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OF

HENRY SEELY WHITE

1861–1943

FOREWORD BY

ARTHUR B. COBLE

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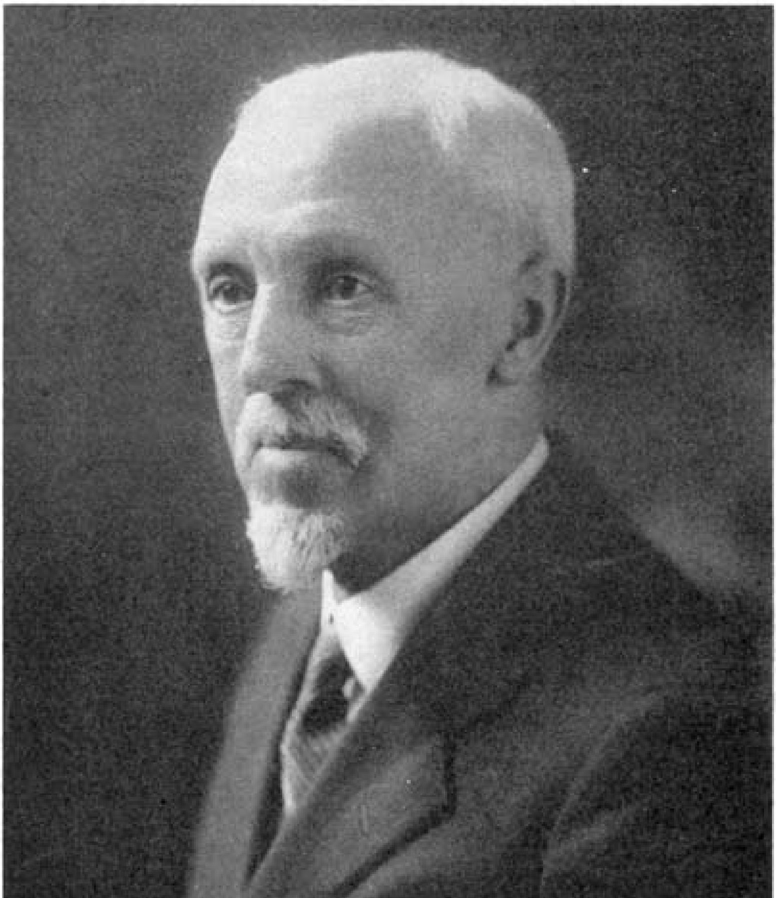

Henry S. White

HENRY SEELY WHITE

1861–1943

FOREWORD

Henry Seely White was one of perhaps a dozen men who furnished the inspiration and set the pattern for the development of the present school of American mathematics. His most important personal contribution to this movement was the establishment of the Colloquium Publications of the American Mathematical Society, now in its twenty-eighth volume. A detailed account of his services and of his contributions to mathematics has appeared in Raymond Clare Archibald's Semi-centennial History of the American Mathematical Society in which forty references to Professor White's activities appear.

Professor White was born May 20, 1861, was elected to membership in the National Academy of Sciences in 1915, and died May 20, 1943.

He had deposited in the archives of the National Academy of Sciences a manuscript autobiography. This charmingly modest account of the way of life of a scholar and gentleman of his period is reproduced here with the exception of some references to details of his own work. It is a valuable supplement to more formal histories. Mathematicians particularly will be interested in the picture it presents both of Professor White's personality and of the time in which he lived.

ARTHUR B. COBLE.

A brief account of the life of

HENRY SEELY WHITE

written by himself (1942) for the records of the

National Academy of Sciences

My father was Aaron White (1824-97) of Paris and Cazenovia, N. Y., who traced his ancestry to Elder John White and his eldest son Nathaniel, early settlers of Cambridge (Newtown) Mass., later founders of Hartford and Middletown, Conn. Aaron was born at Paris, N. Y., the third of six sons of Roderick White and Lucy Blakeslee. Following the example of his next older brother, Moses Clark White (1819-1900), at the age of 23 he left home and studied at Cazenovia (N. Y.) Seminary and Wesleyan University, Middletown, Conn., graduating with the A. B. degree in 1852. From that date he was a teacher of mathematics, sometimes of physics, mostly at Cazenovia Seminary. There he married in 1859 Isadore Maria Haight (1835-1905), a daughter of William Henry Haight and Cornelia Cushing.

