Irving E. Segal

1918-1998

BIOGRAPHICAL *Memoirs*

A Biographical Memoir by
Leonard Gross
and William Segal

NATIONAL ACADEMY OF SCIENCES

IRVING EZRA **SEGAL**

September 13, 1918-August 30, 1998

Elected to the NAS, 1973

University of St Andrews, Scotland.

By Leonard Gross and William Segal

Irving E. Segal was a mathematician renowned for his pioneering developments in quantum field theory, functional and harmonic analysis and for his now ubiquitous concept of C^*-algebra. Almost all of Irving Segal's work was motivated by mathematical problems of fundamental physics. His pioneering works on C^*-algebras, von Neumann algebras, group representations, tensor algebras, infinite dimensional integration theory, and non-linear wave equations, as well as his specific focus on the conformal group of Minkowski space were aimed at, or offshoots of, his drive to understand quantum mechanics, quantum field theory, and cosmology.

Segal was known for his tweed jackets and strong coffee. As with many mathematicians, he was committed, working evenings, weekends, and vacations. He valued originality in everything, maintaining a skeptical attitude towards the "establishment" and what he viewed as conventional thinking. When he saw a sign reading "authorized personnel only," he'd say, "Let's authorize ourselves." One-way streets were not an obstacle, because they could be traversed the wrong way simply by putting the car in reverse. He had a tendency to hold firmly to conclusions he reached deductively. He highly valued bebop, chess, nature walks, and Sara Lee pound cake.

Early Life and Education

Irving Ezra Segal was born on September 13, 1918, in the Bronx, New York. He was the second child of Aaron Segal and Fannie Weinstein, both of whom had immigrated from the Russian empire in the early part of the century and met in New York. Segal attended high school in Trenton, New Jersey, where he became known as the chess champion of the neighborhood. Looking back on his high-school years, Segal explained that he enjoyed passing his free time by conceiving of, and subsequently solving, differential equations. He was admitted to Princeton University at the age of sixteen and commuted each day on the train from Trenton. As one of the few Jewish students in the culture of Princeton in the 1930s, Segal undoubtedly encountered difficulties due to antisemitism. Yet he graduated in three years, winning the George B. Covington Prize in Mathematics, and was admitted to Yale University at the age of nineteen as a Ph.D. student advised by Einar Hille.

Figure 1: Private Segal, 1943. (Courtesy of Segal family.)

Segal completed his Ph.D. thesis in three years and then, at the age of twenty-two, joined the Harvard University Mathematics Department as an instructor for the year 1940–41. At Harvard, he began to understand the dynamics of teaching in front of the lecture hall and inspiring students, for which he would become known in later years. In June 1941, he submitted a short note to the *Proceedings of the National Academy of Sciences* (*PNAS*) describing the content of his thesis.[1] Segal was conscripted into the U.S. Army later that year to work as a scientist. From 1941 to 1943, he worked as a research associate at Princeton University. His work from 1943 to 1945 included classified research at the Aberdeen Proving Ground in Maryland on mathematical models of supersonic aerodynamics of aircraft and ballistics. After the war, Segal published a full version of his *PNAS* note, along with its concomitant ideas and with all their far-reaching ramifications for various parts of mathematics and physics. Four papers on this subject matter, published in 1947 and soon after, will be discussed later.

Figure 2: Irving Segal. (Courtesy of Segal family.)

From 1945 through 1948, Segal was a visiting member at Princeton's Institute for Advanced Study. He was supported in his last year by the first of three Guggenheim fellowships. In 1948 he accepted a position at the University of Chicago as an assistant professor. His famous quote when asked what it was like to be part of the legendary department during this Marshall "Stone age" period, his answer was, "Oh, it's not very good. It's just the best there is."

In 1955, he married artist Osa Skotting. His first son, William, was born in Chicago in 1957. His son Andrew was born in 1959 during Segal's sabbatical to Denmark in 1958-59. In 1960, Segal accepted a full professorship at the Massachusetts Institute of

Technology (MIT) and moved to Cambridge, Massachusetts. His daughter, Karen Birgitte, was born in 1961. He spent his 1964-65 sabbatical at the University of Aarhus, Denmark, and from there visited with colleagues in Moscow, USSR; Malmō, Sweden; and Bures-sur-Yvette, France. In 1968, Segal moved his family residence from Cambridge to the more rural Lexington. He loved to take nature walks, especially around Walden Pond. During his 1973 sabbatical at the University of Copenhagen, he gave lectures in Moscow, Paris, Trieste, Pisa, Bures-sur-Yvette, and Warwick. He won a Humboldt Prize and spent his next sabbatical year, 1981-82, in Germany.

