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# BIOGRAPHICAL MEMOIR

OF

# BENJAMIN OSGOOD PEIRCE

## 1854-1914

BY

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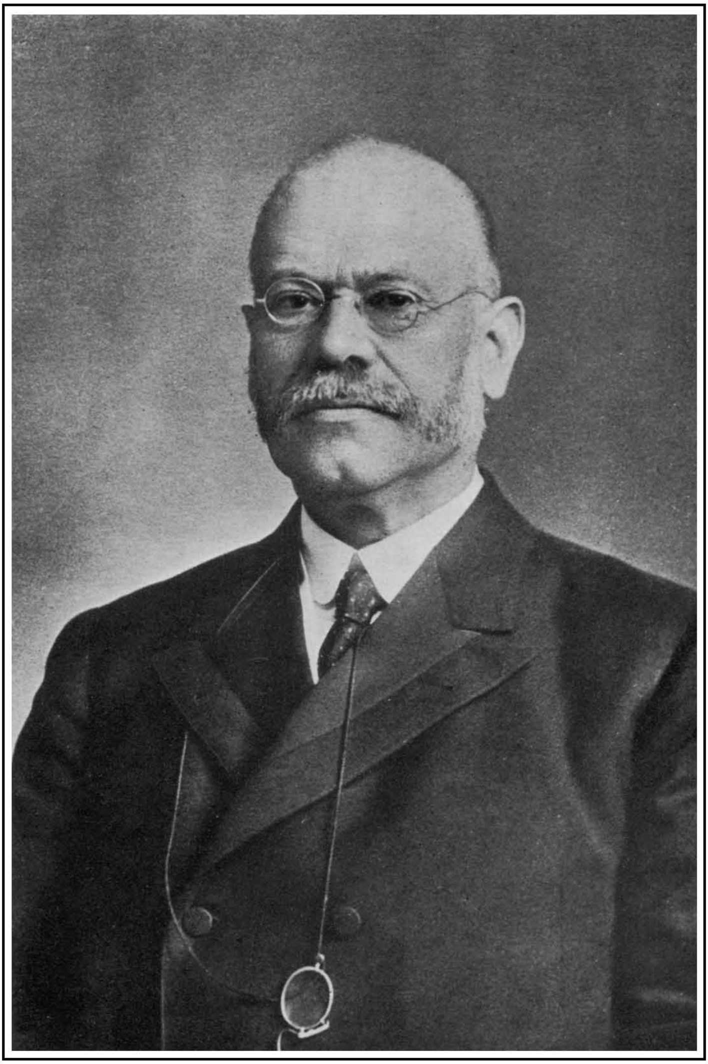

B. O. Peirce

# BIOGRAPHY OF BENJAMIN OSGOOD PEIRCE

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Our colleague, Benjamin Osgood Peirce, who died in Cambridge on the 14th of January, 1914, was born in Beverly, Massachusetts, February 11, 1854. Of his ancestors, Richard Norman came to Gloucester in 1623, John Peirce to Watertown in 1637, John and Christopher Osgood to other parts of eastern Massachusetts before 1640. John Peirce had a son Robert, but after the Cromwellian era first names taken from the Old Testament prevail in the family, and it is hard to refrain from using the robust terms of the Old Testament genealogies in reciting the generations that follow. The son of Robert was Benjamin, and the son of Benjamin was Jerathmiel, and the son of Jerathmiel was Benjamin 2d, who fell at Lexington, and his son was Benjamin 3d, whose son was Benjamin Osgood 1st, the father of our friend.

From Jerathmiel were descended also Jerathmiel 2d, and his son Benjamin, Librarian of Harvard College from 1826 to 1831, and his son Benjamin, Tutor or Professor of Mathematics at Harvard from 1831 to 1880, among whose sons were James Mills, also for many years Professor of Mathematics at Harvard, and Charles Sanders, a brilliant mathematician and projector of the philosophic cult of Pragmatism. Without a break, save perhaps for a few months in 1831, some one of the descendants of Jerathmiel Peirce was in the service of Harvard College from 1826 to 1914. Three of his descendants have been members of the National Academy. In the annals of intellectual achievement in America there is no greater name than Peirce.

Perhaps, too, the name Jerathmiel had a certain potency. It sounds like a whole thunder-peal, the threatening rumble, the climactic crash, the soft and reassuring diminuendo. This name is derived or recreated from the lifeless Jerahmeel of the Old Testament Chronicles by the simple but miraculous change of inserting a *t* and putting an *i* for an *e*. I have wished to claim this transformation as a New England achievement, directly inspired, and have refrained from verifying a suggestion that the greater word merely conforms to the Hebrew pronunciation of the ancient name.

There were contrasts of fortune among the descendants of Jerathmiel Peirce. One of his sons, Jerathmiel 2d, became wealthy as a merchant and built the famous Peirce-Nichols mansion, which stands today as one of the architectural treasures of Salem. The other, a baker in business, killed in the Concord-Lexington fight of April 19, 1775, was followed by a posthumous son who fared ill at the hands of a stepfather, was bound out to a hard master, and so badly treated that he at last ran away to shift for himself.

But the boy came of a sturdy and generous breed. We may be sure that it was no ordinary man who, at the age of thirty-seven, far older[1] no doubt than the great majority of those in arms, hurried across the country from distant Salem to meet his death in that first clash of arms in the Revolution, the only citizen of his town to fall at the hands of the British on that day. So the runaway apprentice, sustained perhaps by pride in the heroic father whose face he had never seen, having at any rate the same blood in his veins, was soon able to make his own way, neither broken by misfortune nor embittered by injustice. At twenty-seven years of age he had built and owned a large house in Beverly, the house in which his son and his grandson, our colleague, and this colleague's first daughter, Jessie, were born. A little later he had mortgaged this house to promote the building of a Baptist meeting-house; for a Baptist he was, and the Baptist tradition remained in the family.

He married well. His wife, Rebecca Orne, a woman of great piety and dignity, counted among her New England an-

[1] Thus in the Danvers Company, which he may have accompanied, though not a member of it, the oldest man killed this day was 33, the ages of the others who fell ranging from 25 to 21. In fact, Peirce was probably too old to be naturally enrolled among the Minute Men. His name is not to be found in the official register of Massachusetts soldiers and sailors of the Revolutionary War. He seems to have been a *franc-tireur*.

cestors a considerable number of clergymen and magistrates, men of note and influence in their time and place. Of these was John Osgood, one of the founders of the first church in Andover, Mass., and the name Osgood, thus brought into the Peirce family, has continued in active use there in three generations. Our colleague, his father, his mother also, as it happens, and his two daughters have all borne it as a middle name. It is small wonder that in England, at the scientific meetings of 1912, the family came to be known as the Osgood-Peirces.

