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GREGOR WENTZEL
1898—1978

A Biographical Memoir by

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BIOGRAPHICAL SKETCH

THE INITIAL LETTER OF GREGOR WENTZEL'S last name has found a solid place in the language of theoretical physics as the W of the fundamental WKB (Wentzel-Kramers-Brillouin) approximation, which describes the semiclassical limit of any quantum system. Beyond this fundamental contribution to quantum theory (1926,1), Gregor Wentzel has played an important role in the theoretical physics of the first half of the 20th century.

Born in Düsseldorf on February 17, 1898, Gregor benefited from a rich and multifaceted education. As the greatest events of his youth Wentzel used to recall the local premières of the symphonies of Gustav Mahler. A lifelong love for music was instilled in the young Gregor. During World War I, he served in the army from 1917 to 1918. At the conclusion of that cataclysmic event he continued his studies, migrating from university to university, as was customary in those days.

First, until 1919 we find him at the University of Freiburg, then at the University of Greifswald, and as of 1920, just like Wolfgang Pauli and Werner Heisenberg and then later Hans Bethe among others, studying with the legendary teacher Arnold Sommerfeld at the Ludwig Maximilians University in Munich, where he obtained his Ph.D. with a thesis on Roentgen spectra (1921). Still in Munich he completed his *Habilitation* in 1922 and became a *Privatdozent* (roughly the equivalent of what today would be an assistant professor). In 1926 Wentzel moves to the University of Leipzig as an *a. o. Professor* (roughly an associate professor).

Then in 1928 he is appointed Erwin Schrödinger's successor as professor of physics at the University of Zürich. That same year Wolfgang Pauli takes over the chair of theoretical physics at the Federal Institute of Technology (ETH) in Zürich. These two former Sommerfeld students become the joint leaders of physics in Zürich, one of the German-speaking world's most vibrant scientific communities.

During World War II, Pauli—a three-quarter Jewish citizen of the Third Reich after the annexation of his native Austria and unable to obtain Swiss citizenship—flees to the United States, and Wentzel remains in Zürich to single-handedly see to the maintenance of the high standards of theoretical physics research and teaching there. After the war, Pauli returns to Zürich, and Wentzel is offered a professorship at the University of Chicago. He moves to Chicago in 1948 and remains there until his retirement in 1970. The Wentzels then move to Ascona, Switzerland. In 1959 the National Academy of Sciences elects Gregor to membership. For his contributions to theoretical physics the German Physical Society awards him the 1975 Max Planck medal, its highest honor. The octogenarian Gregor Wentzel dies in Ascona on August 12, 1978, survived by his wife, Annie, and his son, Donat G. Wentzel, currently a professor emeritus of astronomy at the University of Maryland.

SCIENTIFIC BIOGRAPHY

Gregor Wentzel's scientific life neatly divides into three well-defined periods:

1. his work on erecting the glorious edifice of quantum mechanics,

2. his work on meson theory, and

3. his work on condensed matter physics.

We start with his work on quantum mechanics. The old quantum ”theory” taught by Sommerfeld was too successful to be abandoned altogether and too riddled with contradictions to be able to survive for long. The year 1925 was decisive, being the year when Heisenberg discovered matrix mechanics,[1] and soon thereafter Schrödinger[2] produced his famous equation. From two decades of work with the old theory many of the crucial questions were already clearly in place.

One of these questions asked how the transition from the quantum regime to the classical regime is achieved, in other words, how does one calculate the lowest-order quantum corrections. This question was answered in three now classical papers by Gregor Wentzel (1926,1), Hendrik Anthony Kramers,[3] and Léon Brillouin.[4]

In a brilliant piece of mathematical physics Wentzel recasts the simplest one-dimensional Schrödinger equation, as a Riccati equation. He then expands the function that appears in this Riccati equation in a power series in Planck's constant, and is led to a set of recursion relations for the coefficient functions in this series expansion. These coefficient functions describe the quantum corrections. Wentzel then successfully applies this approximation method to the hydrogen atom and to the Stark effect.

Brillouin's independent paper,[4] presented on July 29, 1926, to the Paris Academy is along the same lines as Wentzel's work, submitted one month earlier (June 18, 1926) to the journal *Zeitschrift für Physik*. Kramers's later work[3] is based on saddle point and steepest descent techniques and provides the famous discussion of turning points. This approximation method was known in various mathematical contexts (see Jeffreys,[5] Liouville,[6] and Carlini,[7] among others). The great importance of its independent discovery by Wentzel, Kramers, and Brillouin consists in the fact that when applied to the then new Schrödinger equation, this approximation scheme solves the major physical problem of systematically calculating quantum corrections.

