.
Groups Formed by Special Matrices. Bull., 24:203-206.
On the α-Holomorphisms of a Group. Proc. NAS, 4:293-294.
Determinant Groups. Bull., 25:69-75.
Principes de geométrie analytique, par Gaston Darboux. Book notice. Science, 47:393.
Definition of the Descriminant of a Rational Integral Function. Monthly, 25:287-290.
A Queer Mistake. School Sci., 18:305.
The Election of Officers by Scientific Societies. Science, 47:191-192.
The Sigma Xi Quarterly. Sigma Xi Quart., 5:117.
An Investigation of Certain Abilities Fundamental to the Study of Geometry, by J. H. Minnick. Book notice. School and Society, 7:683.
Experimental Tests of Mathematical Abilities and Their Prognostic Value, by Agnes L. Rogers. Book notice. School and Society, 8:116.
Mathematics. Year-Book, 9:615-617.
Some Group Theory. School Sci., 18:675.
Scientific Activity and the War. Science, 48:117-118.
Twenty-five Important Topics in the History of Mathematics. Science, 48:182-184.
International Organization. Science, 48:649-650.

Means for the Scientific Development of Mathematics Teachers. Science, 48:553-560.

1919

Sur un point d'histoire des groupes finis discontinus. Bull. des Sciences mathématiques, 43:17.

Groups Containing a Relatively Large Number of Operators of Order Two. Bull., 25:408-413

Form of the Number of Subgroups of Prime Power Groups. Bull., 26:66-72.

Groups Generated by Two Operators of Order Three Whose Product Is of Order Four. Bull., 25:361-369.

Groups Possessing a Small Number of Sets of Conjugate Operators. Trans., 20:260-270.

Groups Generated by Two Operators Whose Relative Transforms Are Equal to Each Other. Amer. J., 41:1-4.

Relations between Abstract Group Properties and Substitution Groups. Annals, 20:229-231.

Bits of History about Two Common Mathematical Terms. Monthly, 26: 290-291.

La teoria de los grupos en la enciclopedia matematica. Revista Mat., 1:229-232.

Cajori's *History of Mathematics*. Bull., 26:79-85.

A History of Mathematics, by Florian Cajori. Book notice. School Sci., 19:768.

Historical Notes in the Mathematics Text-Books. School Sci., 19:414-416.

The Historical Point of View in Teaching Science. Science, 50:489-493.

Apropos of the Proposed Historical Science Section. Science, 49:447.

Marginal Notes on Cajori's *History of Mathematics*. School Sci., 19: 830-834.

Notas sobre el estado de la teoria de los grupos. Revista Mat., 1:148-152.

Dos obros recentes sobre historia de la matematica. Revista Mat., 1:325-326.

A Queer Mathematical Conjecture. School and Society, 10:773.

Professor Ludwig Sylow. Science, 49:85.

Common Numerals. Science, 49:215.

Note on a Singular Historical Error. Math. Gaz., 9:247.

Mathematics. Year-Book, 10:607-608.

1920

Groups Generated by Two Operators, s_1, s_2, Which Satisfy the Conditions $s^m{}_1 = s^n{}_2$, $(s_1s_2)^k = 1$, $s_1s_2 = s_2s_1$. Proc. NAS, 6:70-73.

Groups of Order $2^{m'}$ Which Contain a Relatively Large Number of Operators of Order Two. Amer. J., 42:1-10.
Characteristic Subgroups of an Abelian Prime Power Group. Amer. J., 42:278-286.
Groups Involving Three and Only Three Operators Which Are Square. Proc. London, 19:51-56.
Groups of Order g Containing $g/2$—1 Involutions. Tôhoku J., 17:88-102.
Groups Involving Only Operators Whose Orders Are Divisors of Four. Tôhoku J., 17:332-338.
La notion d'equivalence dans la théorie des groups. L'enseignement, 21:251-254.
Properties of the Subgroups of an Abelian Prime Power Group Which Are Conjugate under Its Group of Isomorphisms. Trans., 21:313-320.
Note on the Transformations of the Sylow Subgroups. Messenger, 50: 149-150.
Transitive Constituents of the Group of Isomorphisms of Any Abelian Group of Order p^m. Messenger, 49:180-186.
An Odd Method for Determining the Year of Birth. School and Society, 12:106; Math. Gaz., 10:208-209.
Mathematische Zeitschrifft. Science, 52:155.
Definicion incorrecta de grupo simple. Revista Mat., 2:192.
Value of the History of Mathematical Ignorance. School Sci., 20:813-817.
Second List of Marginal Notes on Cajori's *History of Mathematics*. School Sci., 20:300-304.