This discloses in my line of descent Enos Cushing, who migrated from Hingham, Mass., and was one of the first settlers in Fenner, Madison Co., N. Y. He lived a long and useful life as land surveyor. Cornelia was one of his four daughters, all of whom lived and died near their birthplace. One of his grandsons, Frank Cushing, attained a reputation as a student of the customs and traditions of the Zuni Indians, and a research member of the Smithsonian Institution.

On my father's side, three brothers went into carpentry, building, and architecture; the other brother died in childhood. Moses Clark, his next older brother, became a physician, missionary to China, and Professor of Pathology and Microscopy at Yale University. Of another brother, Joseph, the eldest son, Andrew Curtis White (1854-1936), graduated at Hamilton College, 1881, and became Instructor in Greek and Assistant Librarian at Cornell University, where he attained the Ph.D. degree in 1888. The five sisters of Aaron White married farmers, some of them teaching in the public schools for a time.

My own two sisters, Cornelia Cushing and Lucy Blakeslee White, were graduated from Cazenovia Seminary; Lucy also from Wellesley College (1891). Cornelia became a librarian, for a time at the Crerar Library, Chicago, and later at Cazenovia Seminary, where, until her death (at age 70), she was also Alumni Secretary. Lucy married Charles Burton Thwing, Ph.D., a physicist, who taught successively at Northwestern University, the University of Wisconsin, Knox College, and Syracuse University, and became finally a manufacturer, chiefly of pyrometers, at Philadelphia, Pa.

Village life was mostly free from excitement, so that school was the chief interest of my early days. Until I entered college, at seventeen, we lived in Cazenovia, Sanquoit, and Canastota, all villages of fewer than 1500 inhabitants. I had begun to read at three, and studied the names on a large Civil War map that hung in the dining room. With improving memory, I learned Bible lessons for Sunday School, and took part in reading aloud a Scripture lesson at daily morning prayers. In my father's private academy at Sanquoit from '65 to '69, and in district school later, arithmetic was my main interest. There also I began algebra under my father's instruction; this was good fortune, for few students have contact with a first class mathematical mind at that early stage. He taught "Natural Philosophy" also, with a fair supply of apparatus, demanding a fair written account of every experiment. At home he let me help a little in preparing specimens for the compound microscope, and introduced me also to rocks and fossils which abounded in those regions. Latin I began at 13, Greek at 14, German at 14. At 17 I had finished preparing for college, having enjoyed the last two years under excellent teachers at Cazenovia Seminary. I doubt whether there was in my secondary school a more efficient teacher of Greek and Latin, for syntax, vocabulary, and memory drill, than Isaac N. Clements, A. M. From college (Wesleyan University) he had gone direct to the army in the Civil War, returned with one wooden leg, and devoted all his active years to teaching. A kindly, sardonic humor and a fine English style made his work doubly effective. Geometry proved easy and fascinating for me, and in my first year at the Seminary I was successful in the final examination for the Watkins prize in that subject. Learning the use of ordinary drawing tools from my father, I spent my leisure hours in vacation in following out the advanced constructions and problems in the old Young's Geometry, with special interest in regular polygons and polyhedra.

I should have mentioned that my father's good judgment had taken me out of school for half of my fourteenth year, setting me to work at carpentry under the tuition of my uncle Frederick S. White, who contracted for and superintended the construction of a large barn near my home in Canastota. I am confident that this manual training was of value, strengthening my intuition of geometric forms in three-fold space. In other summer vacations, up to the end of my undergraduate course, I worked at all kinds of farm labor on the lakeside farm of my grandfather, Wm. H. Haight, a short distance north of Cazenovia.

At seventeen I entered Wesleyan University, Middletown, Conn., as a freshman, following my father at the interval of thirty years. Greek and Latin were required for two years, also mathematics, which I elected for the last two. Elementary courses were required in physics, chemistry, geology, physiology, and astronomy, to which I added qualitative analysis under Atwater, physics with M. B. Crawford, and practical astronomy with Van Vleck, all teachers of wide learning and experience. I had the good fortune to learn something of Van Vleck's work in preparing the tables of the moon's position for the American Nautical Almanac, thus getting some idea of systematic computation. In the first two years I competed successfully for two mathematics prizes and one in Latin. In Greek I was less fortunate.

