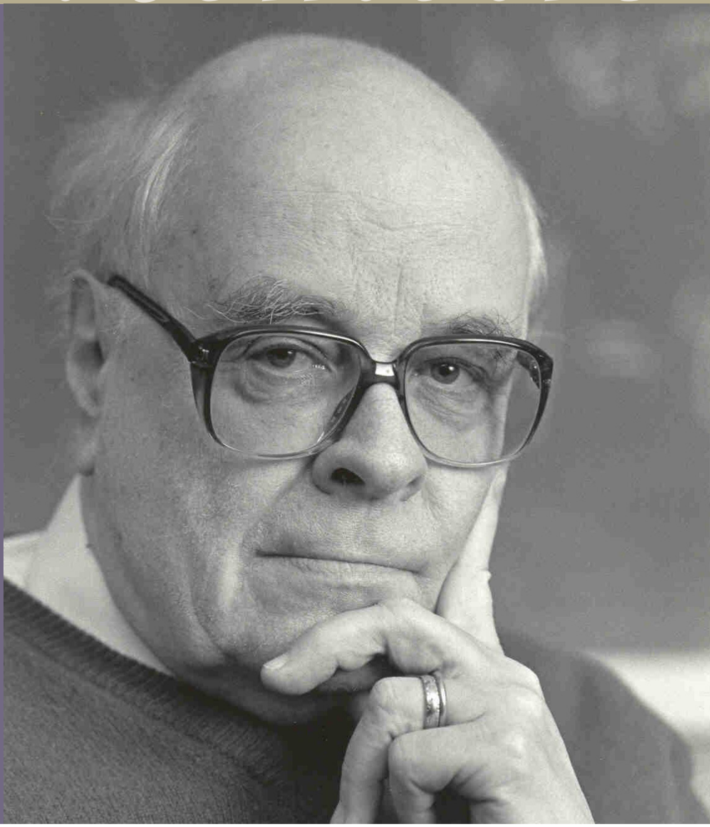
Ulf Grenander

1923-2016

BIOGRAPHICAL *Memoirs*



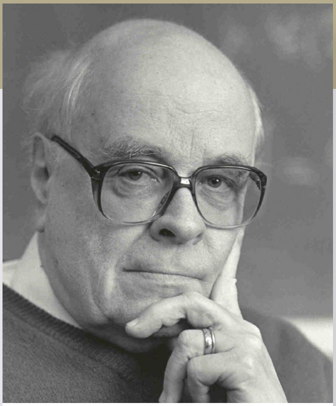
A Biographical Memoir by
Stuart Geman
and David Mumford

NATIONAL ACADEMY OF SCIENCES

ULF **GRENANDER**

July 23, 1923–May 12, 2016

Elected to the NAS, 1996



By Stuart Geman and David Mumford

Ulf Grenander was known for his seminal contributions to time series analysis and to the theory of high-dimensional statistical inference, and for the far-reaching marriage of stochastic and combinatorial structure that he called Pattern Theory. He developed path-breaking methods for modeling complex stochastic systems and pioneered the use of Monte Carlo computational methods in Bayesian statistics. He was as enthusiastic about applications as he was about building mathematical foundations and proving theorems, and his applied work led to many successes across diverse fields, including computer vision, signal processing, the actuarial sciences, and the computer-aided diagnosis of abnormal anatomical structure.

Grenander was born in 1923 in Västervik, Sweden, a small coastal town on the Baltic Sea where he maintained a house to which he and his wife Emma-Stina returned almost every summer for more than forty years. He was educated at a local school in grades three through twelve, following the classical tradition of intense studies in history, culture, and languages. The education suited him, and as a passionate reader, it continued through the rest of his life. Though the curriculum included very little mathematics or science, he also developed an early fascination with statistical and quantum mechanics. He was forced to pursue these topics on his own. This was complicated by Sweden's isolation during World War II, and most of the available up-to-date publications were written in German. Here his classical education came in handy: Grenander was already comfortable reading in German as well as in several other languages.

Grenander entered Uppsala University in 1942 and then transferred to Stockholm University to pursue mathematics, probability, and statistics. At Stockholm, he quickly matured as a mathematician, mentored by Harald Cramér and dazzled by the brilliant analyst and code breaker Arne Beurling, and later by Harald Bohr, mathematician and brother of famed physicist Niels Bohr. (The Bohr brothers, Danish Jews, had fled to

Sweden during the war.) Grenander described Bohr's lectures in functional analysis by quoting painter Henri Matisse, "The purpose of an artist is to decorate the surface," and went on to say,

> *Bohr did not have Power Point, but he had colored chalk. He would start writing at 10:15 in the upper left corner of the chalkboard and then using all sorts of possible colors he would draw his functions and other mathematical structures to fill the whole board. Exactly at 11:00, he would reach the lower right corner of the chalkboard and the lecture would come to an end.*

Grenander with former menthor H. Cramér. (Reprinted with the permission of The Institute of Mathematical Statistics.)

