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ABRAHAM ROBINSON

October 6, 1918–April 11, 1974

BY JOSEPH W. DAUBEN

Playfulness is an important element in the makeup of a good mathematician.

—Abraham Robinson

ABRAHAM ROBINSON WAS BORN on October 6, 1918, in the Prussian mining town of Waldenburg (now Walbrzych), Poland.[1] His father, Abraham Robinsohn (1878-1918), after a traditional Jewish Talmudic education as a boy went on to study philosophy and literature in Switzerland, where he earned his Ph.D. from the University of Bern in 1909. Following an early career as a journalist and with growing Zionist sympathies, Robinsohn accepted a position in 1912 as secretary to David Wolfson, former president and a leading figure of the World Zionist Organization. When Wolfson died in 1915, Robinsohn became responsible for both the Herzl and Wolfson archives. He also had become increasingly involved with the affairs of the Jewish National Fund. In 1916 he married Hedwig Charlotte (Lotte) Bähr (1888-1949), daughter of a Jewish teacher and herself a teacher.

[1]Born Abraham Robinsohn, he later changed the spelling of his name to Robinson shortly after his arrival in London at the beginning of World War II. This spelling of his name is used throughout to distinguish Abby Robinson the mathematician from his father of the same name, the senior Robinsohn. Abby's older brother, Shaul, always used Robinsohn, the traditional form of the family name.

In 1916 their first son, Saul Benjamin, was born in Cologne, Germany. Robinsohn had been appointed the first director of the Jewish National Library in Jerusalem, but the family's plans to emigrate to Palestine were unexpectedly precluded when Robinsohn prematurely died of a heart attack in Berlin on May 3, 1918. Five months later their second son, Abraham (Abby), was born in Waldenburg, Lower Silesia, where Lotte Robinsohn had moved to live with her parents. In 1925 the family moved to Breslau, capital of Silesia, where Lotte Robinsohn worked for the Keren Hajessod, a Zionist organization devoted to the emigration of Jews to Palestine.

Both Saul and Abby were educated at a private Jewish school in Breslau, where Abby was very soon identified as "a genius." He liked to hike, and enjoyed writing short stories, poems, plays, and even a five-act comedy, "Aus einer Tierchronik" (From a Chronicle of Animals). Both brothers attended the Jewish High School in Breslau and looked forward to spending their summers in Vienna with their uncle Isak Robinsohn, a prominent radiologist.

In 1933, however, as Hitler and the National Socialists came to power in Germany, Lotte Robinsohn decided it was time to realize her lifelong dream of settling in Palestine. The family left Berlin by train on April 1, 1933, as Jewish businesses were being boycotted throughout the country. The trip south through Austria to Italy afforded the family a chance to see Rome, where Abby was greatly impressed by the Coliseum and found that he especially liked Italian pastries and espresso. In Naples they boarded the *Volcania*, a ship that sailed for Palestine via Greece. From Piraeus the Robinsohns were able to spend a day in Athens, and the Acropolis naturally made a lasting impression. A day later, when their ship docked in Haifa, as Abby recalled in his diary, everyone on board was singing the Ha-tikvah:

"Our hope is not yet lost, the age-old hope, to return to the land of our fathers. . . ." But under the British Mandate refugees could not establish legal residence, and so the Robinsohns arrived in Palestine as "tourists," with ongoing tickets to Trieste, which they never used.

PALESTINE (1933-1939)

To support her family Lotte Robinsohn ran a small *pension* in Tel Aviv, but when Saul Robinsohn went to Jerusalem in 1934 to attend Hebrew University, within a year she and Abby also moved to Jerusalem, where Abby finished high school before going on to the university as well. To help meet family expenses he began tutoring students in various subjects, including Hebrew. The first evidence of his mathematical interests also dates from this time: a set of notes in German on the properties of conics (Seligman, 1979, p. xii).

Jewish immigration to Palestine grew dramatically in the late 1930s; simultaneously the Arab population increasingly rebelled against the mandate and Zionism. The Jewish response was the creation of an illegal organization for the defense of Palestine, the Haganah. Robinson joined the Haganah and often assumed night watches. From time to time there were also paramilitary exercises in the mountains near Jerusalem that would keep him away from his studies, sometimes for weeks at a time. When six students at Hebrew University were killed on Mt. Scopus in 1936, the immediacy of the danger was apparent. It was not long before Robinson was made a junior officer of the Haganah.

When Robinson entered Hebrew University in 1936, the Mathematics Department (added to the faculty in 1927) was barely a decade old. But, given the exodus of Jews from Europe, a number of impressive mathematicians had settled in Palestine, including Abraham Fraenkel, Michael Fekete, Jacob Levitzki—and Robinson studied with them all. The library was built around the collection of Felix Klein, whose books the university had obtained in 1926. Edmund Landau taught briefly in the newly founded Einstein Institute of Mathematics, and was followed by the appointment of Fraenkel. Robinson was among Fraenkel's first students, but within two years Fraenkel said that he had already taught Robinson, his brightest student, all that he could (Seligman, 1979, p. xv).

In addition to mathematics Robinson also took a number of courses in theoretical physics, an introductory course in Greek, as well as readings in ancient philosophy, especially the pre-Socratics and Plato. He also took a course devoted specifically to Leibniz. One of his fellow students Ernst Straus recalls, "When we did not understand something, we would ask him to explain it to us later" (quoted in Seligman, 1979, p. xvii). Robinson was also active in the university's mathematics club, which he had helped organize and to which he once gave a lecture on the zeta function.

Robinson's first publication appeared in 1939 in the *Journal of Symbolic Logic.* This showed that the axiom of definiteness (the axiom of extensionality, or the axiom that establishes the character of equality within the system) was independent of the axioms of Zermelo-Fraenkel set theory. The paper was reviewed by Paul Bernays, who recommended it with various revisions to Alonzo Church for publication. Another paper accepted in 1939 for publication in *Compositio Mathematica* offered a simple proof of the theorem that for rings with minimal conditions for right ideals, every right nil ideal is nilpotent. This work drew on ideas inspired by courses Robinson had taken with Jacob Levitzky, but when World War II broke out, *Compositio Mathematica* ceased publication, and Robinson's paper, though corrected in page proof, did not appear (although it is included in the edition of Robinson's *Selected Papers*).

At the end of 1939 Robinson was awarded a special French government scholarship. In his application he explained that he needed to broaden his mathematical horizons, especially with respect to "mathematical methodology" and that in France he hoped to be able to read the vast literature on the subject not available in Palestine. And so, despite the war that had already begun in Eastern Europe in 1939, Robinson set off in January of 1940 by ship from Beirut to Marseilles and then went on by train to Paris.