The next problem tackled by Wentzel was that of the photo effect, the phenomenon, which when first studied by Albert Einstein[8] led to the introduction of the light quantum concept, and thus played a pivotal role in the construction of quantum theory. Wentzel (1926,2) and independently P. A. M. Dirac,[9] were the first to give a full-fledged quantum-theoretic treatment of the photo effect. Wentzel (1926,2) finds the angular distribution of the photoelectrons. Wentzel then derives the intensity of the photoemission by writing down the formula known nowadays as Fermi's golden rule, given the subsequent extensive use and emphasis placed on it by Enrico Fermi.

In 1926 Max Born published his celebrated paper[10] about the application of quantum mechanical perturbation theory to scattering processes, the Born approximation. In this paper Born introduced the first explicit statement of the probabilistic interpretation of quantum mechanics. Wentzel often used to remark that "Born did not choose to write a separate paper on the probabilistic interpretation, because at that point this interpretation, though never spelled out in print, was known to everyone working in the field." The obvious application of Born's method would have been to the Coulomb problem. There was the difficulty that a direct application did not yield a finite result on account of the divergence of Born's integral expression in this case. What Wentzel did (1926, 3) was simply to provide the Coulomb potential with an exponential factor (thus turning it into what later would become known as a Yukawa potential), which renders Born's integral convergent. He then carried out the Born integral and in the end result got rid of the exponential by letting the coefficient in its exponent vanish. This famously leads to the well-known Rutherford formula.

It is rarely mentioned in the literature that we owe this result to Wentzel. Born himself was aware of Gregor's paper, and in his Nobel lecture[11] he quotes it as, "Soon Wentzel succeeded in deriving Rutherford's famous formula for the scattering of α-particles from my theory." Wentzel's take: "Born was too mathematical to dare alter the Coulomb potential. I had no such compunctions and Born never forgave me for that." It is amusing that the paper (1926, 3) on the Rutherford formula and the paper on the photo effect (1926, 2), two of Wentzel's three best-known papers from this period, were submitted for publication on the same day and appear next to each other in *Zeitschrift für Physik.*

In fact, Wentzel's interest in scattering processes predates quantum mechanics. Already in the old quantum theory he did the most advanced analysis of scattering processes (1922). Some of the results derived by him have remained in use even after the discovery of quantum theory. Not surprisingly, the first comprehensive review of scattering and radiation processes is due to Wentzel (1933).

We must mention here that in a prophetic 1924 paper Wentzel attempted a sum over paths approach to the construction of quantum amplitudes. He weighted each term in the sum by the nowadays well-known quantum phase factor, which accounts for quantum interference phenomena. From the so obtained quantum amplitudes he then obtained probabilities by taking the squares of their absolute value.

The second phase of Wentzel's work deals with Yukawa's then new meson theory, its highlight being Wentzel's famous strong coupling approximation. Wentzel's first publication on meson theory was a review (1938) describing Yukawa's suggestion[12] that the nuclear force could be due to a field whose quanta would be bosons a few hundred times more massive than the electron, and the demonstration by Neddermeyer and Anderson[13] that particles of such mass constituted the penetrating part of cosmic rays. Soon after this article, it became clear that the observed particles did not have the strong nuclear interactions expected of Yukawa's particle. This serious difficulty for meson theory was not overcome until 1947, when experiments showed that the bulk of the penetrating cosmic rays came in the form of a totally unexpected particle, the mu-lepton, itself a decay product of the charged Yukawa mesons.

Wentzel initiated the strong coupling approximation to the static meson model ("static" here means neglect of nucleon motion) in his 1940 article in which, as suggested by Yukawa, he treated the simplest case of a spinless charged meson field, with an S-wave coupling to nucleons. He begins by saying that the usual perturbation theory that expands in powers of the meson-nucleon coupling constant, g, cannot be used, since the observed strength of the nuclear force implies that g is large. Wentzel therefore turns to the alternative of an expansion in the reciprocal of g and finds that

- for large g the nucleon has low-lying multiple charged excitations, isobars, interpretable either as bound states of a nucleon and mesons or as rotational states (in an internal charge space) of a rigid rotator, and
- the meson-nucleon scattering cross-section does not increase without limit for increasing g. Of course, unitarity (probability conservation) enforces an upper bound on the cross-section in any scattering calculation if done sufficiently correctly; Wentzel's seems to be the first that was.

Wentzel says that unfortunately this limiting cross-section is at least a hundred times what had been observed. For only moderately large g the cross-section will of course be less, but he suspects that the original Yukawa theory is in trouble, and perhaps one should try a spin 1 meson.