Our tutor in psychology and logic was John P. Gordy, later professor in Ohio State University and in New York University. He directed my reading toward Hamilton, Berkeley, Mill, and Spencer, giving me special honors at graduation in philosophy, and a junior year prize in logic, also half of the ethics prize in senior year. This study of course coordinated well with mathematics, in which Van Vleck gave me a start in quaternions as extra work, and special honors at graduation. My average rank in the whole course was just sufficient for first honor in general scholarship, a rank shared with three others in our class of twenty-seven.

The faculty kindly appointed me Assistant in Astronomy and Physics for my first year of graduate study, with a small stipend. I joined undergraduate groups in conic sections and in Hume's philosophy, assisted in preparations for observing the transit of Venus (1882), operated the 12-inch equatorial for undergraduates, and took series of observations for latitude with the meridian transit instrument. Also I assisted my father in collecting scientific notes for the *Northern Christian Advocate* (Syracuse). For mathematical reading I began the Theory of Equations (Burnside and Panton) and Muir's book on Determinants. Under Van Vleck's supervision, with a circle of seniors, I read part of Gauss's Theoria Motus Corporum Coelestium.

This busy and profitable year was broken up in the last three months by teaching, as substitute for my classmate George Prentice, ill health compelling him to resign the professorship of mathematics at Hamline University, St. Paul, Minn.

During the year 1883-4 I taught mathematics and chemistry, with short courses in physics and geology, at a secondary school, Centenary Collegiate Institute, at Hackettstown, N. J. This gave me valuable pedagogic training under a veteran principal, Rev. Geo. H. Whitney. To his recommendation I owe, in some measure, my appointment for the following three years (1884-7) as Tutor in Mathematics and Registrar at Wesleyan University, my Alma Mater. My duties included class room teaching and field practice in land surveying. In the scant leisure time I continued my latitude observations and some computation work with Van Vleck. During one summer vacation, 1884, I made the decennial "charter survey" and map for the village of Manlius, N. Y., and thought hopefully of the surveyor's profession. But the advice of my close friend Wm. J. James and of Professors Van Vleck and Crawford roused my ambition for advanced study of elliptic functions, using the book of Durege, and a visiting fraülein gave me conversation lessons in German (1886-7).

My first intention had been to study mathematics in the University of Leipzig, where Sophus Lie was then developing his Gruppentheorie, and E. Study was lecturing on algebra. But after one summer there, devoted principally to language practice and grammar with a law student from Braunschweig, I shifted to Göttingen, enjoying a foot tour through the Harz Mountains on the way. This I did again under the advice of William J. James (later Instructor in Mathematics and for many years Librarian and Assistant Treasurer at Wesleyan University), who had already studied four years under Felix Klein at Leipzig and Kronecker and Fuchs at Berlin. He held up Klein as not only a leading research mathematician but also as a magazine of driving power, whose students received personal attention and stimulus, and in most cases became themselves productive investigators. This was valuable advice. Klein received me kindly and admitted me to his seminar course, then just beginning, in Abelian Functions. Other Americans working with him at the time were Haskell, H. D. Thompson, H. W. Tyler, Osgood of Harvard, and a year later Maxime Bôcher, all friendly and helpful to the less experienced neophyte. Bolza, Maschke, and F. N. Cole, all whose names are now classic in American mathematics, had left the year before. Klein expected hard work, and soon had in succession Haskell, Tyler, Osgood, and myself working up the official Heft or record of his lectures, always kept for reference in the mathematical Lesezimmer. This gave the fortunate student extra tuition, since what Klein gave in one day's lecture (two hours) must be edited and elaborated and submitted for Klein's own correction and revision within 48 hours.

Within six months we were each set to work on special topics and reading related papers, all in close connection with the seminar and lecturing on our findings before the whole seminar group. After my second year, results proving plentiful, Klein set me to formulation for publication and to reviewing for examination. He kindly published in the *Mathematische Annalen* a condensed account of my problem and chief results (vol. 36), and later procured the insertion of my Thesis *in extenso* in the *Nova Acta* (vol. 57) of the Leopold-Karoline Academy of Sciences at Halle.