PARIS: JANUARY-JUNE 1940

In Paris Robinson lived in a small *pension* in the *Quartier Latin*, not far from the Sorbonne. There is no record of what Robinson may have done with respect to his mathematical studies in Paris, apart from an enthusiastic letter of introduction Fraenkel wrote on Robinson's behalf to the philosopher of mathematics Leon Brunschvicg. While in Paris, Robinson's diary records visits to the museums and galleries, public concerts, the opera, cinemas, and the theatre. He noted in particular a play he saw by Jean Giraudoux, *Ondine*, based on a German novella but presented in Paris with typical French "*esprit*" and "*clarté*," as Robinson said (Dauben, 1995, p. 65). He also commented specifically on an exhibition he had seen by the Belgian artist Frans Masereel, a member of the Association of Revolutionary Artists and Writers, an antifascist group. Masereel had been sympathetic to the Republican cause in Spain and stressed socially progressive and radical themes in much of his artistic work, which Robinson regarded as better known in Palestine than in France.

When the Germans invaded Holland, Belgium, and Luxemburg in May of 1940, Robinson first thought about trying to make his way back to Palestine. But, when Mussolini sided with Germany on June 10, declaring war on France and England, any easy route back to Palestine was effectively blocked. The next day Robinson left Paris, having learned that the German army was only some 30 kilometers northwest of the city. Relying on a combination of suburban trains, trucks, and often making his way on foot, he headed for Bordeaux. Fortunately, traveling on a British passport, Robinson was able to secure a place on a coal tender, one of the last to carry refugees across the channel from France to England. The trip took four days, slowed by intermittent shelling from a German ship and occasional strafings by enemy planes overhead. Everyone slept on the open deck until the boat reached Falmouth in Cornwall. From there Robinson was taken to a holding facility in London for processing along with thousands of other refugees.

LONDON (1940–1946)

Thanks to the Jewish National League in London, Robinson eventually found a place to stay in Brixton, "a quarter of ill repute," as his diary put it. Soon the Germans were bombing London, and the Battle of Britain was underway. For weeks on end the *blitzkrieg* was relentless. One morning, returning from a night's refuge in one of the underground stations, Robinson found his quarters destroyed by a bomb, and for nearly two weeks he was homeless. By November of 1940, however, he had enlisted in the Free French Forces under the command of General Charles de Gaulle.

Despite the war Robinson did his best to keep his mathematics alive and even wrote a short paper on a generalized distributive law for commutative fields. M. H. Etherington, in reviewing the paper for the *Proceedings of the Royal Society of Edinburgh*, described the results as "entirely new," "interesting," and "could not be of any assistance to the enemy," whereupon the article, "On a Certain Variation of the Distributive Law for a Commutative Algebraic Field," was published in 1941.

Meanwhile, Robinson had by a fortuitous set of circumstances been asked to help in writing a report on aircraft design for the Ministry of Aircraft Production. The results were sufficiently impressive that the British government requested Robinson's transfer from the Free French to the Ministry of Aircraft Production at the Royal Aircraft Establishment in Farnborough, just southwest of London. Immediately Robinson began to study aerodynamics in earnest and soon passed a special examination administered by the Royal Aeronautical Society, whereby in June of 1942 he was made an "associate fellow."

In January of 1943 Robinson was visiting friends in London when he met Renée Kopel, a refugee from Vienna who was working in London as an actress and fashion designer. The two soon found that they both enjoyed the theatre, art galleries, nature, walking, and above all, music. Exactly a year after they met, Renée and Abby were married at Temple Fortune in Golders' Green. At first they lived in West Byfleet, nearly equidistant between Farnborough and London, and later Surbiton, somewhat closer to London. In the meantime, Robinson had joined the Home Guard, to have a more active, physical involvement with the war.

At Farnborough Robinson's research was devoted in part to a study of the merits of single- versus double-engine designs for planes on aircraft carriers, but soon he was transferred to the aerodynamics department, where he began research on supersonic aerodynamics. One of the last of Robinson's projects at Farnborough was the reconstruction of a German V-2 rocket from bits and pieces of debris the

Royal Air Force had managed to collect from test firings from Peenemünde that landed in Sweden and Poland.

Once Allied forces had made their beachhead at Normandy in June of 1944, it was another two months before Paris was liberated, on August 25. Within months Mussolini was dead in Italy, Hitler had committed suicide in Berlin, and Churchill finally proclaimed an end to the war in Europe, V-E Day, May 8, 1945. Abby donned his Royal Air Force uniform and went to London, where he and Renée listened to Churchill address the nation from Whitehall, after which they joined the crowds celebrating in central London.

With the war in Europe finally at an end Robinson was assigned to an intelligence reconnaissance task force sent to Germany to debrief scientists in hopes of learning what they had accomplished in aerodynamical research. While in Frankfurt, he also made a special side trip to Breselenz to see the house where Riemann was born.

LONDON (1946–1951)

One of the first things Robinson did after the war was to make a brief return visit to Jerusalem, in part to take his examinations for his diploma from Hebrew University, and to see his mother, brother, and friends whom he had not seen for six years. Robinson was subsequently awarded his M.S. degree, with minors in physics and philosophy. He also used the month he was there to work on a paper with his former instructor Theodore Motzkin, the result of which was "Characterization of Algebraic Plane Curves," published in the *Duke Mathematical Journal* the following year.

Robinson returned to London, having accepted a position as senior lecturer at the newly founded College of Aeronautics at Cranfield (northwest of London), where he taught mathematics in the Department of Aerodynamics. At

Cranfield Robinson spent considerable amounts of time conducting experiments in the wind tunnels and also learned to fly in order to gain the practical experience many theoreticians never acquired. Among Robinson's continuing research interests at this time was the design of delta wings for supersonic travel.

Wanting to further his own mathematical studies, Robinson enrolled as a graduate student at Birkbeck College, University of London, where he studied with Richard Cooke and Paul Dienes. He originally thought to devote a doctoral thesis to the syntax of algebra, but this eventually became "On the Metamathematics of Algebra." He reported some of the early results of his thesis in a brief abstract he sent to the *Journal of Symbolic Logic* in 1949: "Analysis and Development of Algebra by the Methods of Symbolic Logic." Even from this very concise note it was clear that his interests were much more mathematical in a strict sense than were those of other pioneers of the subject like Alfred Tarski and Leon Henkin. Robinson's major interest was algebra, and he regarded logic as a means of obtaining new and more general results—not as an end in itself.

Robinson first came to the attention of a worldwide audience at the International Congress of Mathematicians held in 1950 in Cambridge, Massachusetts. Based on the strength of a proposal he had submitted, Robinson was invited to give a lecture, "On the Applications of Symbolic Logic to Algebra," which presented further results from his just completed Ph.D. thesis (in 1949). Here he was in excellent company; the other invited lecturers in the section on logic, in addition to Tarski, were Stephen Kleene and Thoralf Skolem.

Both Robinson's congress lecture and his thesis dealt with models and algebras of axioms, in which his introduction of diagrams and transfer principles was especially innovative—and where he established a variety of results concerning algebraically closed fields. Philosophically, at this point Robinson was committed as he said to a "fairly robust philosophical realism," meaning that he accepted the full "reality" of any given mathematical structure. The formal languages he drew upon were simply constructs to describe structures, and these he took for granted. His methods above all made it possible to establish results "whose proof by conventional means is not apparent" (Dauben, 1995, p. 175). Later, as a mature mathematician he would adopt a more formalist position with respect to the foundations of mathematics.