Figure 3: Walden Pond. (Courtesy of Segal family.)

Figure 4: Walden Pond, later years. (Courtesy of Segal family.)

In 1967 Segal, along with Paul Malliavin and Ralph Phillips, founded the *Journal of Functional Analysis,* which quickly became one of the most prestigious mathematics journals. In 1971, he was elected to the American Academy of Arts and Sciences, and in 1973 he was elected to both the National Academy of Sciences and the Royal Danish Academy of Sciences. Segal's marriage to Osa ended in 1977. He married Martha Fox in 1985, with whom he had a daughter Miriam, born in 1990. Osa married Saunders MacLane of the University of Chicago in 1987.

Research and Career

In 1947 Segal published three papers developing the notion of a C^*-algebra—an algebra of bounded operators on a complex Hilbert space complete in operator norm and closed under taking adjoints. He used this structure to prove the existence of a separating family of unitary representations of locally compact groups,[2] and, in the same year, published a paper "Postulates for general quantum mechanics," also based on the notion of C^*-algebra.[3] He aimed in the latter paper to provide a mathematical structure within which one might find a formulation of both quantum mechanics and quantum field theory, adequate for incorporating in a meaningful way all the special cases arising from different physical systems. He showed how both classical mechanics and quantum mechanics could be encompassed by this structure.

The method of constructing a representation of a C^*-algebra developed in these papers—the so called GNS construction (Gelfand-Naimark-Segal)—has become a standard tool in C^*-algebra theory and in particular in those extensive parts of quantum field theory in which C^*-algebras provide an environment at the heart of axioms.

Concerning the first 1947 paper, Richard Kadison writes in his memorial article for Segal,[4]

> *With this construction, Segal then gives the most natural proof of the Gelfand-Raikov theorem on the existence of a separating family of irreducible unitary representations of locally compact groups. The importance of what Segal achieved in this short (16-page) article is difficult to overstate!*

Michelle Vergne writes in her memorial article for Segal,[5]

> *Between 1940 and 1952, his concern for group representations was focused on the abstract theory of representations of arbitrary locally compact groups and of algebras related to groups. Some of his abstract theorems are now so much part of our current knowledge that many mathematicians in the field of group representations, including myself before writing this bibliographical notice, may have forgotten that these theorems were discovered by Segal around 1950. Who is not aware that the group algebra of a group is its C^*-algebra? Segal proposed to associate a C^*-algebra to any locally compact group in 1947. Who does not know the Plancherel measure? The existence of the Plancherel measure was proven in 1950 by Segal. Who is not sure that the center of the enveloping algebra of a Lie group acts by scalar operators in any irreducible unitary representation? This was proven by Segal in 1952.*

In 1955, Segal turned his attention to some of the specific structures that arise in quantum field theory. He began work on infinite dimensional integration theory, an understanding of which was demanded from the very beginning of quantum field theory: A wave equation such as $\Delta u = \partial^2 u/\partial t^2$ was regarded by the founders of quantum field theory as just a classical mechanical system consisting of infinitely many harmonic oscillators. Indeed, for n harmonic oscillators coordinatized by a point u in R^n with a linear restoring force, Au, and of mass 1, Newton's equations, $F = ma$, are $Au = (d/dt)^2 u$, with $u(t)$ the position of the system in R^n at time t. Replacing R^n by the infinite dimensional space $H = Real\ L^2(R^3)$ and taking Δu as the linear restoring force we see informally that Newton's equations go over to the wave equation. This was a compelling heuristic for the founders of quantum field theory in 1929 and remains a compelling heuristic in recent physics books. The quantum Hilbert state space for n harmonic oscillators is $L^2(R^n$, Lebesgue measure) and the corresponding quantum mechanical Hilbert space for quantization of the wave equation "should" then be $L^2(H$, infinite dimensional Lebesgue measure). In spite of the fact that infinite dimensional Lebesgue measure has no useable meaning, Segal gave an accurate and meaningful interpretation of this space, along with the operators naturally associated with it, in his 1956 and 1958 papers, implicitly using the ground-state transformation to change from Lebesgue measure to a Gaussian measure.[6,7] Gaussian measures on the space of continuous functions on [0, 1] were introduced by Norbert Wiener in 1923 and were used by Cameron and Martin to develop a kind of advanced calculus over Wiener space. But the central role of the reproducing Hilbert space in their work was first brought out by Segal's 1956 paper. Hundreds of papers have been written by pure mathematicians with no intention to relate to quantum field theory that can be traced back to the concepts introduced in that paper. As so often happens, the origin of ideas gets lost in history, especially if the idea crosses disciplines. In the same paper, Segal introduced a transform, later called the Segal-Bargmann transform, as an intermediate step in proving his main theorem, the unitary equivalence between the Hilbert space of complexified symmetric tensors over H and $L^2(H$, Gauss measure), which intertwines the standard creation and annihilation operators with differential operators.

