George W. Whitehead Jr.
1918–2004

BIOGRAPHICAL *Memoirs*

A Biographical Memoir by
Haynes R. Miller

NATIONAL ACADEMY OF SCIENCES

GEORGE WILLIAM **WHITEHEAD JR.**

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Elected to the NAS, 1972

By Haynes R. Miller

Life

George William Whitehead, Jr., was born in Bloomington, Ill., on August 2, 1918. Little is known about his family or early life.

Whitehead received a BA from the University of Chicago in 1937, and continued at Chicago as a graduate student. The Chicago Mathematics Department was somewhat ingrown at that time, dominated by L. E. Dickson and Gilbert Bliss and exhibiting "a certain narrowness of focus: the calculus of variations, projective differential geometry, algebra and number theory were the main topics of interest."[1] It is possible that Whitehead's interest in topology was stimulated by Saunders Mac Lane, who spent the 1937–38 academic year at the University of Chicago and was then in the early stages of his shift of interest from logic and algebra to topology. Of greater importance for Whitehead was the appearance of Norman Steenrod in Chicago. Steenrod had been attracted to topology by Raymond Wilder at the University of Michigan, received a PhD under Solomon Lefschetz in 1936, and remained at Princeton as an Instructor for another three years. He then served as an Assistant Professor at the University of Chicago between 1939 and 1942 (at which point he moved to the University of Michigan).

The timing was fortunate for Whitehead, but it seems that he spent the academic year 1939–40 at the University of Tennessee as Instructor, and so spent only one year in residence at the University of Chicago studying with Steenrod. In any case, Whitehead received his doctorate in 1941 from the University of Chicago, as Steenrod's first PhD student, with a thesis entitled "Homotopy properties of the real orthogonal groups." As we shall see, the homotopy theoretic properties of Lie groups continued to fascinate Whitehead throughout his career.

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1 https://math.uchicago.edu/about/history.shtml, accessed Sept 27, 2015.

Following his graduation, Whitehead spent four years at Purdue University as an Instructor, followed by two years (1945–47) as Fine Instructor at Princeton University.

In 1947 George Whitehead married Kathleen Butcher. Kay, as everyone always called her, received her PhD in topology that year from the University of Michigan under the direction of Raymond Wilder. Kay was as outgoing as George was reserved. She taught at a variety of Boston area schools—Wellesley, Brown, Smith, Boston University—before settling down, in 1959, at Tufts University, from which she retired in 1985.

Also in 1947 Whitehead was appointed as an Assistant Professor at Brown University, and promoted to Associate Professor the following year. In 1949, however, he moved to the Massachusetts Institute of Technology as Assistant Professor, and was duly promoted to Associate Professor in 1951 and Professor in 1957. He remained at MIT for the rest of his career, becoming Professor Emeritus in 1985 at what was then the mandatory retirement age of 67.

Whitehead acquired his first graduate student already at Brown, in the person of John C. Moore, who followed him to MIT and graduated in 1952 with a degree from Brown. Other early students of Whitehead were Edgar Brown (1954) and Robert Aumann (1955). Aumann wrote a thesis on knot theory under Whitehead's direction, but quickly moved into game theory and economics, and in 2005 won the Nobel Prize in Economics. Altogether Whitehead directed thirteen PhD students, the last finishing in 1978.

George Whitehead was in person gracious, unassuming, and always dressed in a jacket and tartan tie. His office was next door to the department common room, and, in line with the habit of the department, his door was always open. At tea he would often participate with Frank Peterson and others at the bridge table. He and Kay were close friends with another mathematical couple, Richard and Alice Schafer, who shared their passion for bridge. The annual Christmas party George and Kay hosted at their house on Bellevue Road in Arlington always drew a large fraction of the Boston area mathematics community. Acquaintances were renewed over surprisingly powerful "sidecars," and table tennis was played in the basement.

George and Kay loved England and spent every sabbatical there. In 1955–56 Whitehead was a John Simon Guggenheim Memorial Fellow and Fulbright Research Scholar at Oxford University. In 1965–66 he was an NSF Senior Postdoctoral Fellow at the University of Manchester. He spent a sabbatical at Birkbeck College, London University,

in spring, 1973, and at Imperial College, London University, and Oxford University, in 1981. He had a long term interest in genealogy, but found himself frustrated in chasing down his own ancestry in England. As it happened, Kay had Whitehead ancestors as well, who turned out to be easier to trace. They also took a number of cruises with an archeological orientation and British clientele.

Whitehead was elected a Fellow of the American Academy of Arts and Sciences in 1954, and to the National Academy of Sciences in 1972. He was active in the American Mathematical Society, serving as a member of the Council (1953–55), as Vice President (1978–79), and on the Nominating Committee (1982). Whitehead served as Associate editor of the *Proceedings of the American Mathematical Society* (1948–50), and on the editorial board of the *Illinois Journal of Mathematics* (1956–62).