Robinson's thesis from Birkbeck College was published by North-Holland in 1951 as *On the Metamathematics of Algebra.* Robinson was also made deputy head of the Department of Aeronautics at Cranfield. However, in February 1951 he received an invitation to accept a position as an associate professor at the University of Toronto. There he would replace Leopold Infeld, the Polish physicist whose presumed Communist sympathies had created certain difficulties that eventually persuaded him to leave Canada and return to his native Poland.

UNIVERSITY OF TORONTO (1951-1957)

Robinson always tried to write at a constant pace, "three good pages" a day (Dauben, 1995, p. 185). As Abby and Renée sailed from Liverpool to Montreal on a Cunard liner in August of 1951, they not only went first class but in the course of the trip Robinson also completed the 25-page manuscript "On the Foundations of Dimensional Analysis." (The original manuscript dated "RMS *Franconia,* August/ September 1951" is preserved among Robinson's papers in the Yale University archives.) Dimensional analysis, as he explained, was an especially useful tool for engineers and physicists, with important implications for the relation between a model and its full-scale counterpart. In theoretical applications dimensional analysis was useful, because it allowed the transfer of results from an experiment performed under one set of circumstances to another comparable set for which the experiment had not been performed. This all had strong affinities to the work for which Robinson would soon become noted, namely model theory and nonstandard analysis.

Robinson's wife was intent upon returning to the theatre, and she not only did some television work but also made a number of recordings, including a dramatic reading of *Medea*. In addition to working with the Canadian Broadcasting Corporation, Renée was a regular performer on a dramatic radio series, "The Craigs," in which she and the actor Josef Fürst (with whom she also made several films) played an Estonian couple, the "von Hohenfelds." At home the Robinsons would often entertain a very mixed group of mathematicians, actors, and producers.

At the University of Toronto Robinson worked in the Department of Applied Mathematics. The National Research Council took advantage of Robinson's arrival in Canada and invited him to give a short course of lectures on "Supersonic Wing Theory." While at the university his teaching included aerodynamics, fluid mechanics, and differential equations, the sort of courses he had been teaching at Cranfield. He sometimes taught a graduate course on supersonic wing theory and also taught basic introductory courses as well, including calculus and analytic geometry. Robinson also had a number of graduate students who worked on various aspects of mathematical physics.

Most of Robinson's publications during his first few years at Toronto dealt with such applied topics as supersonic airfoil design, especially for delta wings, a subject he pioneered during the war. One of his major efforts at Toronto was a book he wrote with one of his former students at Cranfield, John Laurmann. The book covered both subsonic and supersonic airfoil design under conditions of both steady and unsteady flow and was presented with "that heightened sense of structure and unity that was a characteristic of Abby's work" (Young, 1976, p. 311). Following the appearance of *Wing Theory,* Robinson's interests began to turn increasingly to mathematical logic and the results he had achieved in his dissertation at the University of London.

In August of 1952 Robinson participated in the second *Colloque de logique mathématique*, devoted to "Scientific Appreciation of Mathematical Logic." This was held at the Institut Poincaré in Paris, and Robinson's lecture, delivered in French, was later published as "L'application de la logique formelle aux mathématiques." The basic aim of the paper was to show how the generalized completeness theorem could be applied to algebraically closed fields of characteristic zero, a subject to which Robinson would return in a number of subsequent papers.

In connection with the International Congress of Mathematicians held in Amsterdam in 1954, Robinson also participated in a special independent symposium devoted to the "Mathematical Interpretation of Formal Systems," chaired by Arend Heyting. Robinson was invited to give a special lecture, which he devoted to "Ordered Structures and Related Concepts." Inspired by Tarski, Robinson showed in this paper how metamathematical principles could be used to prove the completeness of a real-closed ordered field without having to use Tarski's elimination procedure. Much of the work Robinson was doing at this time was related to algebraically closed fields, real-closed ordered fields, and model completeness, all of which were developed in a book he published in 1955: *Théorie métamathématique des ideaux.*

This actually proved to be a transitional work for Robinson, as his interests evolved from his thesis on the *Metamathematics of Algebra* to another book that he published in 1956, *Complete Theories*. This later was recognized as "a milestone in the development of model theoretic algebra" (Keisler, 1977, p. vi). The book included such important concepts as model completeness, model completion, and the prime model test. Among the succinct, elegant results Robinson presented in this book was a proof of the completeness of real-closed fields. Although not all theories are model complete, for those that are (for example, the theory of algebraically closed fields may be regarded as a model completion of the theory of integral domains), the model completion of a given theory is unique.

Among the more important results Robinson published while at Toronto was a paper he submitted to *Mathematische Annalen*, "On Ordered Fields and Definite Functions." This provided a model theoretic proof of Hilbert's seventeenth problem, that a positive definite real rational function is a sum of squares of rational functions. Although Artin had proved the theorem in 1927, Robinson not only gave better bounds on the number of squares and their degrees, but as Simon Kochen later described the paper, "the main interest of the model theoretic proof lay in its extreme elegance and simplicity" (Kochen, 1976, p. 314). Robinson came back to this problem a few years later, and in "Some Problems of Definability in the Lower Predicate Calculus" he applied the idea of model completeness to field extensions, again studying uniform bounds on the number of squares in Hilbert's seventeenth problem, which led Robinson to a relativization of the concept of model completeness.

Although Robinson had been hired at Toronto to teach applied mathematics, he nevertheless managed to attract a small coterie of students interested in logic, including Paul

Gilmore and Elias Zakon, both of whom went to Toronto specifically to work with him. Of these, A. H. Lightstone wrote his thesis with Robinson on "Contributions to the Theory of Quantification."

Impressed by the amount and quality of Robinson's work, the University of Toronto promoted him to the rank of professor in June of 1956, but this was not enough to keep him in Canada. Later that same year an offer came from the Hebrew University of Jerusalem. Would Robinson accept the chair in mathematics held by his former teacher Abraham Fraenkel at the Einstein Institute? Robinson could not resist the opportunity to live and work in Israel and promptly accepted the offer.

HEBREW UNIVERSITY, JERUSALEM (1957-1962)

When the Robinsons arrived in Israel in 1957, the country was barely a decade old. The War of Independence fought in 1948 had left Jerusalem divided between Jordan and Israel, with Hebrew University atop Mt. Scopus stranded in a demilitarized zone accessible only by convoy once every two weeks. As a result, until the 1967 Six Day War, which would reunite all of Jerusalem, Hebrew University was scattered throughout West Jerusalem in a patchwork of buildings. The administrative offices were in rented space in a former college attached to the Franciscan monastery of Terra Sancta, while the department of mathematics was located in a building at the northern edge of the King David Hotel. Robinson taught several undergraduate courses on linear algebra and hydrodynamics, as well as an advanced course in logic that he taught with his old teacher Abraham Fraenkel.

Among Robinson's first graduate students was Azriel Levy, who had just finished his master's thesis on "The Independence of Various Definitions of Finiteness" (directed by Fraenkel). Robinson joined Fraenkel as Levy's dissertation advisor, and in 1958 Levy completed his thesis on "Contributions to the Metamathematics of Set Theory." By then, coinciding with the tenth anniversary of the founding of Israel, a new campus of Hebrew University was officially opened on Givat Ram, where Manchester House served both the departments of mathematics and of theoretical physics.