Following his graduation and prior to entering graduate school, Grenander served a year in the military, about which he said, "Military service was not so bad! I discovered the pleasures of outdoor life instead of being a nerd." After his service, he returned to Stockholm University and began his graduate studies as a student of Cramér. This was a remarkably fertile time for the fields of probability and mathematical statistics. Just twelve years earlier, Andrey Kolmogorov had transformed probability with his *Foundations of the Theory of Probability*.[1] The clarity and rigor of the measure-theoretic framework suggested new directions and highlighted pathways to generalizations of existing results. Cramér was among the earliest explorers. Another was William Feller, who resided in Stockholm in the late 1930s after fleeing Germany during the rise of fascism. Cramér had convinced Feller to move into probability theory rather than continue in differential geometry. Cramér himself focused on applying the new framework to classical statistics, which led to his 1945 ground-breaking *Mathematical Methods of Statistics.*[2]

Grenander began his graduate work that same year and eventually launched an ambitious effort to use Kolmogorov's framework to build a new theory for nonparametric statistics and time series. His thesis, "Stochastic Processes and Statistical Inference," was published in 1950, following an earlier paper, "Stochastic Processes and Integral Equations," in 1949.[3,4] Among the notable accomplishments were generalizations of likelihood methods to abstract parameter spaces (e.g., stochastic processes and nonparametric statistics), including a broad generalization of the Neyman-Pearson theory, and the development of new spectral tools for the estimation of "continuously-indexed time series," that is, stochastic processes. Applications have since found their way into control theory, identification for stochastic differential equations and other independent-increment processes, and in general, stochastic filtering and prediction.

Grenander (right) **with A. N. Kolmogorov.** (Reprinted with the permission of The Institute of Mathematical Statistics.)

Kolmogorov himself, who frequently visited Cramér, thought highly of the thesis. He not only encouraged Grenander to continue his work on stochastic processes but also shared his opinion widely. Many invitations followed. Grenander spent the 1951–52 academic year in the United States at the University of Chicago. He shared an office with Charles Stein and rented Leonard "Jimmy" Savage's apartment, where he discovered an excellent mathematics library and developed an unexpected interest in topological groups, which likely seeded his later work on limit theorems for probabilities on algebraic structures.[5]
In addition to Stein, he met many prominent statisticians at Chicago, including Bill Kruskal, Murray Rosenblatt, and Leo Goodman, as well as Joe Hodges, who was on leave from the statistics department at Berkeley.

While at Chicago, Grenander began a fruitful collaboration with Rosenblatt, which ultimately led to their influential book *Statistical Analysis of Stationary Time Series.*[6] Also while at Chicago, Jerzy Neyman invited him to spend the 1952–53 academic year at the University of California, Berkeley, where he met the well-known analyst Gábor Szegő, then-chair of Mathematics at Stanford University, at a joint seminar with Stanford. Szegő was best known for his work on Toeplitz forms and harmonic analysis, which built on earlier developments by Otto Toeplitz, Lipót Fejer, Constantin Carathéordory, and Frigyes Riesz. Grenander had his own reasons for studying Toeplitz forms, because they arise directly from the autocorrelation functions, and indirectly from the spectral representations, of stochastic processes. Those connections first appeared in a paper he published shortly after his thesis and later in a paper with Murray Rosenblatt.[7,8]

Szegő and Grenander struck up a collaboration. The connections between Toeplitz forms and stochastic processes brought new perspective and scope to a theory that had already existed for over forty years. The collaboration between Szegő and Grenander ultimately led to their 1958 book *Toeplitz Forms and Their Applications,* which was praised for both the developments of new theory as well as the unexpectedly rich applications to probability and stationary processes.[9] As Frank Spitzer noted in his review of the book, the synthesis could not have been made without a rigorous theory for prediction and estimation and therefore represented yet another triumph of Kolmogorov's formulation.[10]

Grenander spent most of the years from 1953 until 1966 at the University of Stockholm, eventually becoming the director of the Institute for Insurance Mathematics and Mathematical Statistics. During these years he continued, in the manner of his thesis, to extend the boundaries of probability and mathematical statistics. In 1963, he published *Probabilities on Algebraic Structures*,[11] in which he studied extensions of the standard limit theorems for sums of random variables to sample spaces with more general algebraic structures, including Lie groups, Banach spaces, compact groups, and semigroups. Grenander's motivation almost invariably traced back to a desire to understand physical phenomena and to expand the scope of practical tools. But at the same time, his preferred approach was to seek clarity through generalization and abstraction.

