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EDWARD BURR VAN VLECK

1863—1943

A Biographical Memoir by

RUDOLPH E. LANGER AND MARK H. INGRAHAM

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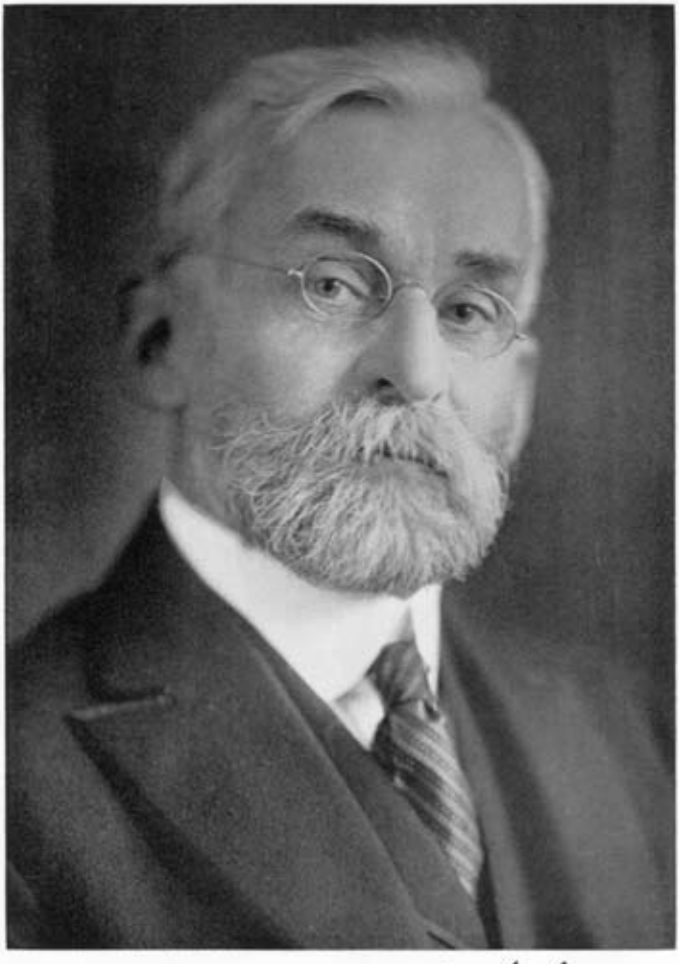

E. B. Van Vleck

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"THE ILLUSTRIOUS SON of a distinguished father and the distinguished father of an illustrious son" was the description given by Dean Holgate, of Northwestern University, of Professor Van Vleck at a dinner of the American Mathematical Society. This description was not only literally true but also symbolically true. Professor Van Vleck was a scholar who to a superlative degree inherited the intellectual and cultural riches of the ages and succeeded in his determination to transmit these enhanced to coming generations.

Edward Burr Van Vleck was born in Middletown, Connecticut, on June 7, 1863. His father, John Monroe Van Vleck, was Professor of Mathematics and Astronomy at Wesleyan University from which he had graduated in 1850 at the age of seventeen and where he taught from 1854 until his death in 1912. Moreover, he frequently acted as president of the University. The Van Vleck Observatory at Wesleyan was named after John Monroe Van Vleck. The Van Vlecks were an ancient family of Maastricht, Holland; and Tielman Van Vleck in 1658 came to America, where he became one of the founders of Jersey City after a period as a notary in New Amsterdam. The family, through the generations, like many other Dutch families, moved up the Hudson Valley—John Monroe Van Vleck being born at Stone Ridge, New York. There was also a large strain of French Huguenot blood in his ancestry. Professor Edward Burr Van Vleck's mother was born Ellen Maria Burr, of Middle-

town, Connecticut, and was chiefly of English descent from stock that had come to New England as early as 1635.

Young Van Vleck's education was in the schools of Middletown and at Wilbraham Academy. He graduated from Wesleyan University in 1884. Endowed with a brilliant mind, blessed with good health, but being quite devoid of athletic skill, he early turned to highly intellectual interests. (His collector's instinct was also shown in youth in his enthusiastic acquisition of stamps.) He found it difficult to decide whether his major interest would be the classics, especially Greek literature, or mathematics. After graduating from Wesleyan, he studied mathematics and mathematical physics at Johns Hopkins University from 1885 to 1887 and taught at Wesleyan from 1887 to 1890. From 1890 to 1893, when he received his Ph.D. degree, he studied at Göttingen, where he formed lifelong friendships with his fellow students (some of them American) and with his major professor, Felix Klein, who had great influence upon Van Vleck as he had upon many others of his students. He always regretted that this period of study had not come somewhat earlier, as from it dates his career as a productive mathematician. The rest of his official career was spent at Wesleyan University, where he was Assistant Professor from 1895 to 1898 and Professor from 1898 to 1905, and at the University of Wisconsin, where he was Instructor from 1893 to 1895 and Professor from 1906 until his retirement as Professor Emeritus in 1929.