In addition to his teaching at the university, Robinson was invited to teach a course on fluid dynamics at the Weizmann Institute (in Rehovot) in the spring of 1959. Robinson still maintained a strong interest in applied mathematics, and while at Hebrew University he also contributed an article on "Airfoil Theory" to Wilhelm Flügge's *Handbook of Engineering Mechanics*. Over the course of his career, nearly one-half of his publications, including one book, were devoted to aerodynamics and the mathematics of structures. As Simon Kochen has suggested, "I believe that the thread that runs through all his work lies precisely, in fact, in this aspect: that also as a mathematical logician, his viewpoint was that of an applied mathematician in the original and best sense of that phrase." By this Kochen meant that the problems set by physical phenomena naturally inspire new ideas in the mathematician, or as Kochen put it, "To logicians, it is the world of mathematics which is the real world" (Kochen, 1976, p. 313).

Among the theoretical areas on which Robinson was working while in Israel was local differential algebra. He was especially interested in expanding upon earlier work of Joseph Ritt, in particular on matters of initial and boundary values. Again, Robinson's successes were due to his model theoretic approach, about which he talked at a meeting of the Union of Italian Mathematicians in Naples in the summer of 1959. Following the meeting in Italy he attended an international symposium on foundations of mathematics in Warsaw, a meeting devoted to infinitistic methods in the foundation of mathematics. There Robinson spoke on “Model Theory and Non-Standard Arithmetic.” This meeting also afforded Robinson a chance to return to his hometown of Walbrzych and visit the grave of his father, who was buried in the Jewish cemetery.

When Fraenkel retired as chairman of the Mathematics Department in 1959, it was Robinson who assumed his position. Robinson was especially interested in curriculum reform and was instrumental in adding a new B.S. degree that had not previously been offered. He was also serious about replacing the old European system of evaluations of students at the end of their studies with course-by-course examinations. He likewise helped to abolish the tradition whereby faculty members worked on their own with one or more teaching assistants, and instead emphasized greater cooperation between faculty with different specialties and in different departments. This was by no means an easy matter, “since there were so many opposed to it . . . and its eventual success was due in large measure to Robinson’s efforts as the ‘living spirit’ behind the new curriculum” (see Dauben, 1995, p. 272).

Meanwhile, Robinson was at work writing up his results on differentially closed fields, which he published in the *Bulletin of the Research Council of Israel.* Using Seidenberg’s elimination techniques, Robinson showed how it was possible to give a model completion for the axioms of differential fields. The differentially closed fields could then be taken as models of the “closure” axioms associated with the completion. As George Seligman has said, reflecting the views of Angus Macintyre, “it would be appropriate to say that he *invented* differentially closed fields” (Seligman, 1979, p. xxiv).

In 1960 Alonzo Church was on sabbatical leave from

Princeton, and Robinson was invited to spend the year as a visiting professor in the Department of Mathematics. The most remarkable result of that year was Robinson's creation of nonstandard analysis. He had been thinking about Skolem's approach to nonstandard arithmetic for some time, when one day as he walked into Fine Hall at Princeton the idea of nonstandard models for *analysis* suddenly flashed into his mind. A plenary lecture he had agreed to give at the silver anniversary meeting of the Association for Symbolic Logic in January of 1961 proved a suitable occasion to make public his new idea, and in the course on "Non-Standard Arithmetics and Non-Standard Analysis" he outlined how it was possible to provide a rigorous foundation for the calculus using infinitesimals. He communicated this almost immediately to Arend Heyting, and soon Robinson's first paper on the subject was published in the *Proceedings of the Netherlands Royal Academy of Sciences.*

In his paper Robinson explained how Skolem had shown the existence of proper extensions of the natural numbers *N* that possessed all of the properties of *N* formulated in the lower predicate calculus. Such extensions provided nonstandard models for arithmetic, and Robinson had the bright idea of taking the same approach to the real numbers *R*. He soon provided a much fuller account of nonstandard analysis in his book *Introduction to Model Theory* (see below).

While in the United States, Robinson spent several months working on an appropriate nonstandard language for nonstandard arithmetic at the University of California at Berkeley. He was also invited to Southern California by the Philosophy Department at the University of California at Los Angeles, where the Robinsons were impressed by both the university and the climate. As Angus Taylor recalls, it was about this time that the idea of offering Robinson a joint appointment in philosophy and mathematics began to take shape. Robinson was especially interested in the "concentration of good people" working in logic at UCLA and elsewhere in California (Robinson to DeLury, April 18, 1961, cited in Dauben, 1995, p. 292). The idea of inheriting Rudolf Carnap's chair was also a powerful factor in the decision to go to UCLA.

Robinson had one last year to spend at Hebrew University before moving to California, and he used it in part to work with several of his graduate students who were finishing their dissertations, including Shlomo Halfin's on "Contributions to Differential Algebra" and Amram Meier's on "Analytic Continuation by Summability and Relations Between Summability Methods." He also finished a revised version of his first book, *On the Metamathematics of Algebra,* to which he added many of the latest model theoretic developments produced in the decade following its publication in 1951. He was especially unhappy that his most basic insight—that important concepts of algebra possess natural generalizations within the framework of model theory—had not gained wider acceptance, something he hoped the new book would rectify. When *Introduction to Model Theory and to the Metamathematics of Algebra* appeared in 1963, the second half was almost entirely new and included a substantial section on nonstandard analysis.

Leaving Israel was not an easy decision for Robinson, for it meant not only leaving his colleagues and students at Hebrew University but also the country to which he was so deeply attached both intellectually and emotionally. Being in Israel had also given him the chance to reconnect with his brother, Saul. When Abby and Renée arrived in Israel in 1957, Saul and his wife, Hilde, were living in Haifa, and the two brothers enjoyed being able to talk again at length about philosophy and education, in which they were especially interested. From time to time they would also make trips together with their wives to interesting archeological sites like Roman Caesarea.

UCLA AND NONSTANDARD ANALYSIS (1962-1967)

Robinson's appointment at UCLA was unusual in that he was a full member of two departments, mathematics and philosophy, which meant that he found himself serving on committees and working with students in both departments. Philosophy, logic, and mathematics had a significant prior history at UCLA, including a year Bertrand Russell spent there in 1939-40. A year earlier (1938) Hans Reichenbach had joined the faculty. Rudolf Carnap followed Reichenbach in 1953, and Robinson succeeded Carnap little less than a decade later.

At UCLA Robinson taught a seminar in logic in the Mathematics Department and an introduction to philosophy of mathematics in the Philosophy Department. He also taught an introductory course on modern logic in the Philosophy Department, and even a general course on the philosophy of science. In the Mathematics Department he also taught a one-year course on axiomatic set theory and another on applications of logic to analysis. From the beginning he was active in the Logic Colloquium at UCLA, founded by C. C. Chang and Richard Montague, which alternated its meetings between the philosophy and mathematics departments. Robinson especially impressed David Kaplan, one of his colleagues in philosophy, because "he talked philosophy the way philosophers did" (David Kaplan, quoted in Dauben, 1995, p. 316).