Grenander (left) and M. Loève. (Reprinted with the permission of The Institute of Mathematical Statistics.)

While still in Sweden, he made many more seminal contributions to statistical inference and indulged an abiding interest in real-world problems by consulting for the insurance industry. In this way, he was following the path of his advisor, Cramér, who had worked for the industry throughout most of his career. Cramér drew much of his motivation for a more rigorous theory of probability from his experience in the industry. (A notable example was his effort to address the ruin problem, which led him to the first results in what was to become large-deviation theory.) For Grenander, the experience included an early encounter with the problem of abstract inference, which would later occupy a great deal of his attention. He had already explored many inference problems arising from observations in abstract spaces, such as parameter estimation from samples of a stochastic process. But what happens when the parameters themselves are abstract objects, such as functions or operators? An actuarial example would be the mortality intensity function, the most important ingredient in pricing a life insurance policy. Can the usual inference methods be applied? In particular, is there a well-behaved maximum likelihood estimator? Certainly not for more general (nonparametric) density estimation problems, but as he showed, MLE for monotone and unimodal density functions is well-defined and consistent.[12,13]

More typically, abstract parameters—such as the shapes and patterns that would arise later in his general pattern theory—require new methods. He introduced many, which were collected and later published together in his 1981 book *Abstract Inference.*[14] A modern-day example is the collection of architectures known as deep neural networks, which Grenander would have called a "sieve," indexed by the hyperparameters. This kind of generality facilitates general proofs, and what is now termed *universality* is one example of the consistency of the method of sieves.

During the 1957–58 academic year, Grenander visited the Division of Applied Mathematics at Brown University. William Prager, who founded the division, had been impressed with Grenander's work on Toeplitz forms and wanted to expand the division from fluids and mechanics to a broader view of applied mathematics. Grenander taught probability and statistics and rekindled a long-standing fascination with computing, thanks largely to the excellent computing resources donated by IBM at the direction of Thomas Watson Jr., a Brown alumnus. At the time, the group at Brown was almost entirely focused on mechanics, dynamical systems, and computation. Nevertheless, Grenander returned in 1966 and remained there as the L. Herbert Ballou University Professor in the Division of Applied Mathematics until his retirement.

Shortly after his return to Brown, Grenander formulated a unique and extremely general approach to modeling and inference that he called pattern theory, which he later referred to as "the intellectual adventure of my life." Grenander first published his ideas in detail in his three-volume *Lectures in Pattern Theory.*[15,16,17] In the third volume, he laid out his full vision of how essentially all human thought comes under the mantle of pattern theory. He began by quoting Hume's principle that instances of which we have had no experience must resemble those of which we have had experience and that the course of nature always continues uniformly the same. He calls these uniformities *regular structures*, the places where chaos is replaced by order, where the usual increase of entropy is kept at bay. The formalism of pattern theory is meant to describe all such regular structures. In order to start a formalism for every kind of pattern, his first major hypothesis is that patterns are compositional, in that they can be constructed by combining simpler patterns according to specified rules. His next basic hypothesis was that rigid rules must be replaced by probabilistic ones (but cautioned that this would require many new non-standard types of probability spaces). There follows a long list of examples, biological shapes, Ptolemaic models of planets, crystals, waves, the Linnaean description of flowers, context-free grammars, and even a theory of the morphology of folk tales.

Having assumed that patterns can be built up by combining simpler ones, one is led to assume that all patterns are built up from atomic ones. Thus, the constructed pattern itself must be a graph whose vertices are the atomic patterns that Grenander called generators and whose edges, where patterns are combined, he called bonds. It is interesting how often graphs have been used as models of cognitive structures. For example, there are the grammars of languages in which, from a pattern-theory perspective, the generators are the words (or their uninflected cores), the bonds are the cases (e.g., nominative, genitive, etc.) or other links (e.g., subordinate clauses), and the graph is the parse tree. On the other hand, quite a different set of bonds between words was given in *Roget's Thesaurus,* where bonds connect any two words with similar meanings. The idea of bonds is very flexible. Early AI researchers introduced the concept of semantic nets in which bonds connected categories of objects when one contained the other (e.g., birds ⊃ robins). A very influential idea, originating with Judea Pearl, was to make graphs with directed edges in which each edge connects one event to another that may contribute to its cause. From the compositional point of view, graphs are indeed a natural formalization of cognitive patterns.