The first Benjamin Osgood Peirce, born in 1812, was in due course of time sent to the Baptist College at Waterville, Me., where he graduated in 1835. He married, in 1841, Miss Mehetable Osgood Seccomb, a native of Salem. She was a lady of excellent family connections and traditions, with one or two reputed witches, of the eminently respectable Salem variety, flitting somewhere in the background of her ancestry. Though born in adjoining towns, Mr. Peirce and Miss Seccomb met for the first time in Georgia, where both were engaged in teaching.

In the case of a man so remarkable as our late colleague it would be interesting, if it were practicable, to trace the influence of each line of inheritance for some generations back; but this is a task beyond the powers of the present writer, and perhaps no one could successfully undertake it. The impress of each parent was strong upon him. He resembled his mother in certain physical and moral aspects; but it seems probable that the father was the dominating influence in his mental inheritance and in his education. An examination of family photographs confirms verbal report in attributing to the mother dignity, force, and poise; to the father like qualities, in so far as they are compatible with great intellectual vivacity and variety of interests. A picture of the mother gives the impression that the photographer could take his time, that his subject would look just the same the next minute or the next hour; a picture of the father, like one of the son, suggests by a certain gleam of the eyes and a certain mock severity of the mouth that the present attitude of stillness is maintained with effort and will vanish into something very different the instant restraint is removed.

After his marriage, in 1841, Mr. Peirce remained for several years in the South as Professor of Chemistry and Natural Philosophy at Mercer. Returning to Massachusetts in 1849, he engaged as a merchant in the South African trade; but evidently business did not wholly engross him. He remained a scholar, had much to do with the Public Library of Beverly, indulged in practical mechanics as an avocation, and took an active part in the early education of his son. Tradition in the family says that the two used to speak Latin in their walks together, and this seems altogether probable to those who knew our colleague's proficiency in that language.

In 1864 Mr. Peirce visited the Cape of Good Hope, going in a sailing vessel and taking with him his son, then about nine years old. This voyage is of interest to us because it gave the boy occasion to write a number of letters which have been preserved and which give more than one indication of the kind of man the writer was to be. They are wholesomely boyish, with an occasional slight error in grammar and a pleasing lack of consecutiveness in the narration of incidents and observations. For example: "It seems queer to see the rebel officers (of the raider *Alabama*) walking about. I am going to try to get another monkey to carry home. We have had wild game several times, but I don't like it very well," etc. But the writing gives promise at least of that fair round hand which is illustrated farther on in these pages, and the lively vigor of the narrative reminds us of the ceaseless activity of later years. He asks his sister not to let certain of his possessions get soiled, a most extraordinary request for a boy, but according well with his lifelong horror of dirt. More significant still, however, are the messages of affection sent to numerous friends at home. To the end of his life Peirce was actively, thoughtfully, friendly toward a host of people. He remembered everybody's birthday, inquired after everybody's health, and strove to contribute to everybody's happiness. He is said to have occupied himself at Cape Town with schemes for blowing up the *Alabama*; but I have no doubt he always made provision for the safety of the crew. There was no hardness in his heart. He has told me that when, as a boy, he sometimes had to fight another boy, he always labored under a certain disadvantage in not being able to get mad enough to enjoy hurting his enemy.

He is said to have retained in his memory so accurate and detailed a picture of Cape Town as to astonish in 1912 a South African whom he met in England. Any one who knew Peirce can well believe this. He was always eager for all sorts of information, and he had the capacity to store it. He acquired it as naturally and assiduously as his father gathered shells, minerals, and coins. Moreover, like most people with extraordinary collections, he had a certain joy in displaying his accumulations.

He was venturesome enough on shipboard to excite the anxiety of his mother, as a normal boy with a normal mother should be. It is related that on one occasion in rough weather he was perched on a boom when it swung outboard. Perhaps a more significant incident was his quiet capture, under his arm, of a large rat which had attacked him in his cabin. For a boy of ten years this was no slight exploit. In general he had a liking for animals, and he was permitted to have a variety of animal pets, though he seems to have formed no favorable opinion of their moral and mental character. Two of his conclusions, based on knowledge, were: "There is no piety in a goat" and "Compared with a turtle, a hen is an intellectual animal."

We here for the first time catch a glimpse of that sportive disposition which was so powerful an element in his make-up. He loved fun, and he made fun, always without malice, all his life. He early developed an inveterate fondness for slang, a turn which shocked his parents, both irreproachable in speech as in other matters. I never heard him utter an "unprintable" word,[2] but he used habitually some expressions that looked queer in print. His parents, after laboring with him

[2] In fact, I think it probable that he adopted his grotesque, but always clean, habit of speech as a substitute for the profanity and general foulness of mouth that he must have been familiar with in some of his boy companions.

in a vain effort to correct his conversation, wisely concluded to let him have his own way, or, as he would have said, "to go his own hooter." Why *hooter*, I never knew.

A more serious perplexity—in fact, a real disappointment and grief—came to his parents when, at the age of sixteen, the boy seemed to lose his ambition for scholarship. There is more than one version or theory of this phenomenon. One is that he was wayward, perhaps through his intimacy with other boys less carefully reared; but it is difficult to think of him as rebellious at any time in his life. It seems more probable that, owing to rapid physical growth and development, he really experienced a temporary mental lassitude. Possibly, too, there was some question, some difference of wish in the family, as to the particular college he should attend, if he went to college at all. Whatever may have been the reason, the fact is that for about two years, from the age of sixteen to that of eighteen, he worked regularly as a carpenter's apprentice at or near his home, taking all the rough experiences that came in the way of this occupation. Probably this life was good for him at this time, though some of his relatives lamented him as a light that had failed. The sturdy labor thickened and toughened the muscles on his big frame, and the methods of a professional workman became a habit with him—that is, the best of these methods; but one injunction from his carpenter "boss," though he doubtless obeyed it while under orders, amused him much in later years. It was substantially as follows: Always keep up an appearance of having plenty to do; if there is nothing else, find a pile of boards and shift it over from one place to another.

I never heard Peirce express any regret for the manner in which he spent these two years. They were pleasant and profitable to him in various ways. Though no longer, it would seem, looking toward a college career, he kept up his Latin and his habit of general reading. He had inherited from his father, who played the flute beautifully, a fine appreciation of good music, and he had an excellent bass or baritone voice. Now he joined the Salem Oratorio Society and vastly enjoyed the singing there. A Beverly organist made him familiar with the fugues of Bach. In short, he was unconsciously fitting himself to serve, as he did serve for many years, in the capacity of member of the Committee on Honors in Music at Harvard. Moreover, during this period of apprenticeship he "made a profession of religion" and became a member of the Baptist Church, to which his father and mother belonged.