Immediately after, Segal proved the analogous unitary equivalence of the Hilbert space for free Fermions with a space of square integrable operators lying in a factor of type II_1, a Clifford algebra.[8] Here he made use of his previously developed non-commutative integration theory.[9] It turned out years later (1975, to be exact) that the analogy with $L^2(H$, Gauss measure) is more than an algebraic similarity. The dimension-independent Sobolev-like coercivity possessed by the Dirichlet form for Gauss measure is shared by the corresponding Dirichlet form for the Clifford algebra and with exactly the same constants.

For a real Hilbert space H, the symplectic group of $H \oplus H^*$ comes up naturally in the context of Boson quantum fields because it is the invariance group for the canonical commutation relations. If H is finite dimensional then a uniqueness theorem of von Neumann shows that the automorphism of the canonical commutation relation (CCR) algebra induced by such a symplectic transformation is implemented by a unitary operator in any irreducible representation of the CCR. Uniqueness fails if H is infinite dimensional, as does unitary implementability. Together with his student David Shale, Segal characterized those symplectic transformations that are so unitarily implementable.[10,11]

This kind of question goes to the foundations of quantum field theory because of the central role the canonical commutation relations play. Shortly afterward, Andre Weil rediscovered and extended their results to a number theoretic context. The theorem, known as the Segal-Shale-Weil theorem, represents an amusing amalgam of quantum field theory with number theory. Vergne discusses this history further in her 2002 tribute to Segal mentioned earlier.

If a classical system to be quantized is specified by a nonlinear wave equation, such as the equation $(\partial/\partial t)^2\varphi = \Delta\varphi + \varphi^3$ or the Yang-Mills hyperbolic equation, then the standard quantization procedure requires one to identify the phase space of this infinite dimensional dynamical system. The initial value and initial time derivative of a solution to the non-linear wave equation play the role of position and momentum (not velocity, sorry) and therefore a point of phase space can conceptually be identified with a classical solution. This is not the usual way of thinking of phase space in Newtonian mechanics. But Segal introduced this viewpoint, and it has caught on in the works of many others. As to what one should do next with this viewpoint for the purpose of quantization remains to be settled. Segal proposed a way to use this viewpoint for quantization, and it attracted a few followers. But to carry out any of the procedures that he had in mind it was necessary to deal first with the problem of existence and uniqueness of solutions to the classical non-linear wave equations. From about 1960 onward, Segal developed very novel techniques to this end and also introduced the notion of scattering for such non-linear equations. Walter Strauss writes,[12]

> *He invented and developed the concept of a semigroup of nonlinear operators, including the definitive formulation of the concept of blow-up in finite time of a solution in a Banach space. It is interesting that his thesis advisor was Einar Hille, of the celebrated Hille-Yosida Theorem on semigroups of linear operators.*
>
> *In 1979 he wrote the first paper that solved the Cauchy problem, locally in time, for the (hyperbolic) Yang-Mills equations.[13] This paper was the first step in the resolution a few years later of the global problem by Eardley and Moncrief.*

Later papers enlarged the allowed initial data space. But at the present time technology has not yet been able to deal with the data of critical Sobolev index 1/2 in three space dimensions.