It soon became clear that the properties of the nuclear force required the Yukawa meson to have i-spin 1 (charge-symmetric theory) and to be p-wave coupled to the nucleon spin, so mesons had to be pseudoscalar or vector (spin$^{\text{parity}}$ 0^- or 1^-). The strong coupling calculation for this kind of meson field was first published by Pauli and Dancoff.[14] It was followed by a 1943 elaboration by Wentzel in Zürich. The isobars were found to have all possible half-odd-integer values of both spin j and i-spin i, with the remarkable restriction $j = i$ (this same feature occurs in the Skyrme model[15] of the nucleon as a topological soliton of a simple nonlinear equation for the meson field). Thus the ground state has $j = i = \frac{1}{2}$ and represents the proton and neutron. The first excited state is predicted to have $j = i = \frac{3}{2}$.

A decade later Brueckner[16] pointed out that the ongoing Chicago synchrocyclotron π-meson-proton scattering results were well fitted by a $j = i = \frac{3}{2}$ resonance peak at ≈ 200 MeV, now known as the Δ(1238) baryon. It had always been assumed in strong coupling theory that isobars had to be bound states (i.e., unable to decay with emission of a meson). Yet for p-wave isobars the bound/unbound distinction is blurred by centrifugal barrier decay suppression. This may explain why no connection was even suggested, whether by Wentzel himself or by anyone else, between the experimentally discovered $\Delta(1238)$ baryon and the isobar predicted by the strong coupling calculation. It should be added, however, that the next isobar, $j = i = \frac{5}{2}$, has never been found. In the quark model its absence from the spectrum is accounted for, since such a baryon cannot be a three-quark state. It is worth pointing out that in 1950 Fujimoto and Miyazawa[17] proposed that the first isobar should appear as a resonance observable in pion photoproduction experiments.

In 1947 Wentzel wrote another review on meson theory, discussing strong coupling and other attempts to get a sufficiently small meson-nucleon cross-section, just before this problem evaporated with the discovery by Lattes, Muirhead, Occhialini, and Powell[18] that the penetrating cosmic-ray particles were only a decay product of what they named the pi-meson ("p" for "primary"), the actual Yukawa meson. Shortly thereafter pi-mesons were produced at the Berkeley cyclotron,[19] which allowed rapid determination of their basic properties.

In his later years Wentzel made strong coupling calculations (1957, 1962, 1963) for K-mesons scattering on Λ - and Σ-hyperons. Some of these exhibited interesting features such as a rapid switch of the nature of the clothed hyperon (the rigid rotator) at a critical value of the ratio of Λ-coupling to Σ-coupling, or a higher symmetry than the Hamiltonian had. However, the results did not have much resemblance to observation.

The third phase of Wentzel's work is devoted to condensed matter physics and many-body problems. Wentzel's activity in these fields started in Chicago in his later years after the war. Considerable progress in the area took place in the 1950s, especially in understanding superfluidity and superconductivity from first principles. So it was natural for Wentzel to get involved. He published several papers (1951, 1957, 1963) during this decade, although they are not among his main contributions to physics. Basically his role was that of promoter and critic. Wentzel appreciated exact results obtained by others, for example, in the properties of an electron gas with interaction, and used his familiarity with field theoretical techniques to generalize or simplify them, at the same time criticizing deficiencies in some papers. In particular, he took great interest in the BCS (Bardeen-Cooper-Schrieffer) theory[20] of superconductivity and the associated Bogoliubov-Valatin[21,22] description of electrons (quasi particles). Like many physicists, however, Wentzel was not satisfied with the lack of gauge invariance in the theory. In a paper on the Meissner effect (1958; see also 1959 and Pines and Schrieffer[23]) he proposed a modification of the BCS procedure, but a better understanding of this issue was left to the developments concerning spontaneous symmetry breaking and mass generation for gauge fields.

GREGOR WENTZEL: TEACHER AND COLLEAGUE

Wentzel's lectures starting in his early Leipzig days, all the way to the memorable courses he taught at the University of Chicago, have always awed listeners by their exquisite elegance. This quality is borne out in his textbook *Einführung in die Quantentheorie der Wellenfelder* (1943), written during the war. This first book ever on quantum field theory was translated into English at war's end (1949), and has been the formative textbook of the postwar generation of theoretical physicists.

Wentzel's list of doctoral students is truly remarkable. On it we find Valentin Bargmann, Markus Fierz, Res Jost, Nicholas Kemmer, A. Houriet, Felix Villars, Fritz Coester, Josef Maria Jauch, Burton Fried, Allan Kaufman, Charles Goebel, Nina Byers, and R. Ramachandran.