Upon his return from Germany in 1893 he married Hester L. Raymond, of Lyme, Connecticut. They had one son, John Hasbrouck Van Vleck, now Hollis Professor of Mathematics and Natural Philosophy and Dean of Applied Science, at Harvard University. Professor Van Vleck's home life was a well from which flowed the quality of his work, his cultural interests, and the influence he had upon his friends—an influence based on intellectual vigor tempered by a fundamental serenity of spirit. Mrs. Van Vleck had much to do not only with her husband's happiness but also with his effectiveness.

Note should be made of three other aspects of his life apart from his research: his interest in literature and the fine arts, his love of travel, and his quality as a teacher.

In connection with the first two of these it must be mentioned that his father late in life had been bequeathed by a brother a considerable estate, part of which Professor Van Vleck inherited. Hence he had means to live graciously, to collect books, etchings, and prints, and to travel extensively. With true Dutch characteristics he was able to combine the love of good living with meticulous care in money matters. He took joy both in giving generously and in investing wisely, but inexactitude, financial or otherwise, went against the grain.

Professor Van Vleck kept abreast of what was published in his field of mathematics, but in spite of this found time for much reading of literature. Often, however, he joked about doing his reading vicariously through Mrs. Van Vleck, who was a prodigious reader. In the graphic arts they shared consuming interest. The etchings of Rembrandt, Seymour Hayden, and Whistler adorned their walls, which however, were always the walls of a home—not those of a museum. Their collection of Japanese prints was notable, and Mrs. Van Vleck became expert in repairing these. Friends from all over America remember with pleasure the occasions when for an hour or so the Van Vlecks would show to small groups some selected prints from their collection.

Travel played a very large role in the life of the Van Vleck family. The guide book and the atlas were ever at hand. (A timetable was not needed in the presence of their son.) The galleries, the churches, and the mountains of Europe were equally familiar. It was perfectly natural for a conversation to turn from point sets to the comparative beauty of the north and south spires of Chartres cathedral. Professor Van Vleck's retirement at sixty-six was associated with both his love of art and of travel for, as he explained to his friends, he wished to retire while he could still enjoy a trip around the world and return to catalogue his Japanese prints. For

each of these programs he set aside a year. He apologized for the fact that, because on his return he missed a connection in Chicago, the trip had taken a year and six hours instead of a year. However he had acquired so many prints during the journey that the cataloguing of this collection was prolonged well past the allotted time.

As a teacher Professor Van Vleck had both natural assets and liabilities. He had the gift of exact expression and of clear organization. However, it was difficult for him to understand a slow mind or to pace himself in accordance with the requirements of an average class. In quizzing a small group or an individual he was superb—discovering any lack of apprehension and clarifying difficult points. He was courteous, yet impatient—one of the few dichotomies of a remarkably integrated personality and related to the conflict between his great tolerance of spirit and his own almost puritanical standards of conduct. He was generous in the extreme with his time, but demanded that he see some results for his effort. He was a stimulating teacher and colleague of the gifted; others surpassed him in getting moderately satisfactory results from the average. As chairman of the Department of Mathematics of the University of Wisconsin, he constantly upheld the highest scholarly ideals.

The qualities of insight, exactitude, and consideration when there was a spark worth fanning made his work as editor of the *Transactions of the American Mathematical Society* in its formative years of great and beneficial influence. A mathematical result was not something to be transmitted haphazardly to the public. It should be a part of a great cultural structure and, as such, it should be expressed with precision and elegance. Many young authors gained much from his kindly but incisive suggestions. Moreover, such standards have been transmitted from scholar to scholar, to become traditional for the *Transactions*.

Not only did Professor Van Vleck believe strongly in the unity of mathematics, but he also believed in the unity of the scholars

who dealt with that subject; and at the time when it seemed likely that they would divide themselves into regional groups, he was a potent force in keeping the American Mathematical Society a truly national organization.

Professor Van Vleck was interested both in the affairs of the University and in those of the community—an interest that was shown through generous giving and through active participation in committees, boards, etc.

There are many who, in their ideal of the scholar and what the life of the scholar should be, have acquired much from Professor Van Vleck and his family.