In 1963 the annual meeting of the Association for Symbolic Logic was held at the University of California at Berkeley, where Robinson presented a paper "On Some Topics in Nonstandard Analysis." This was devoted in part to establishing both old and new results in the theory of functions of a complex variable, including a definition of summability that linked Banach-Mazur limits with the theory of Toeplitz matrices. These results and more were elaborated in a paper Robinson published the following year in the *Pacific Journal of Mathematics*.

Shortly after his arrival at UCLA Robinson's book *Introduction to Model Theory and to the Metamathematics of Algebra* appeared in the spring of 1963, and was described by one reviewer as "the first attempt to write a connected exposition of the new subject of model theory" (Engeler, 1964). The most obvious innovation of the book was the material devoted to Robinson's new creation, nonstandard analysis, and this was the avenue by which many mathematicians first came to appreciate the strength and potential of what he had accomplished. As Ernst Straus put it, nonstandard analysis "reached its next flowering in those years at UCLA." Robinson was quite excited about the potential of his new ideas and often quoted Fraenkel, who said, "Nobody achieves a good mathematical result after the age of 30." Robinson was very pleased, according to Straus, "that his best ideas came to him, at least by Fraenkel's standard, in an advanced age" (Ernst Straus, quoted in Dauben, 1995, p. 319).

Robinson spent a part of the summer of 1963 again at Berkeley, participating in an international symposium on the theory of models. He had helped to organize the meeting, which was intended to offer not only a comprehensive overview of model theory but also to achieve some preliminary consensus about uniformizing terminology and notation for the newly developing field. Robinson's contribution was devoted to "Topics in Non-Archimedean Mathematics," in which he explained how, building on the ideas of Thoralf Skolem, he had succeeded in producing a model of the real numbers that included both infinitesimals and infinite nonstandard numbers. Among applications he offered were theorems devoted to the analytic theory of polynomials, complex functions, topological spaces, normed linear spaces, and spectral theory.

In the summer of 1964 Robinson presented one of the most remarkable early results of nonstandard methods. At a NATO advanced study institute held at the University of Bristol in July of 1964, Robinson explained how he and Allen Bernstein, one of his graduate students at UCLA, had just solved the invariant subspace theorem in Hilbert space for the case of polynomially compact operators. As C. C. Chang notes, this result "instantly rocked the mathematical world, so to speak, although many said a standard proof would soon turn up" (C. C. Chang, quoted in Dauben, 1995, p. 327). Indeed, Paul Halmos shortly thereafter found a standard proof of the theorem by distilling the basic property that "did the trick" for Bernstein and Robinson, which Halmos called "quasitriangularity" (Halmos, 1985, p. 204).

Later that summer Robinson also participated in the International Congress for Logic, Methodology, and Philosophy of Science, held in Jerusalem. Robinson spoke on "Formalism 64" and offered his most recent views on the foundations of mathematics, explaining how he had matured from the Platonic realism he had adopted as a student to favor a more formalistic point of view. The title of his paper was meant to indicate a connection with the formalism of David Hilbert but with revisions that updated the subject to 1964, largely in light of Robinson's experience as a logician and especially in terms of model theory. As Robinson pointed out, when Hilbert advanced the elements of his own particular version of formalism in the 1920s, he could not have known that it was "doomed to failure," which only became apparent thanks to the work of Kurt Gödel in the following decade. Paul Cohen's results in the 1960s, however, which showed the independence of the continuum hypothesis (just a year earlier, in 1963), also influenced Robinson's views considerably, as did ongoing discussion over various axioms of infinity among logicians.

Although Robinson rejected any reference to infinite totalities as meaningless, he nevertheless believed mathematicians should "continue the business of Mathematics as usual, i.e. we should act as if infinite totalities really existed." Here Robinson was clearly comfortable adopting a very Leibnizian position, accepting infinite concepts as "*fictiones bene fundatae*" (Körner, 1979, p. xiii). By this Robinson meant that any statement about an infinite totality was "meaningless" in the sense that "its terms and sentences cannot possess the direct interpretation in an actual structure that we should expect them to have by analogy with concrete (e.g., empirical) situations." He also held that the rules of logic were not arbitrary and that the laws of contradiction and the excluded middle, for example, were "basic forms of thought and argument which are prior to the development of formal Mathematics."

In the fall of 1964, as chair of the University of California's Educational Policy Committee, Robinson was automatically a member of the university-wide Academic Council, chaired by the mathematician Angus Taylor. Primarily responsible for advising the university's president, Clark Kerr, one of the council's major concerns that year was the Berkeley Free Speech Movement, set against the background of both the Civil Rights Movement and the Vietnam War. This eventually pitted faculty and students against the administration, which also had to report to the Board of Regents, which in turn was answerable to then governor of California, Ronald Reagan. Although Robinson was distressed by the anti-intellectual tendency he perceived in the Free Speech Movement, he nevertheless opposed any restriction upon lawful speech or advocacy on any of the University of California campuses. (Robinson in a letter to Theodore R. Meyer, chairman of the Special Committee to Review University Policies, February 3, 1965, cited in Dauben, 1995, pp. 339-40).

Despite the political tensions that dominated the University of California that year, Robinson finished a new book on *Numbers and Ideals,* a slight work of barely 100 pages, yet one he hoped would stimulate interest among students in the simplicity and beauty of abstract algebra. This was followed a year later by *Nonstandard Analysis*, which offered the most powerful demonstration yet of Robinson's basic premise: that mathematical logic could benefit mathematics proper, especially model theory and nonstandard analysis, which provided, as he noted in the book's preface, "a suitable framework for the development of the Differential and Integral Calculus by means of infinitely large numbers."

Infinitesimals, Robinson insisted, "appeal naturally to our intuition." Using nonstandard analysis, he proved a wide variety of results, including basic theorems from the calculus, differential geometry, nonmetric topological spaces, Lebesgue measure, Schwartz distributions, complex nonstandard analysis, analytic theory of polynomials, entire functions, linear spaces (including Hilbert space), along with nonstandard spectral theory of compact operators, topological groups, and Lie groups. He also suggested applications to theoretical physics, and he even suggested that the discovery of nonstandard analysis required a rewriting of the history of mathematics, at least where the history of the calculus was concerned.

A year earlier, in June of 1965, Robinson had been invited to deliver the keynote address at an international colloquium devoted to philosophy of science held at Bedford College in Regent's Park, London. There he presented his views on "The Metaphysics of the Calculus," in which he argued that limits were not the best foundation for the calculus but that infinitesimals were. Robinson included a critique of weaknesses in Cauchy's approach to the calculus based on limits, all of which promoted considerable discussion.