1921

Different Types of Mathematical History. Isis, 4:5-12.
Group Theory Reviews in the *Jahrbuch über die Fortschritte der Mathematik*. Bull., 27:459-462.
An Overlooked Infinite System of Groups of Order pq^2. Proc. NAS, 7:146-148.
Reciprocal Subgroups of an Abelian Group. Bull., 27:266-272.
Group of Isomorphisms of a Transitive Substitution Group. Proc. NAS, 7:325-328.
Note on the Term Maximal Subgroup. Annals, 23:68-69.
Illustrative Examples of Domains of Rationality and Their Groups. Tôhoku J., 19:42-53.
Note on Prime Numbers. School Sci., 21:874.
The Formula $\frac{1}{2}a(a+1)$ for the Area of an Equilateral Triangle. Monthly, 28:256-258.

The Earliest Mathematical Work of the New World, by D. E. Smith. Book notice. Science, 53:458-459.
What Should the Society of the Sigma Xi Preach? Sigma Xi Quart., 9:64-66.
The Mathematical Association of America. School Sci., 21:418-422.
Gerbert's Letter to Adelbold. School Sci., 21:649-653.
The History of Science as an Error Breeder. Sci. Monthly, 12:439-443.
A Few Questionable Points in the History of Mathematics. Sci. Monthly, 13:232-237.
A Mathematical Recreation. Math. Gaz., 11:23-24.
Mathematics in Spanish-speaking Countries. Science, 54:154.
A Notable Mathematical Gift. Science, 54:456.
A New Definition of Pure Mathematics. Science, 54:300-301.
The Group Theory Element of the History of Mathematics. Sci. Monthly, 12:575.

1922

I-Conjugate Operators of an Abelian Group. Trans., 24:70-78.
Contradictions in the Literature of Group Theory. Monthly, 29:319-328.
Substitution Groups Whose Cycles of the Same Order Contain a Given Number of Letters. Amer. J., 44:122-128.
The Simple Group of Order 2520. Bull., 28:98-102.
Substitutions Commutative with Every Substitution of an Intransitive Group. Bull., 28:168-170.
Number of Substitutions Omitting at Least One Letter in a Transitive Group. Proc. NAS, 8:238-240.
Disagreeing with the Text-Book. School and Society, 16:449-454.
A Few Very Popular Mathematics Teachers. School and Society, 15:13-15.
Easy Group Theory. Sci. Monthly, 15:512-519.
Seeming Contradictions in the Theory of Groups. Bull., 28:156.
Comments on the Reply of Professor Cajori. Monthly, 29:304-306.
Mathematical Philosophy. A Study of Fate and Freedom, by C. J. Keyser. Monthly, 29:408-410; Science, 56:229-230.
Plane and Solid Analytic Geometry, by W. F. Osgood and W. C. Graustein. Book notice. Monthly, 29:420-421.
Tangent Lines among the Greeks. School Sci., 22:715-717.
New Mathematical Periodicals. School Sci., 22:276-280.

1923

Inaccuracies in the Mathematical Literature. Sci. Monthly, 17:216-228.

Groups in Which the Number of Operators in a Set of Conjugates Is Equal to the Order of the Commutator Subgroup. Bull., 29:64-70.

Same Left Co-Set and Right Co-Set Multipliers for Any Given Finite Group. Bull., 29:394-398.

Sets of Conjugate Cycles of a Substitution Group. Proc. NAS, 9:52-54.

Form of the Number of the Subgroups of a Prime Power Group. Proc. NAS, 9:237-238.

Groups of Order 2^m in Which the Number of the Subgroups of at Least One Order Is of the Form $1 + 4^k$. Proc. NAS, 9:326-328.

Determination of All the Characteristic Subgroups of an Abelian Group. Quart. J., 50:54-62.

New Applications of a Fundamental Theorem of Substitution Groups. Annals, 25:47-52.

Classification of the Positive Integers as Regards the Ultimate Sum of Their Digits. Math. Teacher, 16:247-248.

A Widespread Error Relating to Greek Mathematics. School and Society, 18:621-622.

Die theorie der gruppen von endlicher ordnung, by A. Speiser, Book notice. Bull., 29:372; Monthly, 30:324-326.

Mathematics, by D. E. Smith. Book notice. Science, 58:288-290.

Mathematical Propaganda. Science, 57:663.

Easy Group Theory. J. Indian, 15:91-97.

Mathematical Errors Sanctioned by Modern Usage. School Sci., 23:853-855.

The Ph.D. Degree and Honesty. School and Society, 17:553.

1924

Number of Cycles of the Same Order in Any Given Substitution Group. Bull., 30:239-246.

Prime Power Substitution Groups Whose Conjugate Cycles Are Commutative. Proc. NAS, 10:166-167.

Historical Note on the Solution of Equations. School Sci., 24:509-510.

Modifications Relating to the *New International Encyclopedia.* Science, 60:382-384.

American Mathematics During Three Quarters of a Century. Science, 59:1-7.

Five Fundamental Concepts of Pure Mathematics. Sci. Monthly, 19: 496-501.