As a mathematician Van Vleck won his spurs with the completion of his doctoral dissertation in 1893. He had spent five semesters at Göttingen, where he had found his primary inspiration in Felix Klein. His thesis subject, "The Development of Hyperelliptic Integrals in Continued Fractions," was in the focal center of interest of the day. The hyperelliptic integrals are of the form

$$\int \frac{W(x)dx}{(x-a_1)^{1-\lambda_1}\cdots(x-a_n)^{1-\lambda_n}}$$

$W(x)$ being a polynomial of the degree $(n-2)$ and the $a_1, \ldots, a_n$, $\lambda_1, \ldots, \lambda_n$, being real or complex constants. Work in this field had been initiated by Gauss in connection with the hypergeometric functions, in particular in connection with the function

$\frac{1}{2}\log\frac{x-1}{x+1}$, which is represented by the integral

$$\int \frac{dx}{(x-1)(x+1)}$$

It had been carried forward by others in connection with studies of the polynomials of Lamé and Stieltjes. Such polynomials appear as solutions of linear differential equations of the form

$$\frac{d^2y}{dx^2}+\left(\frac{1-\lambda_1}{x-a_1}+\cdots+\frac{1-\lambda_n}{x-a_n}\right)\frac{dy}{dx}+\frac{W(x)}{(x-a_1)\cdots(x-a_n)}y=0$$

By an extensive and searching analysis Van Vleck greatly broadened and generalized the existing theory, and threw light upon it from several new angles. His approach was both analytic and geometric. From the analytic standpoint, the convergents of the continued fraction developments yield algebraic approximations to the integral. Van Vleck concerned himself with such approximations, both such as were valid in the neighborhood of a single branch point, and such as were simultaneously valid in the neighborhoods of several branch points. His geometric discussion, which was extensive, was based upon the theory of conformal mapping. The irregularities of the algebraic approximants and the distribution of the roots of the polynomial factors that figure in the integral representations of the remainder terms were investigated. The upshot was an extensive coordination and classification of the integrals, and revelations of some deeper lying connections of their theory with the theories of linear differential equations, of groups, of polynomials, etc.

With this important memoir Van Vleck had opened for himself a number of avenues along which investigations were to occupy him for the ensuing decade. The fruits of these researches were a succession of papers, on the roots of Bessel functions and Riemann P-functions, on the classification along group theoretic lines of differential equations that admit two solutions whose product is a polynomial, on criteria for the radii of convergence of power series, on the roots of hypergeometric series, and, most especially and extensively, on the theory of the convergence of continued fractions. Well-known theorems in this last field are his. His extended preoccupation with this field of analysis well qualified him for the role of "Colloquium lecturer" of the American Mathematical Society. Delivered in 1903, his lectures were on the subject of "Divergent Series and Continued Fractions."

In 1907 and 1908 Van Vleck published papers on point-set theory, his primary concern being the analysis of non-measurable sets. That his appreciation of this field of analysis was not transient is evidenced by the fact that he chose in 1915, as retiring president

of the American Mathematical Society, to direct his address to the subject of "The Role of the Point-Set Theory in Geometry and Dynamics."

Between 1910 and 1916 Van Vleck's research was concerned with the functional equations of the sine and the theta functions, and with linear difference equations. Although he wrote only one paper on the latter subject, he also treated it in a lecture course at the University of Wisconsin, in a manner that was described by George D. Birkhoff, one of his auditors, as "suggestive and stimulating." Birkhoff and his students subsequently achieved notable advances in this field. It is therefore appropriate to observe Birkhoff's remarks, that "one must look upon Van Vleck as an essential factor in American contributions to linear homogeneous difference equations."

The properties and classifications of groups of linear substitutions in any number of variables were treated by Van Vleck in various papers at different times. Another subject of recurring interest to him was the location of the roots of polynomials. On that he wrote in 1899 and 1903, and again in 1925. He made it the subject of his "Symposium lectures" before the American Mathematical Society in 1929.

Van Vleck was a well-informed and discerning mathematician, and a clear and fluent writer. Some essays in which he reviewed various mathematical developments therefore deserve mention, since they were widely read and appreciated. Among these were his address on the role of point-set theory, which has already been mentioned above, his address on "The Influence of Fourier's Series upon the Development of Mathematics," delivered in 1913 on the occasion of his retirement from a vice presidency of the American Association for the Advancement of Science, and his address "Current Tendencies of Mathematical Research," delivered on the occasion of his investiture with the honorary Doctorate of Science by the University of Chicago in 1916.

His honors were numerous: the degrees of Doctor of Mathematics

and Physics from Groningen, Doctor of Science from the University of Chicago, and Doctor of Laws from Clark University and Wesleyan University. He was made "Officier de l'instruction publique" by the French Republic; and, in addition to serving as editor of the *Transactions of the American Mathematical Society,* he was President of the Society, 1913-1915. He was elected to the National Academy of Sciences in 1911.

Dr. Van Vleck died in Madison, Wisconsin, on June 2, 1943, at the age of 80.

KEY TO ABBREVIATIONS

Am. J. Math. = American Journal of Mathematics
Ann. Math. = Annals of Mathematics
Bull. Am. Math. Soc. = Bulletin of the American Mathematical Society
Trans. Am. Math. Soc. = Transactions of the American Mathematical Society

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$$\frac{a_1 z}{1} + \frac{a_2 z}{1} + \frac{a_3 z}{1} + \cdots$$

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