In 1872 the boy, now eighteen years old, decided that he wanted to go to college, to Harvard College, and, as only a few months remained before the opening of the college year, he worked very hard to prepare for the admission examinations. These he passed, as a whole, though I believe he was, curiously enough, "conditioned" in some particular of elementary mathematics. He did not take a room in college, but lived with his family, which had moved to Cambridge in order to be with him, about half a mile away. Thence he ran a telegraph line to the room of two classmates and intimate friends, Le Favour and Pine, in one of the college halls. It is said that his health was somewhat impaired for a time by his too severe labor just before entering college, and it is not improbable that he established this telegraphic communication with his friends by way of diversion during this indisposition. Illness was usually for him an opportunity to do something which he had not found time for in health.

Mr. Pine, now an Episcopal clergyman in Providence, has a vivid recollection of Peirce in his college days. They used to take long summer tramps[3] together with great enjoyment, in which the rollicking humor of the latter was no small element. Percival Lowell was their classmate; the now President Lowell was a year behind them. At one time Peirce, Pine, Le Favour, and the two Lowells were all together in a course of elective mathematics given by Benjamin Peirce, then at the height of his fame. It was a notable company. Mr. Pine disclaims any talent for mathematics, and says he took the course because his friends were in it; but Le Favour

[3] One of these probably furnished the material of a "Theme," which was Peirce's contribution to a little book, *Children's Stories, by 11 Sophomores,* published by Roberts Brothers in 1874.

was the one classmate who for the whole four years of college outranked Peirce, and the career of the others I have mentioned is well known. It is interesting to hear Mr. Pine's observation that Peirce and Le Favour took criticism in docile fashion from their illustrious teacher, but that the Lowells always wanted to argue the point.

Professor John Trowbridge has said that Peirce was his first research student. It was certainly under the influence of Trowbridge that he did the work described in his first research paper, printed in the Proceedings of the American Academy and bearing the date February 9, 1875—that is, the middle of his junior year in college. It has the title, *On the Induction Spark Produced in Breaking a Galvanic Circuit Between the Poles of a Magnet.* It is about ten pages long and is compactly written. It is, all things considered, a remarkable paper. A year or two before the opening of Johns Hopkins University it showed a Harvard junior referring to the work of Becquerel, Rowland, Maxwell, and Thomson, using intelligently and effectively an electromagnet (operated by Grove cells, or, later, Bunsen cells), an induction coil arrangement evidently constructed by himself, a Thomson quadrant electrometer, a Thomson galvanometer in connection with the induction coil and a condenser (said to be of one Farad capacity, but probably the *micro* was accidentally omitted), and applying the integral calculus handily to his experimental problem. There was not, probably, in all America at that time another college junior capable of all this; and there are not many such today.

A few months later appeared the next paper, a very short one, bearing Peirce's name, now coupled with that of his classmate, Le Favour. This dealt with the change of magnetization produced in an electromagnet by the use of an armature under various conditions. Professor Trowbridge was interested in this problem at this time, and undoubtedly he set his two students to work at it. Apparently Rowland's epoch-making research on magnetic induction in closed rings of the magnetic metal, though published a year or two before, had not yet received general recognition, though Trowbridge was one of the first in America to see its importance. It is an interesting fact that the research work which occupied Peirce in his very last years had to do with magnetization, and that it also grew out of an investigation proposed to him by Trowbridge.

The next three papers, which bear respectively the dates March 14, April 11, and October 10, 1877, are of much interest and significance to the student of Peirce's career. During the year 1876-77 he was laboratory assistant to Professor Trowbridge, and the paper of March 14 is doubtless an outcome of this experience. It is entitled *On a New Method of Comparing the Electromotive Forces of Two Batteries and Measuring Their Internal Resistance.* It shows us the competent technician and the vigorous, enterprising teacher. We see in it a presage of the vigorous, masterly course in electrical and magnetic measurements, "Physics 3," which he established at Harvard about ten years later and maintained to the end of his life.

The paper of April 11, 1877, bears the title *Note on the Determination of the Law of Propagation of Heat in the Interior of a Solid Body*, and that of October 10 is an extension and application of this note, describing some experimental work in which Le Favour had a part. We may well consider the two papers as one. This is the first of Peirce's writings in what may be called the large mathematical manner—the masterly and seemingly easy application of great principles to a problem of wide scope.

Peirce was, it is evident, a learned man when he went to Germany for further study in 1877, but his conversational use of German and French at that time was limited. In later years he used to tell of his first visit to the German household in which he was to be domiciled at Leipsic. The landlady and her daughter, after vain attempts to reach an understanding with him by way of any language at their command, suggested waiting for the arrival of the son of the family, a student in the University. When this young man appeared he tried Peirce in Latin. Now, if ever there went to Germany an American student of physics who could speak Latin, Peirce was that student; but the German pronunciation of this language was unfamiliar to him; so again there was difficulty, and the other young man, losing patience, exclaimed, "Have you never been to school?" I have heard no sequel to this story, but I can imagine that this young German had many occasions within the next few months to repent of his rash interrogation; for, though Peirce was humble-minded in a way and habitually called himself an ignoramus, he had no mind to be called that by anybody else. In all probability, under a semblance of asking information from a respected authority, he made a practice of exposing the German's ignorance of various matters till he had brought him to a proper state of mind.

Peirce stayed in Leipsic two years, studying especially under the elder Wiedemann, and gained his Ph. D. there in 1879. The next year he was in the laboratory of Helmholtz in Berlin. He profited, of course, by his sojourn in Germany. He saw the German way of doing things; he began lifelong friendships with notable men among his fellow-students—with Karl Pearson, for example; he enjoyed, as he did in America, long vacation tramps in good company; he heard the kind of music he loved, oratorios and organ recitals, and he sang in a choral society, the Riedelsche Verein. Above all, he met in Leipsic a young Scotch lady, a student in the conservatory of music, Miss Isabella Landreth, who, in 1882, became his wife.