A Widespread Error Relating to Copernicus. Science, 60:82-83.

A Mathematical Black Sheep. Science, 60: 267-268.

Several Greek Definitions of Number. School and Society, 19:410.

Mathematical Shortcomings of the Greeks. School Sci., 24:284-287.
Histoire de cinq concepts fondamenteaux des mathématiques. L'enseignement, 24:59-69.

1925

Simplifications Relating to a Proof of Sylow's Theorem. Bull., 31:253-256.
Operadores caracteristicos de un grupo de orden finito. Revista Mat., 7:280-282.
New Proofs of the Simplicity of Every Alternating Group Whose Degree Is Not Four. Annals, 27:87-90.
The Subgroup Composed of the Substitutions Which Omit a Letter of a Transitive Group. Trans., 27:137-145.
Transitive Groups Involving Direct Products of Lower Degree. Proc. NAS, 11:150-152.
Imprimitive Substitution Groups. Amer. J., 47:176-180.
Elementary Mathematics in 1700. J. Indian, 16:123-129.
Fundamental Facts in the History of Mathematics. Sci. Monthly, 21: 150-156.
Arithmetization in the History of Mathematics. Proc. NAS, 11:546-548; Science, 62:328.
"Mathematics and Eternity." School and Society, 21:299-300.
History of Science in Secondary Education. School and Society, 22:172-173.
Geometric Solution of the Quadratic Equation. Math. Gaz., 12:500-501.
The Group Concept. J. Indian, 15:269-275.
Did the Greeks Solve the Quadratic Equation? J. Indian, 15:153-155.
History of Mathematics, by D. E. Smith. Review. Sci. Prog., 20:340-342.
Mathematics. Year-Book: pp. 779-783.
Two Recent Histories of Elementary Mathematics. Science, 61:491-492.

1926

Subgroups of Index p^2 Contained in a Group of Order p^m. Amer. J., 48:253-256.
Groups Containing a Relatively Small Number of Sylow Subgroups. Proc. NAS, 12:269-273.
Form of the Number of the Prime Power Subgroups of an Abelian Group. Proc. NAS, 12:470-472.
The Transformation of a Regular Group into Its Conjoint. Bull., 32: 631-634.
Lettera al prof. Luigi Bianchi. Annali, 3:239.
Multiply Transitive Substitution Groups. Trans., 28:339-345.
Postulates in the History of Science. Proc. NAS, 12:537-540.

Weak Points in Greek Mathematics. Scientia, 39:317-322.
Historical Note on the Solution of an Algebraic Equation. Tôhoku J., 26:386-390.
John Napier and the Invention of Logarithms. Sci. Prog., 20:307-310.
Der Gegenstand der Mathematik im Licht ihrer Entwicklung, by H. Wieleitner. Book notice. Bull., 32:397.
Synthetische Zahlentheorie, by Rudolf Fueter. Book notice. Bull., 32: 400-401.
Professor Charles Émile Picard. Sci. Monthly, 22:464.
Was Paul Guldin a Plagiarist? Science, 64:204-205.
A Widespread Error Relating to Logarithms. Science, 64:279.
On the History of Logarithms. J. Indian, 16:209-213.
Relative Abilities in Mathematics of Boys and Girls. School and Society, 24:458.
When Certain Mathematical Formulas Became True. Sci. Prog., 20: 436-438.
Biographical Note Relating to J. J. Sylvester. Science, 64:576-577.
The So-Called Naperian Logarithms. Bull., 32:585-586.

1927

Groups Generated by Two Operators of Order Three Whose Product Is of Order Three. Proc. NAS, 13:24-26.
Groups Generated by Two Operators of Order Three Whose Product Is of Order Six. Proc. NAS, 13:170-174.
Groups Generated by Two Operators of Order Three, the Cube of Whose Product Is Invariant. Proc. NAS, 13:708-710.
Groups Whose Operators Are of the Form $s^p t^q$. Proc. NAS, 13:758-759.
On the History of Finite Abstract Groups. J. Indian, 17:97-102.
Determination of the Number of Subgroups of an Abelian Group. Bull., 33:192-194.
Substitutions Which Transform a Regular Group into Its Conjoint. Bull. 33:701-706.
Felix Klein and the History of Modern Mathematics. Proc. NAS, 13:611-613.
Vorlesungen über die entwicklung der Mathematik im 19 *Jahrhundert,* by Felix Klein. Book notice. Science, 65:574-575.
Popular Elements of the Theory of Groups. Sci. Prog., 22:225-230.
Controversial Mathematics. Sci. Prog., 26:423-424.