In a sense, to this list of former Wentzel students one could add the name of Homi J. Bhabha. A student at Cambridge, Bhabha, who later became one of the first internationally recognized Indian theoretical physicists, had been dispatched by his adviser to work with Pauli in Zürich. After reading the adviser's not-all-that-good letter of introduction, Pauli refused to have anything to do with Bhabha. It is, though, in Zürich in this hostile atmosphere that Bhabha wrote his most famous paper, the paper on electron-positron scattering, or "Bhabha scattering" as it is now called. When he tried to show it to the great man, Pauli responded, "If you did this, I am not interested." At wit's end Bhabha went to Gregor, the other senior theoretical physicist in town. Familiar with his good friend Pauli's quirks, Gregor took the paper and read it. He immediately realized its importance and assured its young author that he would convince Pauli of its merit. At first Wentzel's attempt to explain Bhabha's work to Pauli was met with the expected "If Mr. Bhabha did it, I am not interested," but Gregor was prepared for this. "For just a moment," he suggested, "imagine that I had done it." Pauli was willing to listen. Bhabha remained forever in Wentzel's debt.

Beyond teaching, a professor also has to conduct exams. Wentzel had a remarkable technique for doing this. He would start with a question about a simple system, say the classical nonrelativistic rotator. Then he would build up from this to a slightly more complex setting by asking for a quantum mechanical description, then moving up to the relativistic case. Sooner or later the student couldn't answer his question, and where this breakpoint was reached determined the student's grade.

At the University of Chicago just like earlier in Zürich, Wentzel was a central figure among theoretical physicists, able to create a marvelous scientific atmosphere. First of all, there were the frequent and unscheduled private meetings in Gregor's office. Two of us (Y.N. and R.O.) came to Chicago in time to catch some of these meetings. Surrounded by the aromatic smoke of Wentzel's ever present Cuban cigars, stimulating discussions on the latest problems were held in a completely informal setting. Maria Goeppert-Mayer would report on her shell-model calculations, Gregor himself on strong-coupling meson theory and on hyperon decays, and Enrico Fermi on the results obtained in his pion-nucleon scattering experiments at the in-house synchrocyclotron. Reinhard Oehme presented his results on the connection by analytic continuation of particle-particle and antiparticle-particle scattering amplitudes and on the corresponding dispersion relations for pion-nucleon scattering then freshly obtained by him, Murph Goldberger, and Hironari Miyazawa. These dispersion relations agreed nicely with the experiments carried out by Fermi's group. In his own relaxed, spontaneous, and inimitable way Gregor moderated these discussions and kept asking pertinent questions. Sometimes he would invite outsiders—a visit from Hans Bethe comes to mind—in order to explain points in their work.

At the Fermi Institute, or Institute for Nuclear Studies as it was then called, there was a weekly seminar where the institute's members presented their results. In the beginning Enrico Fermi, Willard Libby, Harold Urey, and Gregor Wentzel jointly led this seminar. Gell-Mann's first talk ever about strangeness was presented there. With Fermi's death in 1954 and the subsequent departure from Chicago of Libby and Urey, Wentzel became the seminar's single leader. It was like no other seminar or colloquium. There was a fixed time for it, Thursdays an hour and a quarter before the Department of Physics colloquium, but beyond that absolutely nothing was planned. You went to this seminar prepared to talk for anywhere between 10 and 30 minutes about what you were working on, if and only if called upon. As the seminar started Wentzel would turn around in his front-row seat, look at one of the attendees, and invite him to speak (at the risk of admitting to political incorrectness, after Maria Goeppert-Mayer's departure from Chicago, there were no women in attendance in those ancient times). This was before the age of the laptop and even of the transparency, and people had to make do with chalk and blackboard, and being spontaneous, they had no notes along either. It was the most exciting seminar many of us ever attended. Chandrasekhar often reported the sensational news about quasars. As the *Astrophysical Journal*'s editor in chief he knew about these new spectacular findings long before they were made public. Very excited, he prefaced each report with a demand of confidentiality; he swore the audience to silence, as it were. It was high drama at the very frontier of science.

Gregor was a modest man and at all times very much the gentleman. He made his mark on physics both through his own important work and through his legacy as a teacher. This legacy, not unlike that of his own teacher, Arnold Sommerfeld, is remarkable for the brilliant physicists it helped shape. It is also remarkable for the great role Wentzel's book has had in setting the direction of postwar research in theoretical physics.

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