By 1965 Robinson had decided that trying to work effectively in two departments at once was too much of a strain and demanded too much of his time. He thus gave up his position in philosophy, and moved full-time to the Mathematics Department. Meanwhile, he had been invited by John Crossley to visit St. Catherine's College, Oxford, for the fall semester of 1965, where logic as Crossley put it was "booming" (John Crossley in a letter to the author, July 5, 1991, cited in Dauben, 1995, p. 369). In the course of the term Robinson gave a regular series of lectures on nonstandard analysis. As John Bell recalls, "The lecture hall was packed—the audience included Moshé Machover, Alan Slomson, Peter Aczel, John Wright, Frank Jellett, John Crossley, and Joel Friedman. These lectures were very absorbing—it was obvious that Robinson was presenting something of fundamental importance—and were delivered with, what I can only describe as, an endearing lack of slickness" (John Bell to the author, March 12, 1994, cited in Dauben, 1995, pp. 370-71).

Following his term at Oxford, Robinson lectured on nonstandard analysis at the University of Paris, where he was appointed a *Professeur associé* at the Institut Henri Poincaré. Interest in nonstandard analysis was growing in France, especially in Strasbourg, where an active group of logicians formed around Georges Reeb, who was known to proclaim "in every corridor that nonstandard Analysis was 'something really new, an actual revolution'" (Lutz and Goze, 1981, p. vi). Robinson also lectured on nonstandard analysis at the Castelnuovo Institute at the University of Rome, which resulted in two papers in the *Proceedings of the Accademia dei Lincei*, both devoted to Dedekind domains in the theory of algebraic numbers. He also published "On Some Applications of Model Theory to Algebra and Analysis" in the *Rendiconti de Matematica e delle sue Applicazioni*.

Back at UCLA, the administrative turmoil and constant department meetings began to take their toll, as did the continuing financial woes of the entire university. Enrollments at UCLA had increased to nearly 30,000 students. Demonstrations against the Vietnam War continued, and Robinson joined Donald Kalish for one of the silent vigils organized by UCLA's Vietnam Day Committee. Financially the university was facing severe cutbacks, and faculty salaries were no longer competitive on a national scale. While still at Oxford Robinson had received a letter from Nathan Jacobson at Yale. A position in logic was being transferred from philosophy to mathematics, and on the advice of an ad hoc committee (consisting of Church, Kleene, Wang, and Montague), Jacobson wrote to say, "we feel very strongly that you would be the ideal person to help us to develop in this new direction for our Department" (Nathan Jacobson to Robinson, November 29, 1965, cited in Dauben, 1995, pp. 371-72). After some initial hesitation Robinson eventually decided to accept Yale's offer.

In the summer of 1966 Robinson returned to Tübingen where Peter Roquette had arranged a guest professorship. This resulted in a month-long course on the fundamentals of model theory, which included applications to both algebra and nonstandard analysis. When he returned to UCLA for the fall term, Robinson taught a course on set theory, another on lattice theory and Boolean algebras, and a three-quarter survey of model theory, decidability and undecidability, and recursive functions.

The highlight of Robinson's last year in California was an ambitious international symposium on applications of model theory to algebra, analysis, and probability, organized by W. A. J. Luxemburg as the culmination of a semester-long seminar on nonstandard analysis at Caltech. This meeting brought together for the first time a large number of mathematicians, all of whom were making use of nonstandard analysis, and the list of participants was impressive. Robinson used the occasion to talk about "Topics in Nonstandard Algebraic Number Theory," which concentrated on the further development of class field theory of infinite algebraic number fields, which he related to earlier classical results of Chevalley and Weil. Robinson finished his final summer in Los Angeles with an institute on axiomatic set theory at UCLA, which he co-organized with Paul Cohen and Dana Scott. This was actually the fourteenth in a series of annual summer research institutes sponsored by the American Mathematical Society and the Association for Symbolic Logic, with financial support from the National Science Foundation. The meeting was dedicated to Kurt Gödel, who had been invited to attend but declined for reasons of ill health.

While at UCLA Robinson had a diverse group of graduate students, and of those who wrote their dissertations with him, the ones who obtained the most significant results were Allen R. Bernstein, "Invariant Subspaces for Linear Operators" (1965); and William M. Lambert, "Effectiveness, Elementary Definability, and Prime Polynomial Ideals" (1965). Subsequently, Robinson worked with Larry E. Travis on "A Logical Analysis of the Concept of Stored Program: A Step Toward a Possible Theory of Rational Learning" (1966); Joel Friedman, "A Set Theory of Proper Classes" (1966); and Lawrence D. Kugler, "Nonstandard Analysis of Almost

Periodic Functions" (1966). Robinson's last three students at UCLA were Peter G. Tripodes, who wrote on "Structural Properties of Certain Classes of Sentences" (1968); Robert G. Phillips, "Some Contributions to Non-Standard Analysis" (1968); and Diana L. Dubrovsky, "Computability in *p*-adically Closed Fields and Nonstandard Arithmetic" (which she defended in 1971).

Robinson's decision to leave UCLA for Yale after the spring term of 1967 was influenced by many factors, but among them certainly were issues of prestige—and style:

> A smaller private university offered him close contact with all members of a distinguished senior faculty in mathematics, a regular flow of talented postdoctoral researchers and graduate students, and the opportunity for easy contact with scholars in fields other than his own. No doubt his awareness of tradition lent appeal to the case of a university nearly twice as old as the oldest of his previous affiliations (Seligman, 1979, pp. xxvii-xxviii).

YALE UNIVERSITY, 1967-1974

At Yale Robinson joined a distinguished faculty in the Department of Mathematics, which at the time included, among others, Walter Feit, Nathan Jacobson, Shizuo Kakutani, William Massey, George Mostow, Oystein Ore, Charles Rickart, and George Seligman. Michael Rabin had prepared the way for Robinson by teaching a course the previous semester on model theory. With Robinson now at Yale he served as a magnet for other logicians, and soon he had established a strong group of postdoctoral students, including Jon Barwise, Paul Eklof, Manuel Lerman, James H. Schmerl, Stephen Simpson, Dan Saracino, and Volker Weispfenning. Established mathematicians also came to Yale for visits of a semester or longer, like Azriel Levy, Gerald Sacks, and Gabriel Sabbagh. As George Seligman has described Robinson's influence at Yale: "Graduate students were charmed and excited by the promise held out for model theory and nonstandard methods in analysis and arithmetic by his courses, and many sought him out as adviser. He refused none and gave generously of his attention to all" (Seligman, 1979, p. xxviii).

It was not long after his move to Yale that Robinson was elected to a two-year term as president of the Association for Symbolic Logic (1968-70). One of the major responsibilities of the Association was overseeing publication of its *Journal of Symbolic Logic*. Robinson was also interested in ways the Association could serve to promote logic in parts of the world where it had not as yet been established. Among the efforts Robinson championed during his presidency was development of mathematical logic in Japan and Latin America, where the Association, with support from the National Science Foundation, sponsored a number of special logic seminars and summer schools. Thanks in part to Robinson's efforts and these early meetings the subject began to win a progressively stronger foothold in Japan and throughout Latin America.