1928

Number of Systems of Imprimitivity of Transitive Substitution Groups. Proc. NAS, 14:82-84.
Transformation of Conjugate Elements or of Conjugate Subgroups. Proc. NAS, 14:518-520.
Determination of All the Groups Which Contain a Given Group as an Invariant Subgroup of Prime Index. Proc. NAS, 14:819-822.
Groups Involving a Cyclic, a Dicyclic, or a Dihedral Group as an Invariant Subgroup of Prime Index. Proc. NAS, 14:918-921.
An Infinite System of Complete Groups. Tôhoku J., 29:231-235.
Commutative Conjugate Cycles in Subgroups of the Holomorph of an Abelian Group. Proc. Toronto, 1:365-371.
Definitions of Abstract Groups. Annals, 29:223-228.
Groups of Plane Symmetries. J. Indian, 17:208-212.
Possible Orders of Two Generators of the Alternating and of the Symmetric Group. Trans., 30:24-32.
A Characteristic Property of a Co-Set. Bull., 34:307-309.
Origin of Our Present Mathematics. Sci. Monthly, 26:295-298.
Harmony as a Principle of Mathematical Development. Proc. NAS, 14:214-217.
The Algebraization of Mathematics. School and Society, 28:363-364.
History of Intangible Advances in Mathematics. Scientia, 44:81-88.
The So-called Sieve of Eratosthenes. Science, 68:273-274.
Archimedes and Trigonometry. Science, 67:555.
Laws Relating to Mathematical Operations. Science, 67:104.
Note on the History of Logarithms. Tôhoku J., 29:308-311.
The Development of the Function Concept. School Sci., 28:506-516.
The Development of the Group for Expressing Functionality. School Sci., 28:829-834.

1929

Possible α-Automorphisms of Non-Abelian Groups. Proc. NAS, 15:89-91.
Groups Which Admit Three-fourths Automorphisms. Proc. NAS, 15:369-372.
Automorphism Commutators. Proc. NAS, 15:672-675.
On the Number of Cyclic Subgroups of a Group. Proc. NAS, 15:728-731.
Non-Abelian Groups of Odd Prime Power Order Which Admit a Maximal Number of Inverse Correspondencies in an Automorphism. Proc. NAS, 15:859-862.

Groups Which Admit Automorphisms in Which Exactly Three-fourths of the Operators Correspond to Their Inverses. Bull., 35:559-564.
Number of Abelian Subgroups in Every Prime Power Group. Amer. J., 51:31-34.
Determination of All the Abstract Groups of Order 72. Amer. J., 51:491-494.
Group Theory and Applied Mathematics. Science, 69:217.
History of Algebra. School Sci., 29:404-410.
History of Several Fundamental Mathematical Concepts. Proc. Toronto, 2:959-967.
Mathematical Deserts and Some of Their Oases. Sci. Monthly, 29:407-410.
Origin of Our Common Positional Arithmetic. School and Society, 30:431-432.
The So-called Hindu-Arabic Numerals. School and Society, 29:390-391.
Graphical Methods and the History of Mathematics. Tôhoku J., 31:292-295.
On the History of Our Common System of Numerical Notation. J. Indian, 18:121-125.
Large Numbers Used before the Christian Era. Science, 70:282.
Mathematics and the Truth. Science, 70:608-609.
On the History of Mathematical Ideas. School Sci., 29:954-960.
Did John Napier Invent Logarithms? Science, 70:97-98.

1930

Marginal Notes on Volume II of Smith's *History of Mathematics*. J. Indian, 18:265-270. (Review.)
Mathematics in the Fourteenth Edition of the *Encyclopedia Britannica*. J. Indian, 18:169-175. (Review.)
Early Definitions of the Mathematical Term *Abstract Group*. Science, 72:168-169.
Babylonian Mathematics. Science, 72:601-602.
Determination of All the Groups of Order 64. Amer. J., 52:617-634.
Determination of All the Groups of Order 96. Annals, 31:163-168.
Inverse Correspondences in Automorphisms of Abelian Groups. Bull., 36:277-281.
Groups Which Admit Two-thirds Automorphisms. Proc. NAS, 16:86-88.
Non-Abelian Groups Admitting More than Half Inverse Correspondences. Proc. NAS, 16:168-172.
Groups Generated by Two Given Groups. Proc. NAS, 16:305-308.
Groups of Isomorphisms of Abelian Groups. Proc. NAS, 16:398-401.

Groups Which Are Decomposable into Two Non-invariant Cyclic Groups. Proc. NAS, 16:527-530.
Extension of the Concept of Group of Isomorphisms. Monthly, 37:482-484.
On the History of Determinants. Monthly, 37:216-219.
Introduction a la théorie des congruences au moyen de la théorie des groupes. L'enseignement, 29:7-17.
Inverting the Denominator of a Fraction. School Sci., 30:881-883.
Finality in Mathematics. Sci. Monthly, 31:531-534.
Other Side of Mathematical Statements in the New Edition of the *Britannica*. School Sci., 30:295-300.