Meantime I had followed lectures by H. A. Schwartz, his regular course on function-theory; by Schönflies on projective geometry and curve-theory; by Georg Elias Müller on psychology and *naturphilosophie,* and Baumann on history of philosophy. Müller used me also for a subject in his new psychological laboratory, and helped me with a seminar lecture on Hume's "Untersuchungen über den menschlichen Urstand" (a translation). But equal profit also accrued from my daily study of Clebsch's Vorlesungen über Geometrie (by Lindemann), and Reye's Geometrie der Lage. Those both contributed a secure background to the university lectures. The biweekly sessions of the Mathematische Gesellschaft also were open to us. Kant's *Kritik der Reinen Vernunft* of course was prescribed reading, and in this I had the good fortune to combine with the late John C. Schwab, later Professor and Librarian at Yale, who was preparing for his degree in economics.

My examination came early in March, 1890, with Klein and Müller as chief examiners. Psychology was my second department. The Pedell, conveying keyhole information, calmed my apprehensions by citing the advocacy of those two good friends, and I suppressed as useless the regrets over the more brilliant success that I might have secured by more extended coaching. To the training and encouragement of Felix Klein and G. E. Müller I owe a large part of my good fortune in attaining both desirable teaching positions thereafter, and the "equanimity" (Sir Wm. Osler's name for it) and intellectual satisfactions that accrued in my long experience as teacher and investigator. Many American students can testify similarly of their indebtedness to Klein; not so many to Müller's influence, since psychology had in those years fewer devotees. Yet he was a worthy successor to Lotze and Herbart, and produced work of permanent effect in renovating his chosen science.

Returning home in March, 1890, I took for the spring quarter a temporary position in the preparatory department of Northwestern University at Evanston, Ill., secured for me by my friend and classmate Joseph R. Taylor, who taught Greek in the same school. (He afterwards filled for more than forty years a professorate at Boston University).

Clark University was opening in that year its career, full of high aims and hope, at Worcester, Mass. Its president, Dr. G. Stanley Hall, had spent the previous year visiting universities and scholars in Europe, and looking up available men for his institution. Klein had introduced me and given some flattering testimonials, and Dr. Hall offered me now a place as Assistant in Pure Mathematics under Prof. Wm. E. Story, with Henry Tabor and Oskar Bolza as colleagues. Though the salary was hardly adequate for subsistence, I accepted it eagerly in spite of kind offers from Evanston and Middletown. My teaching was mainly algebraic and projective geometry, and the invariant-theory of linear transformations.

In 1891 I refused an invitation to do similar work at Johns Hopkins University, but in 1892 accepted an associate professorship at Northwestern. The inducements were, first a better salary with assured permanency, and second, proximity to the new University of Chicago and my highly valued friend E. Hastings Moore, its new Head Professor of Mathematics. He indeed tried to bring me into his department but could not secure sufficient appropriation. For my labors at Clark I had secured two short papers that were accepted by the *American Journal of Mathematics,* and a considerable number of new friends among our ambitious younger mathematicians. Some few new problems, too, I had begun to formulate. In particular, in the seminar of Professor Wm. E. Story we had discussed the epoch making paper of David Hilbert of Königsberg (later Göttingen) on Algebraic Forms, embodying the theorem never before enunciated in so general form, that every given sequence, finite or infinite, of algebraic forms in any domain of rationality must have a *finite* number of such forms constituting a "Basis" in terms of which all the rest are linearly expressible with entire algebraic forms as coefficients. Gordan's proof of the finiteness of the form-system of covariants of any set of binary ground forms was subsumed as a primary corollary. Along with this, Story presented the enumerative work of Sylvester and Franklin on the same (binary) subject. This was fundamental and fruitful.

The new president of Northwestern University, Henry Wade Rogers, had called a number of younger scholars to important chairs, all of about the same age, men with the Ph.D. degree, trained at Johns Hopkins or Harvard or in German universities; and the "atmosphere" was favorable to research and writing. Most of the older professors were of similar fiber, and the college enjoyed an era of marked growth.