But in the form of intellectual impulse he owed but little, I believe, to Germany. He published but two papers as the visible product of his studies there, and neither of these gave room for the play of his best powers. The first, *Ueber die Emissionsspectra der Haloïdverbindungen des Quecksilber,* printed in the *Annalen der Physik und Chemie* for 1879, is purely descriptive, giving in two pages the results of some observations he had made with the assistance, in some form, of E. Wiedemann. It is notable only for the extreme compactness of the style, a quality Peirce's scientific papers always had, which is well exhibited by the first paragraph: "The emission spectra of the mercury-haloid compounds are most convientIy obtained by conducting the stream of an induction coil through a Geissler tube, in which a small quantity of the salt is placed, and then heating it. If the tube is cold, the air spectrum appears. If it is slowly warmed with a Bunsen burner, this vanishes and the ordinary mercury spectrum is seen. By further heating, a light-band, different for each salt, comes out." Brief as it is, this paper reports on four different salts, and it probably represents a good deal of laboratory work. It doubtless made a genuine contribution to scientific knowledge and gave the experimenter useful experience in a field of research new to him. It appears to have been a kind of try-out, preceding the long grind of the dissertation research.

The second paper, the Inaugural Dissertation, is also entirely, or almost entirely, descriptive, containing only a trace of theory and no mathematics beyond the simplest arithmetic. It is entitled *Ueber die Electromotorischen Kräfte von Gaselementen.* It was, I cannot help thinking, a rather unhappy turn of fate that set Peirce, a born mathematician and lover of theorems, to spend a year or more of immense labor on gas batteries at a time when physical chemistry was floundering through a bog of experimentation, directed only, or rather misdirected, by the false proposition that the electromotive force of a battery should be calculable from the heat yielded by the chemical operations occurring in it. He tested more than four hundred cells, of six different kinds, taking exhaustive pains. His results did not agree with the supposed principle just mentioned, and he sorrowfully states this fact. A few years later Willard Gibbs and Helmholtz had disproved this general theorem; but at this time and in Wiedemann's laboratory it was, apparently, unquestioned. It seemed a direct deduction from the law of conservation of energy; it was supported by the opinion of Sir William Thomson; experiments in some cases had seemed to confirm it.

Peirce was not temperamentally the man to challenge on the strength of his own observations a proposition so well supported. He had no passion for originality, for dissent; he had rather a passion for conformity,[6] provided it involved

[6] Thus, in speaking with an Englishman he would be likely to substitute for the moment the English for the American pronunciation of certain words. But, on the other hand, being on moral grounds opposed to the use of alcoholic drinks, he passed through his German experience without adopting the habit of drinking beer; and, being by religion a sabbatarian, he refrained strictly from work on Sunday.

nothing that seemed to him a moral sacrifice. He absorbed with consummate ease the teachings of the great leaders of science, and, so long as he could believe these teachings, he had no desire to question them. He was much more likely to suppose his own vastly painstaking work to be somehow wrong than to suppose that Sir William Thomson had made an error in his reasoning.

I never heard him blame Wiedemann or any one else for the disappointing outcome of his research work; but he formed and retained for many years the conviction that physical chemistry was a most unpromising field to labor in.

Returning to America in 1880, Peirce became for one year a teacher of mathematics in the Boston Latin School. In 1881 he returned to Harvard as Instructor in Mathematics, a title which he held for three years. In 1884 he was made Assistant Professor of Mathematics and Physics, and in 1888, on the retirement of Professor Lovering, Hollis Professor of Mathematics and Natural Philosophy. I also went to Harvard as a teacher in 1881, and so I was witness to his whole subsequent career there.

Though he had left Harvard only a few years before, he was a stranger to many of the fellow-teachers with whom he now came into close relations, and it was interesting to see the impression he made on them. They saw in him from the start something big and powerful, something genial and inspiring, and he soon came to be called by his initials, B. O., as an affectionate nickname. Though a most zealous and conscientious teacher, he seemed to do his necessary work with the greatest ease and to have his mind free half the time for stimulating and assisting others, always kindly, with great care not to seem patronizing or meddlesome. He would throw out casually, and as if he were merely anticipating by a moment his friend's thought, some suggestion which might never have come into the mind of the other, but which, once given, would have influence for years to come. For example, I once, finding that the subject of Probability was not treated by any Harvard teacher, proposed to offer a half-course in this subject. "Yes," said Peirce instantly, "Probability and the Kinetic Theory of Gases." This was about thirty-five years ago. I am still giving the elements of the Kinetic Theory, but have long since given over lecturing on Probability as such. It was, I fancy, in a somewhat similar way that he started Byerly to give the course on Trigonometric Series, famous now for a generation as Mathematics 10.

He had whimsical turns of speech, odd gestures and attitudes. Thus, he called his intimates Deacon, or Colonel, or Uncle habitually. When he said Deacon Peirce or your Uncle Peirce, he meant himself. A favorite simile with him was, like a *cat in a fit* (pronounced *catnafit*). Some of his expressions—for example, *howl of suppressed emotion, true inwardness of all outdoors*—were probably not of his own invention; but they struck his fancy and resounded there. Others, like *jasm*, meaning vigor and power, seemed to be original with him, though his intimates soon adopted them. When I was about to be married, I took Peirce with me to look at a house. It had a winding stairway, behind which was a rather dark and secluded corner. He peered into this and remarked, "Boss place to sweep your dirt." Only one who knew his horror of dirt and of all shiftlessness could appreciate the comic effect of his suggestion.

In his later years his hair was thin, and he made frequent references to his bald spot. One day he said, "I have wondered for a long time why I couldn't part my hair in the middle, but now I know. I got between a lantern and a screen the other day, and then I saw that I had an odd number of hairs."

When leaving the laboratory at night he would stop for a moment at my door, always with some characteristic word or gesture. Sometimes he would tell me he wished I were a better man or urge me to "try to be decent"; sometimes he would shake his fist at me with a most evil grimace and go away without a word.

All this is enough, perhaps, to explain why Byerly in a burst of enthusiastic appreciation one day exclaimed, "Peirce is a cuss. He knows more mathematics than anybody else in the Mathematical Department and more physics than anybody else in the Physical Department, and in addition to all that he is a cuss."

Nevertheless, he was essentially a shy, deeply reserved man. The "antic disposition" that he wore was partly the spontaneous product of his good fellowship and sense of fun; but it served often, like the pretended lameness of a partridge, as a measure of protection, to prevent a possible intrusion.

Peirce was married, as I have already said, in 1882, in Scotland, but on account of illness in her family his wife did not come to America till 1884. Then, partly, it may be, for economy, but partly, I have no doubt, for privacy, they took a house in Waverley, three or four miles distant from Cambridge. Here they lived for some years, and here their second daughter, Emily, was born.

Peirce was generous of his time and labor for others, but when he was working for himself he wanted to be entirely undisturbed. Perhaps the very quickness and intensity of his sympathies made this necessary. The Waverley house was small, and he put his study table in the front hall; for there were few callers in that remote habitation. There he worked, facing a windowless wall. In after years, when he lived in Cambridge, few, even among his intimate friends, were ever invited into his study.