1931

Groups Which Admit Five-eighths Automorphisms. Proc. NAS, 17:39-43,
Inverse Commutator Subgroups. Proc. NAS, 17:144-147.
Groups Generated by Two Operators Whose Squares Are Invariant. Bull., 37:270-275.
A Few Theorems Relating to the Rhind Mathematical Papyrus. Monthly, 38:194-197.
Automorphisms of Order 2 of an Abelian Group. Proc. NAS, 17:320-325.
Theorems Relating to Pre-Grecian Mathematics. Monthly, 38:496-500.
Groups Involving a Small Number of Conjugates. Proc. NAS, 17:691-694.
Representation of a Group as a Transitive Permutation Group. Bull., 37:857-860.
Theorems Relating to the History of Mathematics. Proc. NAS, 17:463-466.
The Mathematical Weakness of the Early Civilizations. Sci. Monthly, 33:419-423.
Twelve Fundamental Mathematical Concepts. Tôhoku J., 34:230-235.
A Mathematical Proof. Science, 73:476.
A Mathematical Review. Science, 74: 366-367.
Elementary Theory of Finite Groups, by L. C. Mathewson. Book notice. Sci. Prog., 25:529.
Number, the Language of Science, by Tobias Dantzig. Book notice. Bull., 37:9.
Useful Mathematics. School and Society, 41:646.
On the History of Common Fractions. School Sci., 31:138-145.
Marginal Notes on Sanford's *History of Mathematics.* J. Indian, 19:81-85.

1932

Complete Sets of Conjugates under a Group. Amer. J., 54:110-116.
Group Theory and the History of Logarithms. J. Indian, 19:169-172.

A Few Desirable Modifications in the Literature of Group Theory. Annals, 33:319-323.

Orders for Which a Given Number of Groups Exist. Proc. NAS, 18:472-475.

Non-Group Operations. Proc. NAS, 18:597-600.

The Commutator Subgroup of a Group Generated by Two Operators. Proc. NAS, 18:665-668.

Orders for Which There Exist Exactly Four or Five Groups. Proc. NAS, 18:511-514.

Sets of Distinct Group Operators Involving All the Products but Not All the Squares. Proc. NAS, 18:100-102.

Babylonian and Egyptian Mathematics. Sci. Prog., 26:270-271.

Definitions of a Mathematical Group. Science, 75:102-103.

Meaning of the Word Billion. School and Society, 33:591.

A New Mathematical Encyclopedia. School and Society, 34: 756-758.

Enciclopedie delle matematiche elementari. Review. Bull., 38:157-158; Monthly, 39:168-171; School Sci., 32:226-228.

A Few Uncertainties in the History of Elementary Mathematics. Rev. Bull., 38:838-844.

Recent Discoveries in the History of Mathematics. J. Indian, 19:225-231.

Groups of a Given Order. J. Indian, 19:205-210.

1933

Groups Whose Orders Involve a Small Number of Unity Congruences. Amer. J., 55:22-28.

Groups Involving Only Operators Whose Orders Divide 4 and Whose Operators of Order 4 Have a Common Square. Amer. J., 55:417-420.

Sylow Subgroups and the Number of Operators Whose Orders Are Powers of the Same Prime. Proc. NAS, 19:555-558.

Number of Operators of Prime Power Orders Contained in a Group. Proc. NAS, 19:780-783.

Groups Whose Operators Have No More than Three Distinct Squares. Proc. NAS, 19:848-851.

Groups in Which Every Operator Has at Most a Prime Number of Conjugates. Trans., 35:897-902.

Widespread Errors Relating to Laplace. Science, 78:531-532.

Groups Generated by Two Operators of Orders 2 and 4 Respectively Whose Commutator Is of Order 2. Tôhoku J., 38:1-6.

A Crusade against the Use of Negative Numbers. School Sci., 33:959-964.

Historical Notes on Felix Klein's *Elementary Mathematics.* Math. Student, 1:49-51.
Notas historicas de Felix Klein sobre matematica elemental. Revista Mat., 8:184-187.
Historical Note on Negative Numbers. Monthly, 40:4-5.
Geschichte der Elementar-Mathematik, von Johnannes Tropfke. School Sci., 33:689.
Characteristic Features of Mathematics and of Its History. Sci. Monthly, 37:398-404.
Mathematical Confusions. School and Society, 37:494-495.
A New Aid for Mathematics Teachers. School and Society, 38:120-121.
Groups Generated by Two Operators of Order 3 Whose Commutator Is of Order 2. Proc. NAS, 19:199-202.
Groups in Which either All the Operators or All the Subgroups of the Same Order Are Conjugate. Proc. NAS, 19:332-335.