Strauss also writes,

> *He was always centrally interested in what he called fundamental physics, which for him mainly included quantum theory, quantum field theory, and cosmology. To him non-linear wave equations were a key ingredient in the mathematical construction of such physical theories. His ideas have played a crucial role in the development of the fundamental theory of non-linear waves.*

The explicit construction of quantum fields for specific, relatively simple models of quantum field theory began with the work of Segal's student Edward Nelson in 1966. A key early step in the construction of the Hilbert spaces and operators thereon, needed to show the internal consistency of these theories, consisted in showing that the Hamiltonian operator of the theory is bounded below. It is well understood that if it were not bounded below then the universe described by the theory would collapse. To this end Nelson first proved an unusual kind of boundedness property of the semi-group generated by the Hamiltonian of a non-interacting field (think, for example, of an electromagnetic field without any charges around.) James Glimm improved this bound in 1968 so as to make it nominally independent of quantum dimension. Segal then showed that the bound is indeed independent of quantum dimension.[14] The circle of ideas surrounding these bounds have come to be known as hypercontractivity bounds. After a co-author of this memoir showed the equivalence of these bounds with certain dimension-independent Sobolev-like inequalities, so-called logarithmic Sobolev inequalities, this family of inequalities became an influence in distant regions of mathematics, including classical statistical mechanics, large deviations, concentration of measures, optimal transport, random matrices, computer science and Perelman's solution to the Poincaré conjecture. Because the papers of Nelson, Glimm, and Segal were written in the format of the creation and annihilation operators fundamental to quantum field theory, a mathematician working in any of the above areas would find it difficult to trace the history of logarithmic Sobolev inequalities back to its proper origins in the work of these three authors.

In 1972 Segal turned most of his attention from construction of quantum fields to cosmology. Maxwell's equations and other wave equations corresponding to mass zero are not only invariant under the ten-dimensional Lorentz group but also under the 15-dimensional conformal group of Minkowski space. The conformal group is generated by Lorentz transformations, dilations, and inversion in the light cone. Whereas three of the 10 dimensions of the Lorentz group have the physical interpretation of transformation to a coordinate system in Minkowski space moving with constant velocity, three of the other five dimensions of the conformal group have the physical interpretation of transformation to a coordinate system in Minkowski space moving with constant acceleration. The invariance of Maxwell's equations under the conformal group was discovered by Bateman and Cunningham in 1910. The group has played a role in various theories within general relativity. In 1967, Segal proposed a way to combine it with quantum field theory and enhance its role in cosmology.[15] Because of its role in a quantum theory, the classification of unitary representations of the conformal group received much stimulation. Because of its role in cosmology, Segal pursued statistical analyses of brightness versus recession velocity of stars with the aim of showing that Hubble's generally accepted linear recession law is wrong, and the quadratic recession law predicted by his use of the conformal group is commensurate with the data. He was one of the first to offer alternatives to the Hubble paradigm of the Big Bang, arguing that there was a lack of support from statistical observational data of redshifts of the most distant observable objects. Most astronomers have not accepted his statistical interpretation of the data. It is generally regarded by astronomers that the only stars whose intrinsic brightness can be reliably estimated are Type 1a supernovas or Cepheid variable stars. For these stars, the recession velocity vs. brightness data supports Hubble's law. Aside from disagreement over the interpretation of data, general relativists were split on its correctness as a physical theory on more theoretical grounds, with discussion even reaching the *New York Times* in 1990.[16,17,18]

Segal's 1976 book, *Mathematical Cosmology and Extragalactic Astronomy,*[19] is an exposition of the concepts and the large amount of work already done by 1976 in pursuit of this theory. During the last 25 years of his life, more than half of the 225 papers that he published in his lifetime were devoted to the mathematics of his chronometric theory and the data analysis that supported it. Quite apart from whether the theory was physically correct, its pursuit generated a tremendous amount of understanding of representations of the conformal group of Minkowski space and of the adjacent structures. The review article by Michelle Vergne discussed previously gives a survey of this very extensive work by Segal and others.

Segal died in 1998 of a heart attack while out for a walk near his house in Lexington. He is survived by his second wife, Martha, four children and five grandchildren. Segal's first wife, Osa, passed away in 2014. In his lifetime, Irving Segal produced forty Ph.D. students. A list can be found online at The Mathematics Genealogy Project.

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