Moreover, as I have said, he was shy, fearful of being conspicuous. As an active Baptist Church member, he had sometimes to "boss a prayer-meeting," but it was always a painful effort for him. In the discussions of the Harvard Faculty his voice was almost never heard. Even when he was lecturing to a small class, made up of students who regarded him with admiration and awe, he would speak rapidly, in a low tone, with an occasional slight gasp suggestive of mental or physical distress. It was, perhaps, as a kind of relief from nervous tension that he would vary the ordinary precise speech of his lecture by occasional humorous twists, while maintaining a facial expressoin of unbroken gravity. Thus he frequently said *down stairs* for *in the denominator,* and if he had occasion to illustrate viscosity, for example, he might, instead of referring to Lord Kelvin's famous experiments with shoemaker's wax, mention *Mr. Geddes his bucket of glue,* Mr. Geddes being one of the laboratory janitors, better known as *Willum.*

In various other ways he showed this curious combination, a Gothic combination it may be called, of great sensibility with a love of the grotesque. He was so much affected by music that he sometimes dared not attend symphony concerts lest he should be made sleepless for the night. He once wrote home from Germany, "I have been listening to some chorales, Messiah, and passion music, played by —— ——, until my back almost crawled out of my skin. The laws of heat conduction will not account for the 'cold streaks' that run up and down a fellow's back when he is listening to some pieces of church music." He played music of this kind on the piano in the privacy of his own family. He sang well and gladly, we are told, in various choral companies. But the only songs I remember him as singing were *"The Little Brown Jug"* and a most absurd ditty, *Woollomooloo,* heard in a London music hall, where, suppressing for the moment his objection to such places, he had gone to hear Harry Lauder. He would beat out a tune with his forefinger on his cheek, varying the pitch by opening his mouth more or less widely, or, seizing the tail of the family dog and turning it as the hand of a hurdy-gurdy, he would grind out wild airs, singing through his nose.

His capacity for serious reading was immense. A volume of the Encyclopedia Britannica was to him like a novel to another man—a book to be read through with eager pleasure. Yet there was room in his mind for trifles and mere oddities of information. I remember hearing one of his intimate friends, Mr. H. N. Wheeler, express something akin to exasperation that Peirce should know the number of nails in a horseshoe. It seemed to him almost unbecoming that such a mind should occupy itself with such a bit of lumber. But it was probably easier for Peirce to know this thing than not to know it. It is likely that a distinct effort of self-repression would have been needed to prevent him from knowing the number of nails in a horseshoe. Yet he had a habit, a very obvious habit, of professing ignorance and asking for enlightenment. Now the desire for enlightenment was genuine. He really did want to know anything new one could tell him, and his "Thank you; I have learned something," was never sarcastic, though it often might have seemed so. But his professions of ignorance were a kind of game. If he had really felt himself to be as ignorant as he often declared himself to be, he would have resigned his professorship. There were at least two motives that moved him to talk in this way; first, caution, lest he should be led into some trap; and, next, an amiable, though sometimes ineffectual, desire to encourage and draw out his interlocutor. It is probable that he got into this habit of self-depreciation when he was a boy, and felt apologetic toward other boys for knowing so much more than they did. If he found that either party to the conversation was likely to suffer from his pretense of ignorance, he dropped it immediately. No one could be more generous than he in sharing his knowledge or his other possessions with any one in need.

Among his useful accomplishments, by which others profited, was his ability to use tools and machinery. He was a practical printer; he could handle lathes, large or small. He had, perhaps, no marvelous skill in any one of these particulars, and he had no vain ambition to surpass the professional craftsmen at their own work, but he was possessed of a general, all-around, ability to do things with his hands and to estimate the skill and labor of others. This enabled him always to establish sympathetic relations with good workmen of any vocation. There was at Gloucester, where he often went in the summer, a little shop where one or two men made excellent fish knives in an old-fashioned way. To help trade, Peirce once gave them an order, to which they responded by making for him one of the most tremendous "weepuns" ever seen off a pirate ship, shaping the handle out of a treasured piece of teak from a famous vessel, the *Hotspur*. This knife, though too big for any domestic use, delighted Peirce, for he liked big things. He had, indeed, always a manner, a habit, of copiousness. In his household he bought sugar by the barrel; for his laboratory wooden furnishings he used, while they could still be found in the market, planks, not boards, of clear white pine; he made out problems years ahead for his courses; the plate of iron on which he made the study of heat-flow already described was about three feet wide and five feet long. He laid in long ago so large a stock of a certain kind of wire, made in Germany, that nearly four years after his death a manufacturer of apparatus was negotiating with the Jefferson Physical Laboratory for a supply, which he could not get elsewhere.

For some years after his return to Harvard in 1881 Peirce did but little original research. There were other things that seemed to him of more immediate importance. He wanted to get his own courses well established, to get other courses in mathematics or physics well started, and, after 1883-'84, to do his part in putting the work of the Jefferson Physical Laboratory on a solid basis. Through all his life he could neglect no labor that seemed to be his duty, and his sense of duty was far-reaching. There was, I think, little feeling of self-sacrifice in this. His passion was for being helpful, rather than for doing things men would call great. Many a man of far less ability, far less capacity for the broader undertakings of mathematics and physics, would have repined at the immense labor that he put into his laboratory course, Physics 3, a severe course of exact measurements in electricity and magnetism, an undertaking with no showy possibilities whatever. It was not, to be sure, the kind of course an ordinary man could have created and maintained. It was a rock of safety for some of the rest of us. It furnished us with standards, tangible and intangible; with examples of care and precision; with portable, loanable, not always returnable, constant cells and reliable resistance coils. Yet we understood well enough that we were not to go into the quarters of Physics 3 and take things away without his knowledge. He asked for and received at the start sole possession of certain rooms nobody else wanted. He filled these rooms with apparatus, much of it made with his own hands or by his particular direction, devoted exclusively to the uses of his course, except in so far as he might expressly give permission for its use elsewhere. He installed in charge of this apparatus a green chore boy, and trained him to great skill and usefulness; then let him go out to some wider career, and took another, to be treated in the same way.

It was in his willingness to do this kind of thing that Peirce showed his real humility, a humility dignified by moral force and ennobled by a spirit of devotion. It was all in keeping with, if not actually a part of, his religious life. If he had possessed one particle of that intellectual hauteur we sometimes can perceive in able men, he might well have been more widely known, but he would not have been so widely beloved. Who shall say that he did not choose wisely?