1934

A Mathematical Solution Developing during More than Three Milleniums. (Discussion.) School and Society, 39:211-212.
Minimum Number of Squares in a Group When Not All of Them Are Relatively Commutative. Proc. NAS., 20:127-130.
Definitions of the Mathematical Term Group. Science, 79:291-292.
Groups Involving Four Operators Which Are Squares. Amer. J., 56:47-52.
Historical Note on the Determination of All the Permutation Groups. Tôhoku J., 39:60-65.
Groups Generated by an Operator of Order 2 and an Operator of Order 3 Whose Commutator Is of Order 3. J. Indian, 20:145-147.
Confusions in the Use of the Mathematical Term Group. Proc. NAS, 20:288-291.
Groups Involving Three and Only Three Elements Which Are Squares. Proc. NAS, 20:243-246.
Groups or Order 2^n Whose Squared Elements Constitute a Cyclic Subgroup. Proc. NAS, 20:372-375.
Groups Whose Sylow Subgroups of Order p^m Contain No More than p^{m-2} Operators Whose Orders Are Powers of p. Quart. J., 5:23-29.
Vague Historical Views Relating to Negative Numbers. Math. Student, 2:1-6.
Distinct Groups Whose Subgroups Are Simply Isomorphic. Proc. NAS, 20:430-433.
History of Mathematics in America. School and Society, 40:96.

Background of Mathematics in America. Science, 80:356-357.
Evolution of a Modern Definition of the Mathematical Term Group. Sci. Monthly, 39:554-559.
Groups in Which the Squares of the Elements Are a Dihedral Subgroup. Trans., 36:319-325.
Mathematics in *Webster's New International Dictionary,* Second Edition, 1935. School and Society, 40:728.
Definitions of Mathematical Terms in General English Dictionaries. Science, 82:248-249.

1935

A Widespread Error Relating to Egyptian Mathematics. Science, 81:152.
Groups Involving a Set of as Many Conjugates as Commutators. Proc. NAS, 21:41-44.
Sets of Group Elements Involving Only Products of More than n. Proc. NAS, 21:45-47.
Groups Which Are the Products of Two Permutable Groups. Proc. NAS, 21:469-472.
Largest Groups Determined by the Squares of Their Operators. Proc. NAS, 21:479-481.
Formulas Giving the Number of the Groups Determined by Squares. Proc. NAS, 21:671-674.
History of Mathematics in America. School Sci., 35:292-296.
Motifs pour l'introduction des nombres negatifs. L'enseignement, 33:221-226.
Differences between Arithmetic and Algebra. Science, 81:513-514.
The Backwardness of Early American Mathematics. Math. Student, 3:8-11.
Groups Containing Five and Only Five Squares. Amer. J., 57:615-622.
A Wide-spread Error Relating to the Pythagoreans. Science, 82:126.
Evolution of the Use of the Modern Mathematical Concept of Group. Sci. Monthly, 41:228-233.
A Characteristic Feature of Babylonian Mathematics. School and Society, 42:436-439.
Mathematics and the Question of the Cosmic Mind, and Other Essays, by C. J. Keyser. Review. Math. Mag., 10:68-69.
Correcting Errors in the Histories of Mathematics. School Sci., 35:977-983.
On the History of Negative Numbers. Science, 82:517.
Historical Note on the Determination of All the Permutation Groups of Low Degrees. Collected Works, 1:1-9.

Historical Note on the Determination of Abstract Groups of a Given Order. Collected Works, 1:91-98.

History of the Theory of Groups to 1900. Collected Works, 1:427-467.

1936

General Theorems Applying to All Groups of Order 32. Proc. NAS, 22:112-115.

Groups in Which the Squares of the Operators Generate a Cyclic Group. Proc. NAS, 22:288-291.

Regular Subgroups of a Transitive Substitution Group. Proc. NAS, 22:375-377.

Groups Containing a Relatively Large Number of Operators of the Same Order. Proc. NAS, 22:541-543.

Number of the Abelian Groups of a Given Order. Proc. NAS, 22:654-655.

Enumeration of the Abelian Groups Whose Order Does Not Exceed a Given Number. Proc. NAS, 22:700-703.

Groups Whose Prime Powers Generate a Cyclic Subgroup. Proc. NAS, 22:703-706.

Basal Facts in the History of Mathematics. Sci. Monthly, 42:230-235.

Recent German Mathematics. Science, 83:230.

Group Theory Definitions in *Webster's New International Dictionary*. Math. Gaz., 20:143-144.

Fundamental Ideas in the History of Mathematics. Math. Student, 3:121-126.

Group Theory in the Secondary Schools. School and Society, 44:344-345.

A Dozen Mathematical Errors in *Webster's Dictionary*. Science, 84:418-419.

Factor Congruences in the History of Mathematics. Tôhoku J., 42:362-365.

Erreurs des mathématiciens. Scripta, 4:270-272. (Review.)

1937

Groups Which Contain an Abelian Subgroup of Prime Index. Proc. NAS, 23:13-16.

The Groups of Order p^m Which Have $m - 1$ Independent Generators. Proc. NAS, 23:234-236.