In the summer of 1968 Robinson spent a month at the University of Heidelberg, where he offered a series of lectures on "Model Theory and its Applications." This also gave him an opportunity to work with his colleagues Gert Müller and Peter Roquette. By then the Six Day War had reunited Jerusalem, and Abby and Renée were pleased to spend most of August back in Israel. At the end of the month Robinson presided over a two-day meeting of the Association for Symbolic Logic in Warsaw, during a week's conference devoted to "Construction of Models for Axiomatic Systems." From Poland they went on to Italy for a meeting at Varenna, where Robinson gave a one-week introduction to model theory. Building on the work of Ax and Kochen with respect to Artin's results on *p*-adic zeros of forms over *p*-adic fields, Robinson used nonstandard analysis to produce "one of the most striking applications of model theory to date" in order to prove in a direct way the Ax-Kochen theorem.

Back at Yale one of the first courses Robinson taught was "Chapters in the History of Mathematics," offered in the spring of 1968. Subsequently, in a Festschrift to honor Arend Heyting he chose to consider in partly historical terms the "ultimate foundation" for mathematics. Just as non-Euclidean geometry destroyed faith in Euclidean geometry as the one true geometry of space, so too, Robinson held, did the results of Gödel and Cohen destroy any faith one might have had in the existence of a single, absolutely true set theory. Thus both standard and nonstandard versions of arithmetic and analysis were possible, which served to reinforce the reasonableness of a formalist foundation for all of mathematics.

The following summer, 1969, Robinson was back in Heidelberg, to work again with Peter Roquette on nonstandard number theory. By now the two had been collaborating for some five or six years and had found that they could greatly simplify parts of Siegel's work, which the nonstandard approach made "manageable" (G. D. Mostow, quoted in Dauben, 1995, p. 419).

In the spring of 1970 while on leave from Yale, Robinson was invited to give three Shearman lectures back at his alma mater, the University of London. These he devoted to "Logic as the Science of Mathematical Reasoning." He also gave a second series of lectures for the mathematics department on nonstandard analysis. At this time the Mathematical Association of America released a film it had made with Robinson, an hour's introductory lecture on nonstandard analysis. Following a straightforward account of formal languages and mathematical logic, whereby he introduced the nonstandard, non-Archimedean continuum R* of nonstandard real numbers, Robinson went on in the second part of the film to show how derivatives, integrals, and limits of sequences could all be expressed in terms of R*.

Early in June 1970 the Robinsons flew to Norway, where the Second Scandinavian Logic Symposium, sponsored by the Association for Symbolic Logic, was being held in Oslo. Robinson was among the 30 participants, and talked about recent developments due to "Infinite Forcing in Model Theory." What Robinson did was to develop Paul Cohen's method of forcing in set theory within the context of model theory. What was new in his own approach, Robinson explained, was that he was able to introduce "a kind of compactness theorem for forcing," along with an axiomatization of classes of generic structures by infinitary sentences. This was related to work he had been doing with his colleague at Yale, Jon Barwise. Somewhat later Jerome Keisler summarized the significance of all this as follows:

> Robinson showed that the generic models constructed by his forcing are closely related to model completions. If a theory has a model completion, then it must be the set of sentences true in all generic models of the theory. Whether the theory has a model completion or not, all generic models are existentially closed in the theory. These results created new interest in model completeness and suggested many questions in particular areas of algebra. As a result there has been a substantial increase recently in research activity in the whole area of model-theoretic algebra (Keisler, 1979, p. xxxv).

From Oslo Robinson went on to Chile, where he was a major figure in the first Latin American meeting on logic. From Latin America he returned to Europe to spend a few days in Heidelberg seeing Peter Roquette and Gert Müller before going on to the Mathematical Research Institute in Oberwolfach for the first international meeting on nonstandard analysis. This had been organized by W. A. J. Luxemburg and Detlef Laugwitz, and brought together virtually all of the pioneers of the subject, including Robinson, of course, as well as Larry Kugler, Jerome Hirschfeld, Moshé Machover, Keith Stroyan, and Peter Loeb, who found Robinson's performance at the meeting "charismatic" (Peter Loeb, quoted in Dauben, 1995, p. 434). Following Oberwolfach, Robinson was in Nice for the international congress of mathematicians, where he gave an invited hour-lecture on "Forcing in Model Theory."

Among Robinson's graduate students at Yale, Gregory Cherlin was working on infinite forcing, developed from the point of view of generic hierarchies that resulted in his thesis, "A New Approach to the Theory of Infinitely Generic Structures." Cherlin's thesis also drew upon results of another of Robinson's graduate students, Carol Wood, who was also working on forcing but in terms of higher-order languages rather than first order logic. Her thesis, "Forcing for Infinitary Languages," dealt with differentially closed fields of characteristic $p \neq 0$, for which she established the existence of a model companion.

Tarski's seventieth birthday was celebrated at Berkeley in June of 1971 with a special two-day symposium, where Robinson talked about model theory in relation to an early paper of Tarski's, "A Decision Method for Elementary Algebra and Geometry," published in 1948. As a tribute to Tarski, Robinson wanted to show how "the successive widening of our model theoretic point of view has shed new light on Tarski's result." Robinson was able to do so through the introduction of the theory of existentially complete and generic structures.

At the end of the summer Robinson not only delivered the twentieth series of Hedrick lectures at the summer meeting of the Mathematical Association of America (on "Nonstandard Analysis and Nonstandard Arithmetic") but he also attended the Fourth International Congress for Logic, Methodology, and Philosophy of Science in Bucharest, where he spoke on "Nonstandard Arithmetic and Generic Arithmetic," which presented work he and his graduate students at Yale had been doing on model companions and related topics.

When Robinson returned to New Haven in the fall of 1971, he had been made a Sterling Professor, a prestigious named professorship at Yale. Among the applications of nonstandard analysis that had begun to interest Robinson in the early 1970s were results he and his colleague at Yale Donald J. Brown had obtained for "nonstandard economies," where the effect of any individual in an infinite economy of traders might be taken as infinitesimal. This led to several joint papers, including "A Limit Theorem on the Cores of Large Standard Exchange Economies" and its sequel "Nonstandard Exchange Economies," which appeared in *Econometrica* and summarized virtually all of the results Robinson and Brown had achieved.

Among those whom Robinson invited to Yale as a visiting professor was Gabriel Sabbagh, who spent the fall of 1972 in New Haven. Angus Macintyre had also accepted a position at Yale that same year as an associate professor, which meant that Yale was fast becoming one of the most stimulating centers for mathematical logic in the world. Robinson was away for a part of the year at the Institute for Advanced Study.

When Robinson retired from his presidency of the Association for Symbolic Logic in 1973, he was asked to give an hour lecture at its annual meeting, which he devoted to "Metamathematical Problems." In his lecture Robinson sought to pose 12 open problems or areas in mathematics that he believed would require logic and model theory for their solutions. Meanwhile, he was meeting with Gödel at Princeton, where they discussed their mutual interests in mathematics and logic. Gödel was especially impressed by nonstandard analysis and its potential applications in other parts of mathematics. He had suggested in fact that Robinson come to the Institute for an extended period of time, and even hoped that Robinson might one day be his successor (Robinson to Gödel, April 14, 1971; Gödel papers #011957, Princeton University archives; cited in Dauben, 1995, p. 458). On the subject of nonstandard analysis Gödel had the following to say:

> In my opinion Nonstandard Analysis (perhaps in some non-conservative version) will become increasingly important in the future development of Analysis *and* Number Theory. The same seems likely to me, with regard to all of mathematics, for the idea of constructing "complete models" in various senses, depending on the nature of the problem under discussion (Gödel to Robinson, December 29, 1972; Gödel papers #011962, Princeton University archives; cited in Dauben, 1995, p. 459).