Not all of his teaching, however, was of this restricted and elementary character. He lectured for a time on thermodynamics, and afterward on the advanced theory[7] of electricity and magnetism and on hydro-mechanics. His book on the Newtonian Potential Function, of which the first edition (143 pages) appeared in 1886 and the third (490 pages) in 1902, grew out of his coöperation with Byerly in Mathematics 10.

$$\iiint h_u h_v h_w \frac{\partial}{\partial u}\left(\frac{\Omega}{h_v h_w}\right) d\tau = \iint \Omega \cos(u,n)\, dS$$

$$h_u h_v h_w \left\{ \frac{\partial}{\partial u}\left(\frac{Q_u}{h_v h_w}\right) + \frac{\partial}{\partial v}\left(\frac{Q_v}{h_u h_w}\right) + \frac{\partial}{\partial w}\left(\frac{Q_w}{h_u h_v}\right) \right\}$$

$$\iiint h_u h_v h_w \left\{ \frac{\partial}{\partial u}\left(\frac{\mu h_u D_u V}{h_v h_w}\right) + \frac{\partial}{\partial v}\left(\frac{\mu h_v D_v V}{h_u h_w}\right) + \frac{\partial}{\partial w}\left(\frac{\mu h_w D_w V}{h_u h_v}\right) \right\} d\tau$$

$$D_u^2 V\, h_u^2 + D_v^2 V\, h_v^2 + D_w^2 V\, h_w^2 + D_u V \cdot \nabla^2(u) + D_v V\, \nabla^2(v) + D_w V \cdot \nabla^2(w) = -4\pi\rho$$

$$\iiint \mu \left\{ D_x U\, D_x W + D_y U \cdot D_y W + D_z U \cdot D_z W \right\} d\tau$$

$$\iint \mu \cdot U \cdot D_n W \cdot dS - \iiint U \cdot \Omega(\mu W)\, d\tau$$

Not many students were fitted to take the more advanced of his courses, but there were always a few eager to take anything he offered. That is all the following a teacher in the higher regions of mathematical physics can expect or even wish for.

In 1883 Peirce published in the American Journal of Science a paper of three pages *On the Sensitiveness of the Eye to Slight Differences of Color,* giving an account of an investigation suggested by Prof. Wolcott Gibbs. It is a character-

[7] Copies of certain formulæ which stood for years constantly on his blackboard are shown on this page. His beautiful formal handwriting is well exemplified here.

istically terse, conclusive statement of the object, method, and results "of a series of observations made by a number of different persons and extending over several months." The main general conclusions were: "In all cases the eye was most sensitive to changes in a color slightly less refrangible than that of the sodium line, though this color varied somewhat with different persons, being in some cases more orange and in others more yellow. In all cases the eye was more sensitive to changes in the color corresponding to the *F* line than to changes in colors lying half way between *b* and *F*." That is, in the part of the spectrum used, from *Li* $\alpha$ to *G*, the curve of sensitiveness showed, for every eye examined, two maxima. Some eyes were found to be ten times as sensitive to small changes as other eyes. An interesting fact noted is that an observer may be able to see that two slits of light are of slightly different color without being able to tell which of the two colors is the more refrangible. The acoustic analogue is that even a good musical ear may be unable to decide which of two tones, perceived to be different, is of the higher pitch. I dwell somewhat upon these particulars because it seems to me that this careful little paper, based on a considerable body of observations, may be of interest to some investigators of the present time who may have overlooked it. I think, though of this I am not sure, that Peirce was the first to make such a study of retinal sensitiveness by direct use of the spectrum, instead of using revolving colored disks.

Many years later he published two other papers of a kind that might well come from a psychological laboratory. One of these, printed in *Science* for September 29, 1899, is on *The Perception of Horizontal and of Vertical Lines*. The opening paragraphs read as follows:

"Almost every person is occasionally called on to decide by the eye whether some straight line is horizontal or some other line vertical. It usually happens, as, for instance, when one has to set a picture straight on the wall of a room, that the judgment is helped by the presence, in the neighborhood, of other lines, known to be nearly horizontal or vertical, but sometimes all standards are lacking and then the decision is a little more difficult to make.

"In order to find out whether such training as a student in physics gets from several years of laboratory work is likely to improve his judgment in such matters as these, and whether astigmatism affects the results materially, I have experimented in the Jefferson Physical Laboratory upon forty persons who kindly consented to make observations for me."

Some of the interesting conclusions from this study are given thus:

"Trained observers have smaller ranges [among several trials, all by the same individual] than other people, but their deviations [average errors] are not noticeably small. Astigmatism, so severe as to require the constant use of spectacles [discarded here], does not seem to affect the readings much."

The other paper which I have referred to, published in 1906, was *On the Length of the Time of Contact in the Case of a Quick Tap on a Telegraph Key*. About twenty persons participated in the experiments here described, for Peirce was never satisfied with a small number of any kind of observations. Some of the conclusions were that the average person, pressing down a key and then lifting it, can make a contact as short as one-thirtieth of a second, and that, striking with a thimble on a block, he can make a contact of rather less than one two-hundredth of a second.

I find no research paper from Peirce during the interval between 1883 and 1889, but in the latter year he published, with Prof. R. W. Willson, an article *On the Charging of Condensers by Galvanic Batteries*. The following are two of the opening paragraphs:

"We shall begin by considering the behavior of different batteries when they are suddenly called on to furnish definite quantities of electricity in definite short times, and in this first paper we shall give some results which we have obtained in using water cells.

"These results are interesting, because the water battery possesses in an exaggerated degree some properties which are common to all batteries, and which may seriously affect the quantity of electricity furnished to a large condenser by a cell with which it is connected for a short time only."

The paper describes in detail the method used for making and measuring contacts lasting a small part of a second, and then follow several pages of results and discussion. These results are striking enough. Any one familiar with a water-cell—a strip of zinc and a strip of copper placed opposite each other in a small vessel containing ordinary tap-water—knows that it is highly subject to polarization, but many may nevertheless be surprised to read that 240 water-cells in series, connected for two seconds with a condenser, put into this condenser less than twice as much electricity when the latter had a capacity of 11.74 microfarads as when it had a capacity of 0.5 microfarad.

Evidently resistance has an important part in such experiments as those here described, and accordingly this paper is followed after a few months by another, from the same authors, entitled *Note on the Measurement of the Internal Resistance of Batteries.* This is very short, only three pages, and, taken with its predecessor, which is eighteen pages long and bears a Roman numeral I, as if it were to be followed by other corresponding parts, gives the impression that the original undertaking of this research was not fully carried out at this time. In fact, I think it may have been the relation and relative size of these two papers that suggested to Peirce the phrase *a portico to my hen coop;* for this expression, or one very like it, he used with reference to some work of his own.