Groups of Order Less than 2^m Having $m - 1$ or $m - 2$ Independent Generators. Proc. NAS, 23:280-285.

Sets of Independent Generators of a Finite Group. Proc. NAS, 23:494-496.

Groups in Which Every Subgroup of Composite Order Is Invariant. Proc. NAS, 23:549-552.

Groups Having a Maximum Number of Independent Generators. Proc. NAS, 23:333-337.

Groups Which Contain a Hamiltonian Group of Odd Prime Index. Proc. NAS, 23:587-589.

Groups in Which Every Set of Independent Generators Is a Maximum Set. Proc. NAS, 23:446-448.

Group Theory in Mathematical Education. School and Society, 45:183-185.

Basal Facts in the History of Mathematics. Scientia, 31:66. (Resumé.)

False Group Theory. Sci. Monthly, 44:280-283.

The Quadratic Equation. School and Society, 45:684-685.

The Handmaiden of Science, by E. T. Bell. Review. Math. Mag., 12:102-103.

Group Theory for the Million. Sci. Monthly, 45:562-565.

Groups of Order 64 Whose Squares Generate the Four Group. Amer. J., 59:57-66.

1938

Groups Having a Maximum Number of Independent Generators. Proc. NAS, 24:17-20.

Groups Having a Maximum Set of Independent Generators of the Same Order. Proc. NAS, 24:91-94.

Groups Having a Maximum Number Set of Conjugate Independent Generators. Proc. NAS, 24:94-97.

Relative Numbers of Operators and Subgroups of a Finite Group. Proc. NAS, 24:199-201.

Minimum Degree of Substitutions of Highest Degree in a Group. Proc. NAS, 24:202-204.

Largest Degree of a Substitution in a Group of a Given Degree. Proc. NAS, 24:293-296.

Groups of Degree n Containing Only Substitutions of Lower Degrees. Proc. NAS, 24:333-336.

The Relative Numbers of the Subgroups and Operators of Certain Groups. Proc. NAS, 24:473-476.

Groups of Order p^m Containing $m - \alpha$ Independent Generators. Proc. NAS, 24:561-565.

Groups Whose Commutator Subgroups Are of Order Two. Amer. J., 60:101-106.

Implications in the History of Mathematics. School and Society, 47:275-277.

Mathematical *pia fraus*. School and Society, 47:637-638.

Mathematical Myths. Math. Mag., 12:388-392.

The First Known Long Mathematical Decline. Science, 87:576-577.

Group Theory in the History of Mathematics. Sci. Monthly, 47:24-27.

Unpardonable Errors in the History of Mathematics. School and Society, 48:78-79.

The History of Mathematics Forty Years Ago. Science, 88:375-376.

Kleinsche Vierergruppe. Math. Gaz., 22:486; also Bolletino di matematica, Anno 1, (1930):68.

Progress in the History of Mathematics during the Last Forty Years. Math. Student, 6:71-73.

Note on the History of Group Theory during the Period Covered by This Volume. Collected Works, 2:1-18.

Primary Facts in the History of Mathematics. Collected Works, 2:493-528.

1939

Groups of Degree *n* in Which the Largest Degree of a Substitution Is a Minimum. Proc. NAS, 25:43-46.

Number of the Subgroups of Any Given Abelian Group. Proc. NAS, 25:3-262.

Independent Generators of the Subgroups of an Abelian Group. Proc. NAS, 25:364-367.

Groups Having a Small Number of Subgroups. Proc. NAS, 25:367-371.

Groups Containing a Prime Number of Non-invariant Subgroups. Proc. NAS, 25:431-434.

Groups Which Contain Less than Ten Proper Subgroups. Proc. NAS, 25:482-485.

Groups Which Contain Ten or Eleven Proper Subgroups. Proc. NAS, 25:540-543.

Prime Power Groups Determined by the Number of Their Subgroups. Proc. NAS, 25:583-586.

Groups Having a Small Number of Sets of Conjugate Subgroups. Proc. NAS, 25:640-643.

Solution of Equations by the Ancients. School and Society, 49:178-179.

Forty Years of Mathematics. Sci. Monthly, 48:268-271.

A First Lesson in the History of Mathematics. Math. Mag., 13:272-277.

Primary Facts in the History of Mathematics. Math. Teacher, 32:209-211.

Approximations in Mathematics Regarded as Correct. Math. Student, 6:137-142.

The First Articles on Group Theory Published in America. Science, 90:234.

A Dozen Mathematical Errors in the *Encyclopedia Britannica*. Science, 90:512-513.

A Second Lesson in the History of Mathematics. Math. Mag., 14:144-152.

1940

Changes in Modern Mathematics. Science, 91:289-290.

Fundamental Laws of Operations in Mathematics. Science, 91:571-572.

Groups Which Contain Less than Fourteen Proper Subgroups. Proc. NAS, 26:129-132.