My second year in Evanston, 1893-4, was memorable for the Columbia World's Fair, held at Jackson Park on the lake shore at the southern end of Chicago; and for the so called "Congresses" held in connection with it, mostly, at the Fine Arts Building on the lake front at Randolph Street. I served on the committee for the Congress of Mathematics, along with the three professors from the University of Chicago: Moore, Bolza, and Maschke. Professor Felix Klein was deputed by imperial authority to accompany the German exhibit of mathematical books, photographs and apparatus, and he graciously accepted the hospitality of my wife and myself, commuting daily by railroad to the city, 12 miles. A full account of this Congress and the participants was published by the Macmillan Company for the Mathematical Society. Klein gave in the following two weeks twelve lectures, chiefly on the topics covered in his courses at Leipzig and Göttingen. It had been planned that these should be given for one week at the University of Chicago, and one week at Northwestern University. But poor drains and wet weather had flooded the campus at Chicago and all were transferred to Evanston. The University furnished auditorium and reference library and opened a dormitory on the campus for the convenience of listeners. Professor Alexander Ziwet of Michigan kept a full report, which with Klein's emendations was published under the title "The Evanston Colloquium," and achieved wide circulation. A French translation also had a large sale. The whole series, with the Congress and the exhibits at the Fair, gave a strong stimulus to mathematical research and reading in this country.

In the comparative leisure of the following year, '93-'94, I worked out a theme and a paper for the *American Journal of Mathematics,* for Semi-combinants and Affiliants (this latter term a new one). . . .

In the next ten years I worked out some problems of minor interest, connected with plane cubic curves, elliptic functions, and apolarity. Extending the Euler polyhedron theorem to surfaces of positive deficiency, I tabulated regular systems of divisions upon them, admitting of course curved lines as boundaries. These were based on existence theorems due to Riemann, concerning canonical "Querschnittsystems." The distinction of primitive and derivative systems proved interesting, and I made models, with the help of Professor O. H. Basquin, of all regular sets of divisions upon surfaces of low deficiency. Of my interest in the American Mathematical Society there is a fairly full account in the history, by Professor R. C. Archibald, of the first fifty years of that Society. Chiefly useful was the scheme for holding frequently a colloquium, or week of advanced lectures on special theories, in connection with summer meetings of that Society. The first was held in '96 and I was asked to conduct a course in 1903, this was published as part of "The Boston Colloquium," by the American Mathematical Society, the first of an extensive series. Osgood, Bolza, Bocher, Pierpont, and Webster, by their generous labors, gave this Colloquium series a high standing at the start and made it a valuable feature of American mathematics.

Becoming interested in the work of Picard and Poincare on integrals upon algebraic surfaces, an extension of Riemann's work on Abelian Functions to functions of two independent variables, I secured leave of absence for a half-year's study in Europe, Feb.–Aug. 1901. With this I combined the plan of writing an introductory book on Plane Cubic Curves, a long task which was not completed until 1925. By Klein's advice, I went to Turin and listened to part of a course by Professor Corrado Segre upon Algebraic Surfaces, treated by synthetic geometric methods. This required a hasty acquisition of some slight speaking knowledge of Italian, a valuable addition to my range. In Turin also I enjoyed a slight acquaintance with D'Ovideo, Peano, and Severi. After the close of the summer semester there, I spent the balance of my free period in Göttingen, busied principally with my manuscript on Cubics. Returning via Liverpool and Boston, I saw for the first time a few points in England, a brief glimpse of London and Cambridge, with a bicycle tour through Oxford, Stratford, Nantwich, and Chester (with Hawarden). This was the third such tour I had enjoyed, bicycles being then at the acme of their popularity before the advent of automobiles. The first two were shared with my friend E. H. Moore, one from Michigan City via Indianapolis, Brookville, Cincinnati to Columbus (1899) and return via Marion, Lima, and South Bend; the other from New York (1900) to Hudson, Great Barrington, Williamstown, Amherst, Worcester, Boston, Providence, and New Haven. Some few other trips I had made alone, far less pleasant and profitable.

During the decade, 1890-1900, occurred my marriage to Mary Willard Gleason of Hartford, whom I had known in college days at Middletown, daughter of the banker Frederic Lathrop Gleason; and the birth of my three daughters. The very efficient domestic and social ability of my wife, a musical and poetic genius, furnished stimulus and sheltered environment for my professional activities. Whatever we accomplished in this ideal partnership was due very largely to her ambition and management.

In 1905, in order to be nearer the home of my mother, who was seriously ill, I accepted the offer of the headship of the mathematics department in Vassar College, under the presidency of Dr. James Monroe Taylor. This availed little, so far as my mother was concerned, for she died during that summer, before we had settled in Poughkeepsie. Besides the usual undergraduate courses we conducted advanced courses and research with a few graduates who were attracted by endowed fellowships. After a few years, however, the trustees altered the conditions, believing it more profitable for graduate students to pursue such studies in larger groups at some of the larger institutions which were then in process of evolution. Our small departmental staff, however, cooperated effectively in promoting advanced reading and research.