George Whitehead showed great courage in dealing with a disfiguring cancer in his last years. He died quite suddenly, on April 12, 2004. His widow Kay Whitehead died five years later, on April 18, 2009.

Work

George Whitehead was instrumental in shepherding homotopy theory through its troubled adolescence, and played a key role in the gradual algebraization of the subject.

According to Whitehead himself [1983], homotopy theory may be said to have had its origins in the discovery, by Heinz Hopf in 1931, of essential maps between spheres of different dimension. In this construction Hopf exploited the number systems given by the complex numbers, the quaternions, and the Cayley octonions, and provided an invariant—generalizing the linking number—by which to detect them.

In his thesis Whitehead reinterpreted and generalized Hopf's construction, showing how homotopy groups of the orthogonal groups could be mapped into homotopy groups of spheres, via the "*J*-homomorphism." Whitehead's early work showed a genius for writing down explicit formulae, and the orthogonal groups and Lie groups more generally afforded a great arena for that talent. George Whitehead's *J*-homomorphism played a central role in geometric as well as algebraic topology in the 1960's and 1970's, and its study provided an organizing principle throughout that era.

The pioneering work of Hans Freudenthal (1937) identified the "stable range" of homotopy theory. Whitehead took the next step, and found means to analyze the "metastable range." He extended the scope of Hopf's invariant in the form of a map between

homotopy groups of different spheres. He introduced the "EHP sequence" relating Freudenthal's suspension (or Einhangung), his extension of the Hopf invariant, and a form of J. H. C. Whitehead's product, and eventually proved its exactness in the meta-stable range. (This set-up was later extended to all dimensions by Ioan James.) George Whitehead skillfully combined the detection mechanism provided by his Hopf invariant with the constructive device afforded by the *J*-homomorphism to produce many new nontrivial maps between spheres—enough, for example, to show that spheres may have non-cyclic homotopy groups.

In 1938 Lev Pontryagin announced, without giving details, that his geometric theory of framed bordism implied that $\pi_5\left(S^3\right) = 0$. It is interesting to watch the evolution of Whitehead's attitude towards this claim: from acceptance and use (in his thesis), to statement with reservation, to flat out disproof (in 1950). Whitehead's first ICM talk, in Cambridge, 1950, was on this topic. While Pontryagin published a correction at about the same time, Whitehead's resolution of the issue was regarded as a victory for the "algebraic method" in topology.

Whitehead was fascinated by the algebraic structure on homotopy groups provided by the Whitehead product. From his point of view, it determines the homotopy of a bouquet of two spheres in the metastable range (a result later extended to all dimensions by Peter Hilton and John Milnor). He proved that it is killed by suspension, and combined it with his Hopf invariant to describe the failure of distributivity of addition over composition in homotopy groups. In 1954 George Whitehead proved that the Whitehead product satisfies a graded form of the Jacobi identity. While several other proofs emerged at the same time, Whitehead's proof is universally acknowledged to be most perspicacious of the lot, relating Lyusternik-Schnirelman category of the space in the source to the nilpotence class of the group of homotopy classes of maps to a topological group.

In his early work Whitehead preferred explicit methods over more indirect approaches using homology. However, as new homological tools appeared—Steenrod's construction of cohomology operations and the French revolution led by Jean Leray, Henri Cartan, and their students—he adopted them with enthusiasm. He defined the tower of connective covers of a space, independently of the work of Cartan and Serre, and used them to interpret Eilenberg's subcomplexes of the singular complex of a space.

As Cartan and his student Jean-Pierre Serre, and then Hirosi Toda, took over the computational aspects of homotopy theory, Whitehead's attention turned to more structural

issues. His magisterial 1962 paper "Generalized Homology Theories" set the framework for stable homotopy theory—using the notion of "spectra" that had been introduce by Elon Lages Lima a few years earlier—that has been in use ever since. He supplanted the "join" operation used in his earlier work with the "reduced join" or "smashed" (now simply "smash") product, and used this formalism to describe and prove a represent-ability theorem for generalized homology theories by spectra. Duality now played a fundamental role, and was set in its proper context. The representability theorem of Whitehead's student Edgar Brown found its natural place in this theory.

Another novelty of the late 1950's was the advent of simplicial methods in algebraic topology. Whitehead's first student John Moore was a pioneer in this work, as was Daniel Kan, who joined Whitehead on the MIT faculty in 1959. By 1961 Kan and Whitehead were publishing joint papers developing a simplicial theory of spectra and using it to extend the smash product to spectra. They defined a hierarchy of degrees of orientability, using the lower central series from group theory. This work was at the center of White-head's second ICM talk, in Stockholm (1962). I think it's fair to say that it has yet to be incorporated into contemporary stable homotopy theory.