The shorter paper, after stating cautiously the rather surprising conclusion to which the evidence led, ends as follows: "It would be easy to suggest explanations for the results noted here, but we content ourselves with drawing attention to the facts." Such an attitude was highly characteristic of Peirce. Though profoundly versed in scientific theory, he was not a theorizer. His constant effort was to add to our certainties of knowledge. He did not care to contribute to our uncertainties. He was not willing to run the risk of misleading or of being obliged to retract. There is something admirable in this caution, yet I cannot help wishing that in this case, and in some others, he had ventured more.

Many years later, and probably at his suggestion, one of his students, Mr. C. H. Ayres, made the resistance of galvanic cells the subject of his doctorate thesis, and another, Mr. Shuddemagen, studied the phenomenon of residual charge in condensers. It may be that in this way Peirce's original purpose was carried out.

It is a curious fact that no research paper written by Peirce between 1877 and 1891 makes any use of the higher mathematics, and any reader knowing him through these papers only might suppose him to be purely an experimentalist. But the next paper, *On Some Theorems Which Connect Together Certain Line and Surface Integrals,* printed in 1891, shows him in a different light. Doubtless these theorems were discovered and used in his teaching. They were first published in the London *Educational Times,* an arena in which those mathematically endowed are in the habit of exhibiting their own powers and testing the powers of others by solving or proposing ingenious and difficult problems. The first theorem is stated as follows:

"Let $U$ be any function of the two polar coördinates, $r$ and $\theta$, which, with its first space derivatives, is finite, continuous, and single-valued throughout that part of the coördinate plane which is shut in by the closed curve $T$. Let $\delta$ be the angle between the radius vector, drawn from the origin to any point $P$ on $T$, and the normal to $T$ drawn from within outwards at $P$. Then, if $T$ does not include the origin, the line integrals of $U \cos \delta$ and $U \sin \delta$, taken around $T$, are equal respectively to the surface integrals of $\frac{D_r\,(r.\,U)}{r}$ and $\frac{D_\theta\, U}{r}$, taken over the area enclosed by $T$."

I shall make no attempt to analyze this or any other of Peirce's purely mathematical papers, for such an effort would be laborious to me and unsatisfactory to my readers. I have, however, quoted this one passage in the hope that it may somehow convey, even to those not versed in the higher mathematics, some idea of the manner in which he moved and made his way in that element. It was a manner of proficiency and power, instantly recognized in any company of eminent mathematicians as proof of his initiation to their fraternity.

We have now seen that Peirce wrote three classes of scientific papers, the purely experimental, the purely mathematical, and a third class in which mathematical and experimental investigation went hand in hand; and we have at least looked at some example of each class. These papers were, as we have seen, infrequent during the first ten years of his teaching at Harvard; but thereafter they were more numerous, too numerous to be mentioned exhaustively, even by title, here. It will be enough if I indicate the range and the general character of his productiveness during the last twenty years of his life.

He continued to publish from time to time brief papers giving the results of observations which he had made incidentally in the course of his teaching or his experimental research, perhaps purely informational, relating, for example, to the properties of certain alloys much used in electrical work, or to the effectiveness of wood and certain other materials as electrical insulators, or to the merits of cast-iron as a material for permanent magnets. A few illuminating sentences from such papers, showing the breadth of the experience on which they are based, are the following:

"I have been obliged, during the last three years, to procure several hundred more or less complicated switchboards, and many of these had to be used in making accurate measurements of electrical quantities. It has been necessary, therefore, to determine under what circumstances hard dry wood or red vulcanized fiber may safely be used, and when marble or ebonite, or even a block of freshly scraped paraffine, is required."

"In preparing a large number of deflecting magnets for the use of students in measuring by Gauss's method the intensity, *H*, of the horizontal component of the earth's magnetic field, I have had occasion to make several hundred measurements of the induction coefficients of seasoned magnets of different sizes and shapes."

"During the last six or seven years a large number of d'Arsonval galvanometers, in which the permanent fields are due to hardened and artificially seasoned cast-iron magnets, have been used in the Physical Laboratory of Harvard University, in competition with similar instruments furnished with hardened forged-steel magnets from the shops of well-known makers."

So all the immense labor of his routine laboratory teaching, vigorously, alertly, carried, was made to yield lessons of experience communicated to fellow-toilers throughout the world. And it was the same with his teaching of mathematics and mathematical physics. His *Short Table of Integrals*—containing in the edition of 1899 no less than 897 forms, but still, according to him, a *short* table—was a book that, under his hands, wrote itself, to the vast advantage of the rest of us. I think that all of his purely mathematical papers, having no experimental element though applicable to physical problems, grew out of his teaching, except perhaps the *Table of the First Forty Roots of the Bessel Equation* $J_0(x) = 0$, etc.; and this appeared as part of an experimental research paper which, with Professor Willson, he published in 1898.

In the decade 1893-1903 he returned to and was largely occupied with an experimental study of heat-flow, but now heat-flow in poor conductors, especially slabs of stone. To this difficult and laborious undertaking he brought all the resources of his mature knowledge and skill, together with the remarkable ingenuity of his coadjutor, Professor Willson. The main results obtained are given in two articles—the first, published in 1898, *On the Thermal Conductivities of Certain Poor Conductors;* the second, published in 1900, *On the Thermal Diffusivities of Different Kinds of Marble.* The first of these is a mighty paper. It opens with a tremendous attack, reminding one irresistibly of the heavy artillery fire with which a great modern battle begins, and then follows, what is often wanting in the military analogue, a movement for which every means and instrument has been maturely prepared. If this paper is not widely famous, it is because the scientific world during the past twenty years has not been keenly interested in the mathematical theory of heat-flow or in the specific thermal conductivities of the materials here dealt with.

The mathematics deal broadly with the problem of the final distribution of temperature along the axis of a prism of any uniform material, kept hot at one end and cool at the other, but with a variety of assumed conditions for the temperature of the sides. The results of the experiments are shown in the following quotation:

"Arranging the results in the order of the conductivities of the specimens, we get the subjoined table. We call attention to the two groups of fine-grained marbles, which have conductivities of about 0.0068 and 0.0076 respectively, at about 30° C.

| | |
|---|---|
| Carrara statuary | 0.00501 |
| " " | 0.00509 |
| Mexican onyx | 0.00556 |
| Vermont statuary | 0.00578 |
| American white | 0.00596 |
| Egyptian | 0.00623 |
| Sienna | 0.00676 |

| | |
|---|---|
| Bardiglio | 0.00680 |
| Vermont cloudy white | 0.00681 |
| Vermont dove-colored | 0.00684 |
| Lisbon | 0.00685 |
| American black | 0.00685 |
| Belgian | 0.00755 |
| African rose ivory | 0.00756 |
| Tennessee fossiliferous | 0.00756 |
| Knoxville pink | 0.00757 |
| St. Baume | 0.00761 |

We reserve for a second paper the results of observations made upon other materials."