My chief interest, increasing for many years after 1910, was in so-called Triad Systems and their groups. This subject became of importance in the theory of substitution-groups on a finite number of elements, either $6n + 1$ or $6n + 3$, where n is an integer. Up to $n = 2$ there is only one way to arrange the triads (in a set of 3, 7, or 9, elements); but for 13 elements it was known that two different, dissimilar arrangements are possible. The definition prescribes namely that every possible pair of the given elements shall occur in one, and in no more than one, triad of the system. Two systems are considered duplicates when each can be derived from the other by simply re-naming the elements. Beyond this, Moore had proved that for all larger values of n there are at least two different triad systems, distinguished by non-conformable groups. . . .

A geometrical application that proved to be important involved duality in three-space. Seven elements of a triad system being represented by points, the triads may naturally be pictured as planes, each determined by 3 points. Now 6 points determine a twisted cubic curve which contains them. If the seventh point of a set lies also on that curve, I found that the seven planes representing any triad system on those points will all osculate a second twisted cubic curve. This gives a theorem for 7 points of a gauche cubic, analogous to the Pascal theorem for 6 points of a plane quadric curve. But further, the two sets, points and planes, have the poristic property. With one degree of freedom, the 7 points may move along their cubic in such a way that the 7 planes continue to osculate their cubic curve. This gives us a link between two gauche cubics, quite similar to Poncelet's polygons connecting two conics in a plane. Coble (A. B.) gave an extensive theoretic treatment of this situation, calling it the only important generalization of Poncelet's problem. (See citations for both Coble and White in Archibald's History, l.c.).

Other brief geometrical studies have been intended rather to fill gaps or to supplement the work of earlier writers than to open new lines of research. Thus Hurwitz's theorem on two tetrahedrons inscribed to one twisted cubic and circumscribed to another, was extended to two $(n + 1)$-gons inscribed to a norm-curve in n-space. A well known set of points in 3-space are the 8 points in which three independent quadric surfaces intersect; from any seven of them the eighth can have its coordinates calculated rationally. Though this was proven, the explicit formulae had not been given. The result of somewhat tedious labor revealed new relations.

Curiously enough, while algebraic curves and surfaces have been staple objects of study, the degenerate forms, sets of planes or lines, have been neglected, beyond five lines in a plane or six planes in three-space. With Miss Cumming's cooperation I examined the divisions that could result from 6 or 7 real lines, or from 7 real planes. The methods were not less interesting than the results. More recently I have found special interest in skew hexagons and their close connection with the permutations of six marks or symbols. The desideratum is, to find means of skeletonizing the picture, or substituting a framework for the less easily apprehended, fully explicit intuition. For example, in the case of polyhedral division of space by planes, faces of fewer than 5 sides can be ignored in the diagram; i.e., only pentagons and hexagons need be studied with their connectivity.

In 1933, at the age of 72, I was, nominally at least, relieved of part of my teaching obligations, being made Professor Emeritus and Senior Lecturer. Three years later the latter title was withdrawn. Since that time I have been living on a small farm with the family of my youngest daughter, Mary White Perez, devoting some thought to gardening and apple culture, and to admiration of the noble range of the Catskill Mountains.

What further work or experience I may have that will be worth noting, I cannot foresee. A list of honors of various kinds that have fallen to me is given in Archibald's History, l.c. It omits the compliment of an assignment to write two articles for the Encyclopædia Britannica. (1926).

After my decease, if the custom of printing memorial biographies is continued, I think some competent reporter or friend can extract and condense from this rambling narrative three or four pages which will suffice.

April, 1942.

KEY TO ABBREVIATIONS USED IN BIBLIOGRAPHY

AJM = American Journal of Mathematics

AM = Annals of Mathematics

AMM = American Mathematical Monthly

AMS Bull. = American Mathematical Society Bulletin

AMS Colloq. Pub. = American Mathematical Society Colloquium Publications

AMS Trans. = American Mathematical Society Transactions

Astrophys. J. = Astrophysical Journal

Enc. Brit. = Encyclopædia Britannica

MA = Mathematische Annalen

NAS Mem. = National Academy of Sciences Memoirs

NAS Proc. = National Academy of Sciences Proceedings

Sci. Mo. = Scientific Monthly

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