An important part of Whitehead's influence on the development of homotopy theory was his generous sharing of his hopes for the future. In particular the "Whitehead conjectures" have played a seminal role. One of his conjectures dealt with his J-ho-momorphism: he hoped that the J-homomorphism induced a surjection from the stable homotopy groups of the orthogonal group to the stable homotopy of spheres, in positive dimensions. The 2-primary part was shown to follow from a conjecture due to Mark Mahowald, asserting that the "transfer map" provides a surjection from the stable homotopy of projective space onto the 2-primary stable homotopy groups in positive dimensions. Mahowald's conjecture was proven by Stewart Priddy and Whitehead's student Daniel Kahn. Kahn and Priddy proved an odd-primary version as well, but it does not imply Whitehead's conjecture in that case; and indeed Karlheinz Knapp showed in 1978 that Whitehead's conjecture is wrong at odd primes.

The Kahn-Priddy theorem is the first of a sequence of surjectivity results conjectured by Whitehead, involving the symmetric product filtration for the sphere spectrum. This "Whitehead conjecture" was eventually proved by Nick Kuhn in 1982, and has deep connections with currently fashionable topics in homotopy theory such as Goodwillie's calculus of functors.

Books

 Much of George Whitehead's later effort went into describing his subject in books and historical surveys. Whitehead's first book, *Homotopy Theory*, represented a faithful transcription, made by Robert Aumann, of a graduate course at MIT. While the book was only published in 1966, the course was given in the fall of 1953, and bore the imprint of that era. It ended with a brief treatment of spectral sequences; this must have been one of the earliest treatments of that subject in a course.

Whitehead gave a lecture series at New Mexico State University at Las Cruces in 1969–70, under the auspices of the Conference Board of the Mathematical Sciences, and followed up with *Recent Advances in Homotopy Theory*. This slim monograph gave an authoritative summary of the state of homotopy theory at that point in time, and captured the excitement of the moment. (The impact that Novikov's and Quillen's work on complex cobordism was to have on homotopy theory was not yet evident at that time, and no mention was made of it.) The Adams spectral sequence had entered the picture, and very recently had been extended to the unstable case by an assortment of Whitehead's colleagues at MIT. Whitehead described this, and reproduced computations for the spheres, stable and unstable. He himself also took up the computational opportunities opened up by this work, and reported on his extensive hand computation of the stable term for projective space. This computation, largely carried out during a transcontinental road trip with Kay, served as a check on the Mahowald-Whitehead conjecture.

Whitehead modestly entitled his magnum opus *Elements of Homotopy Theory* (1978), and in the first sentence of the introduction he asserted that "this book is concerned with the elementary portion of the subject of homotopy theory." This hefty book provides beautifully crafted and articulated treatments of the Hurewicz theorem, obstruction theory, the Blakers-Massey theorem, Postnikov systems, Whitehead products, the Hilton-Milnor theorem, and an introduction to the Serre spectral sequence. It is to this day the best place to learn about the Bott-Samelson theorem, the non-distributivity formulae for composition, and many other landmarks of classical homotopy theory. In an appendix Whitehead returned to some of his roots, describing the topology of Lie groups, with special attention to the Spinor groups and the exceptional Lie groups and . In the introduction, Whitehead promised a second volume, but, sadly, this was not to be.

ACKNOWLEDGEMENTS

STUDENTS

1952	John C. Moore
1954	Edgar H. Brown
1954	Arthur H. Copeland, Jr.
1955	Robert (Yisrael) J. Aumann
1955	Alan J. Goldstein
1958	John E. Kimber, Jr.
1958	Robert E. Mosher
1961	Michael Ginsburg
1964	Daniel S. Kahn
1966	Joel M. Cohen
1975	Jacques Labelle
1976	Leonard J. Nissim
1978	Peter J. Welcher

SELECTED BIBLIOGRAPHY

1950   A generalization of the Hopf invariant. *Ann. of Math.* 51(1):192–237.

1953   On the Freudenthal theorems. *Ann. of Math.* 57(2):209–228.

1954   On mappings into group-like spaces. *Comment. Math. Helv.* 28(1):320–328.

1962   Generalized homology theories. *Trans. Amer. Math. Soc.* 102(2):227–283.

1965   With D. M. Kan. The reduced join of two spectra. *Topology 3, Suppl.* 2:239–261.

1966   *Homotopy theory.* Compiled by R. J. Aumann. Cambridge, MA: MIT Press.

1970   *Recent advances in homotopy theory.* Regional Conference Series in Mathematics, No. 5. Washington, DC: Conference Board of the Mathematical Sciences.

1978   *Elements of homotopy theory. Graduate Texts in Mathematics 61.* New York: Springer-Verlag.

1983   Fifty years of homotopy theory. *Bull. Amer. Math. Soc.* 8(1):1–29.