The paper on diffusivities, of which the title is given above, dealt with these same marbles. There was no subsequent paper on conductivity bearing the name of both Willson and Peirce. Professor Willson had now become deeply engaged in teaching astronomy, and Peirce went on alone with his researches, publishing in 1899 a paper *On the Thermal Conductivity of Vulcanite,* and undertaking, at the request of the late Prof. Alexander Agassiz, a study of the conductivities of certain pieces of rock from the Calumet and Hecla mine.

The results of this study were published in a brief article in 1903, with the quiet remark, "The determinations involved steady work for several months." In fact, this "steady work for several months," coming at the end of steady work at high pressure for many years, was too much for even his great strength. In 1900 Peirce completely broke down, and for two years he was out of the harness. He has told me that during this period he could do work in mathematics, but not in physics. Indeed, the third edition of his Newtonian Potential Function, a book very largely mathematical, came out in 1902.

He suffered terribly and almost constantly from nausea during these two years—"felt drunk," as he expressed it. After his return to work in 1902, he for a long time walked carefully with a cane and with feet wide apart, when out of doors, and in the laboratory he would move about with his hands against the walls for support; but he gradually regained his strength and poise. Though he never had again the abounding physical energy he once possessed, he was, in the number of papers coming from his pen, more prolific than he had ever been before. In 1910 he returned to the theory of heat conduction in a brief article from which I take a few sentences:

> "The work is straight forward enough, but the computation when the slab is relatively broad is very laborious, and in view of the practical importance of the wall method in determinations of the conductivities of poor conductors of heat, it seems well to record some of the results." "When the ratio of $a$ [the width] to $c$ [the thickness] is large, the double series, which defines $W$, converges very slowly. Thus to obtain the last number in the table more than one hundred and fifty terms of the series were needed."

I reproduce these sentences to show once more the spirit in which Peirce worked. May the labor that shortened his life lighten the burden of others! This would have been his prayer, and it may well be ours.

Peirce's research work after 1903 relates, for the most part, to magnetism, and he published on the average more than one paper a year in this field. Along with these came a series of papers regarding the instruments of his investigation, ballistic galvanometers especially. It is unnecessary for me to say much here about this work, for the very reason that during the later years of his life Peirce was beginning to be discovered by physicists at large in his true quality, as an investigator whose findings, whether mathematical or experimental, were of fundamental solidity and strength. He was growing in reputation at home and abroad.

He was elected to membership in the American Philosophical Society, the Société Française de Physique, and the Circolo Matematico di Palermo. In spite of his habitual self-effacement and his reluctance to accept the responsibility of office, he became by a sort of inevitableness president of the American Physical Society in 1913. In 1912 he was the representative of Harvard University at the celebration of the two hundred and fiftieth anniversary of the foundation of the Royal Society.

But once more he was working too hard. In fact, he was by temperament incapable of sparing himself. He worked tremendously most of the time, and he never learned the wholesome practice of complete mental idleness during the rest of the time. When there was nothing that must be done, he would find something to do—some exercise of wits, some prank, perhaps. He was fond of Walt Whitman's phrase, "Loaf and invite my soul"; but he paraphrased it to "Loaf and invite your soul," and the very way in which he uttered it, swiftly and with animation, showed that he had no realization of what loafing is. In the spring of 1913 he was compelled by the state of his health to leave Cambridge a few weeks before the end of the academic year, but he was able to visit England with his family. He went also to Scotland, visiting there the two brothers of Mrs. Peirce, Rev. James Landreth, of Logie-Pert, and Rev. Peter Landreth, of Perth, with whom he had cordial relations of intellectual and moral fellowship.

During the summer news of an ominous character concerning his physical condition reached some of his friends in America. An attack of phlebitis had prostrated him, giving rise to alarming symptoms of the heart. He was urged in a telegram from President Lowell to make his health the first consideration; but he wanted to come back, and he did come back, taking up his regular work in the fall. Evidently he was ill, but he said little about his health, and he seemed even more cheerful than usual. At times after his first great breakdown he had shown evidence of solicitude regarding himself; he had, for example, kept pretty close watch of his changes of weight; but now there was, I fancied, a change in his bearing. I think he knew perfectly well the serious character, the probable outcome, of the stroke that had fallen upon him during the summer in England, and with this grave certainty upon him all nervousness and anxiety seemed to pass away. He planned for future work—he could not tell, of course, just how much he might still be able to do—but he faced all the solemn possibilities of his condition. I will not say that he put his house in order; it had never been out of order; but he steadied himself.

As to the verity of his religious faith and religious life there can be no doubt. The question is whether he ever had anything that could be called religious doubt. At least one who knew him well holds it to be preposterous to suppose that he could have had any trouble of this kind. But to claim for him such an exemption is to put him into a company where, as a thinking, intelligent man, he would stand alone, Saint Paul himself not being able to qualify for membership therein. In fact, his sympathetic nature must have been peculiarly subject to one kind of religious perplexity, the problem of the existence of evil, of suffering that seems needless, a question that has vexed the devout of all ages.

During the Christmas recess of 1913 he once more, and for the last time, fell acutely ill. He bore the terrible suffering of the next two weeks with unbroken fortitude, with unwearied consideration for his attendants, and he met death in a spirit of serenity and courage.

I cannot better sum up the impression which his personality and his life made on those who knew him well than by adding here an extract from the *Minute* placed on the record of the Faculty of Arts and Sciences of Harvard University, February 17, 1914:

> "He has left among us a large place, which no other man can fill, and when the question is asked, how we shall now fare without him, we can only reply, better than if we had never had him; for he was constructive, and he builded well. He was one of those of whom it can be said: 'They may rest from their labors and their works do follow them.'"

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2. The effect of leakage at the edges upon the temperatures within a homogeneous lamina through which heat is being conducted.

3. The magnitude of an error which sometimes affects the results of magnetic tests upon iron and steel rings.

4. The magnetic permeabilities at low excitations of two kinds of very pure soft iron.

5. The resistivity of hardened cast iron as a measure of its temper and of its fitness for use in permanent magnets.

1911. The effects of sudden changes in the inductances of electric circuits as illustrative of the absence of magnetic lag and of the von Waltenhofer phenomena in finely divided cores. Certain mechanical analogies of the electrical problems.

1912. The anomalous magnetization of iron and steel.

1913. The maximum value of the magnetization in iron.