The Groups Which Contain Exactly Fourteen Subgroups. Proc. NAS, 26:283-286.

Abelian Groups Which Contain No More than 25 Proper Subgroups. Proc. NAS, 26:350-354.

Subgroups of the Groups Whose Orders Are Below Thirty. Proc. NAS, 26:500-502.

Minimal Cross-cut Subgroups Relative to the Product of Their Orders. Proc. NAS, 26:621-625.

Every Two Equal Order Subgroups Having Only the Identity in Common. Proc. NAS, 26:652-655.

The First Thousand Mathematical Works Printed in America. Science, 92:216-217.

Multisensual Mathematical Terms. Science, 92:40-401.

Enumerations of Finite Groups. Math. Student, 8:109-111.

1941

Maximal Subgroups of a Given Group. Proc. NAS, 27:68-71.

Maximal Subgroups of a Finite Group. Proc. NAS, 27:212-216.

Groups Containing Maximal Subgroups of Prime Order. Proc. NAS, 27:342-345.

Maximal Subgroups Whose Orders Are Divisible by Two or Three. Proc. NAS, 27:399-402.

Groups Containing a Maximal Proper Subgroup of Order 4. Proc. NAS, 27:445-448.

Maximal Invariant Proper Subgroups of a Finite Group. Proc. NAS, 27:587-590.

Arithmetic and Algebra. School and Society, 53:84.

A Third Lesson in the History of Mathematics. Math. Mag., 15:68-71.
General or Special in the Development of Mathematics. Science, 93:235-236.
Pre-Euclidean Greek Mathematics. Science, 94:89-90.
The First Mathematics Section of the National Academy of Sciences. Science, 94:345-346.

1942

Nature of Group Theory. Science, 95:96.
A Group Theory Dilemma of Sophus Lie and Felix Klein. Science, 95:353-354.
Maximal Sylow Subgroups of a Given Group. Proc. NAS, 28:80-83.
Certain Direct Products of the Groups of Self-Isometries. Proc. NAS, 28:141-144.
Some Deductions from Frobenius' Theorem. Proc. NAS, 28:251-254.
A Second American Mathematics Dictionary. School and Society, 56:19-20.
Automorphisms of the Dihedral Group. Proc. NAS, 28:368-371.
The Permutation Groups of a General Degree. Proc. NAS, 28:407-410.
A Fourth Lesson in the History of Mathematics. Math. Mag., 17:13-20.
Letter to the Editor. Math. Mag., 17:72.

1943

Early Mastery of the Group Concept. Science, 97:90-91.
Implications Involved in Mathematical Advances. Science, 97:38-39.
Determination of the Subgroups of Small Index. Proc. NAS, 29:25-28.
Possible Groups of Automorphisms. Proc. NAS, 29:29-32.
Groups Containing a Prime Number of Conjugate Subgroups. Proc. NAS, 29:104-107.
Groups Containing Four and Only Four Non-invariant Subgroups. Proc. NAS, 29:213-215.
Groups of Transformations of the Non-invariant Subgroups. Proc. NAS, 29:240-242.
Special Invariant Subgroups. Proc. NAS, 29:308-311.
Subgroups Transformed According to a Group of Prime Order. Proc. NAS, 29:311-314.
A Fifth Lesson in the History of Mathematics. Math. Mag., 17:212-220.
A Sixth Lesson in the History of Mathematics. Math. Mag., 17:341-350.
A Seventh Lesson in the History of Mathematics. Math. Mag., 17:67-76.

1944

Relative Number of Non-invariant Operators in a Group. Proc. NAS, 30:25-28.

Possible Number of Non-invariant Operators of a Group. Proc. NAS, 30:114-117.

Groups Containing Less than Twenty-eight Non-invariant Operators. Proc. NAS, 30:275-279.

Groups Containing a Small Number of Sets of Conjugate Operators. Proc. NAS, 30:359-362.

Starring Subjects in *American Men of Science*. Science, 99:386.

Solution of the Cubic Equation. Science, 100:333-334.

An Eighth Lesson in the History of Mathematics. Math. Mag., 18:261-270.

A Ninth Lesson in the History of Mathematics. Math. Mag., 19:64-72.

Webster's Biographical Dictionary. Review. Math. Mag., 19:105-106.

1945

Groups Having a Small Number of Sets of Conjugate Subgroups. Proc. NAS, 31:147-150.

Fanciful Portraits of Ancient Mathematicians. Science, 101:223-224.

A Tenth Lesson in the History of Mathematics. Math. Mag., 19:288-293.

1946

Prime Number of Conjugate Operators in a Group. Proc. NAS, 32:53-56.

Prime Number of Operators in Sets of Conjugates. Proc. NAS, 32:149-152.

1947

Comments by Readers. Science, 105:232.

An Eleventh Lesson in the History of Mathematics. Math. Mag., 21:48-55.