While at the Institute, Robinson was working on a paper dedicated to Andrzej Mostowski for his sixtieth birthday. Of the "metamathematical problems" Robinson had raised in his Association for Symbolic Logic presidential address, he now turned to answer some of the questions he had posed on the "emerging field" of topological model theory. He also wrote another commemorative article that he dedicated to his colleague A. I. Mal'cev, "On Bounds in the Theory of Polynomial Ideals."

The most significant honor of Robinson's entire career was conferred in April of 1973 when he was awarded the L. E. J. Brouwer Medal by the Dutch Mathematical Society. The first recipient, three years earlier, had been the French pioneer of catastrophe theory, René Thom. Robinson's Brouwer lecture was devoted to "Standard and Nonstandard Number Systems" and constituted a mathematical tour of the major highlights of his best-known results, showing the power of nonstandard approaches to mathematics in general. In his Brouwer lecture Robinson also articulated further his views on the foundations of mathematics, which now reflected his experience with nonstandard number systems in particular:

> [Nonstandard analysis] does not present us with a single number system which extends the real numbers, but with many related systems. Indeed there seems to be no natural way to give preference to just one among them. This contrasts with the classical approach to the real numbers, which are supposed to constitute a unique or, more precisely, categorical totality. However, as I have stated elsewhere, I belong to those who consider that it is in the realm of possibility that at some stage even the established number systems will, perhaps under the influence of developments in set theory, *bifurcate* so that, for example, future generations will be faced with several coequal systems of *real numbers* in place of just one.

Robinson spent part of the summer of 1973 back in Heidelberg. Gert Müller had invited him to spend a week with the model theoreticians in Müller's seminar, but this also gave Robinson a chance to meet with Roquette's group, which took advantage of his visit as well. This also allowed Robinson and Roquette to continue their collaboration on nonstandard number theory. Together they were working on nonstandard approaches to diophantine equations, in particular C. I. Siegel's theorem on integer points on curves. Kurt Mahler had generalized the theorem, allowing for certain rational as well as integer solutions. By exploiting the idea of enlargements, specifically of an algebraic number field in a nonstandard setting, Robinson and Roquette hoped that nonstandard methods would help them to go beyond the results Siegel and Mahler had obtained. As Roquette later explained, "These ideas of Abraham Robinson are of far-reaching importance, providing us with a new viewpoint and guideline towards our understanding of diophantine problems" (Roquette and Robinson, 1975, p. 424).

From Heidelberg Robinson flew to Bristol for a European meeting of the Association for Symbolic Logic, where he discussed problems of foundations. Despite the great advances made in contemporary mathematics, where technical developments in particular had been "spectacular," he was concerned about how little the essential nature of mathematics itself had been illuminated with respect to the problem of infinite totalities ontologically. His own response was basically a formalist one:

> I expect that future work on formalism may well include general epistemological and even ontological considerations. Indeed, I think that there is a real need, in formalism and elsewhere, to link our understanding of mathematics with our understanding of the physical world. The notions of objectivity, existence, infinity, are all relevant to the latter as they are to the former (although this again may be contested by a logical positivist) and a discussion of these notions in a purely mathematical context is, for that reason, incomplete.

Robinson ended the summer of 1973 back in Princeton with a brief visit, again to see Gödel. When the fall term began at Yale, Robinson offered a course with the philosopher Stephan Körner on the "Philosophical Foundations of Mathematics." He later confided to Körner that he doubted if the students were enjoying the seminar, "because we are enjoying it too much" (Stephan Körner in a letter to Renée Robinson, cited in Dauben, 1995, p. 471).

In the end Robinson expressed his philosophy of mathematics in his usual light-hearted way in an account he wrote for the *Yale Scientific Magazine* (47, 1973), "Numbers—What Are They and What Are They Good For?"

> Number systems, like hair styles, go in and out of fashion—its what's underneath that counts.

He explained this in part as follows:

> The collection of all number systems is not a finished totality whose discovery was complete around 1600, or 1700, or 1800, but that it has been and still

is a growing and changing area, sometimes absorbing new systems and sometimes discarding old ones, or relegating them to the attic.

Nonstandard analysis and Robinson's nonstandard real numbers were just another step in the continuing evolution of mathematics, which served to broaden and deepen the number systems available to mathematicians and logicians alike.

When Robinson returned to Yale in the fall of 1973, he had been experiencing stomach pains and finally underwent a series of tests at the end of November. "These gave abundant grounds for suspecting cancer of the pancreas, and an exploratory operation revealed that the disease was beyond surgical remedy" (Seligman, 1979, p. xxxi). Robinson began to cancel commitments, lectures he had agreed to give, and meetings he had hoped to attend, but he continued to meet with his students.

The class was removed to his modest office, where a dozen or so hearers crowded in. The disease and the drugs forced him to struggle to concentrate, but his wit still could flash out, and his listeners' laughter would then fill the narrow corridor outside his office (Seligman, 1979, p. xxxi).

Robinson was not able to withstand the progressive advance of the cancer, and in April he was forced to cancel his one class and return to the Yale Infirmary. Shortly thereafter he died quietly in hospital on April 11, 1974. A few days earlier he had just been elected a member of the National Academy of Sciences.

CONCLUSION

Robinson once said that "playfulness is an important element in the makeup of a good mathematician," and he was certainly a mathematician who enjoyed his work to the fullest. He was also happy to remind people that his own career was the perfect counter-example to the old myth that mathematicians do their best work before they are 30, at the beginning of their careers. Robinson had indeed produced excellent work at the beginning of his own career, but his best-known and most often discussed work was done well after he was 40. It can even be said that Robinson was only just beginning to develop the potential of nonstandard analysis and model theory when he died so prematurely at the age of 55.

Robinson was in many respects a universal mathematician, at home in many fields and thus able to exploit the power of model theory in many different areas. As one sympathetic to the work of applied mathematicians as well as the most theoretical, he was also interested in finding applications of nonstandard analysis in a host of disciplines, from quantum physics to economics. And yet as the work he did at Yale clearly shows, he was not only aware of its powerful applications in certain contexts, but he appreciated the fact that it was historically revolutionary as well. As a tool, however, it required an experienced hand, and he was among the few who knew other parts of mathematics well enough to know where nonstandard analysis might be most helpful, or even essential. As he once told Greg Cherlin, “At first it was easy to get results—now you have to do more” (Gregory Cherlin, cited in Dauben, 1995, p. 492).

In the course of his 55 years Robinson accomplished more than most can claim to have accomplished in far longer lifetimes. Indeed, he was a man who made mathematics a thing of beauty, and equally important, he had the remarkable ability to reveal that beauty to all who wished to learn from his example.

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