When probabilities are introduced, in what Grenander called metric pattern theory, the key role of the graph is to represent when the likelihoods of different parts of the pattern are conditionally independent. This is central both to assigning probabilities in context-free grammars and in Pearl's analysis of causation. In vision, the conditional independence defined by the graphical structure made clear how the problem of image segmentation was related to the problem of phase change in statistical mechanics, especially the Ising model. Grenander recognized that his models went far beyond traditional statistics, and in his 1981 monograph *Abstract Inference,* he developed new methods for nonparametric inference applicable to very general classes of probability distributions.[18]

A key way in which pattern theory differed from and deepened the work in statistics known (confusingly) as pattern recognition is through what Grenander called the principle of realism. This requires that, in addition to a "pure" underlying reality, a pattern must describe the "deformed" observable, in which the pure pattern may be hard to recognize. This generalizes, for example, Chomsky's idea of the deep structure of an utterance vs. its surface structure, where *deep* corresponds to *pure* and *surface* to *deformed*. Grenander implicitly criticized contemporary speech recognition algorithms for pre-processing speech in an attempt to eliminate speaker variation before making a mathematical model to analyze the speech. He insists that a proper stochastic model must include the actual utterance. Of course he was right, and state-of-the-art speech recognition models include speaker variation. Once you model everything, you can sample your model and synthesize new patterns. Do these samples have the look and feel of the actual data? If not, your model needs improvement. He summed up this point of view with the maxim "pattern synthesis = pattern analysis." (State-of-the-art variational autoencoders and diffusion models are wonderful examples, which he would likely have celebrated.)

In the 1980s, he began to explore applications of the theory, including image restoration, synthesis, and analysis, language processing, and even musical composition. Early on, he assigned a student the project of modeling the music of marching bands sufficiently well that the algorithm could create new tunes in this distinct genre. By the 1990s, a major direction of his team became the study of shape, especially the shape of human bodies. Inspired by D'Arcy Thompson's famous book *On Growth and Form*,[19] Grenander considered a variety of ways to apply pattern theory to shapes, either using diffeomorphisms on the ambient space, or on an encoding of the shape's boundary.[20,21] An especially fruitful application resulted from a collaboration with Michael Miller of Johns Hopkins University. Grenander and Miller undertook the construction of a library of models of all structures in the human body: a "digital anatomy" equipped with measures of normal shape and normal variability and including all its organs, vascular system, and nerves.

He initiated this with a July 2004 email, writing Michael Miller and Laurent Younes, and cc'ing the authors, opening with: "Buckle up! Fasten your seatbelts! You are in for a rough ride." He went on to propose the following:

> *Construct fully automatic computational procedures for analyzing MRIs: image understanding as labelling anatomical components, detecting abnormalities, quantified determination of growth properties. The algorithm shall be biologically based in that it will incorporate detailed numerical knowledge of anatomies…and based on pattern theoretic ideas….*

Constructing a diffeomorphism from a template shape to a given variation of this shape is a quintessential example of what Grenander called deformations from a pure pattern to a deformed pattern. There is now an extensive literature on different mathematical ways of formalizing the rather elusive concept of shape. The diffeomorphism approach drew in his co-author Miller, and many followed, notably Anuj Srivastava of Florida State, Laurent Younes of Johns Hopkins, and Alain Trouvé of the École Normale Supérieure de Cachan (now ENS Paris-Saclay).[22,23,24] Collectively, they broke new mathematical ground and the approach is now used routinely as a tool in clinical research on neuro anatomy and pathology, such as in the early detection of Alzheimer's disease.[25]

With M. I. Miller, circa 1995.

Grenander's scientific vision and intellectual daring never waned. Early in the 2000s, he turned to the ultimate application—analyzing thoughts, and more specifically, his own brain! The new adventure started in 2003, announced again by email. As he put it, he sought to build a GOLEM, a program that would contemplate the sort of things he liked to contemplate and produce stochastically a stream of consciousness resembling his. He made remarkable progress, leading to his final publication, *A Calculus of Ideas,* in 2012.[26] In it, he speculated about the nature of human thought within the context of two thousand years of approaches, from Greek philosophers to modern theories in the cognitive and neuro sciences, and built an explicit generative model using the formal tools of his pattern theory. In a section on free association, he produced a stochastic stream of consciousness, consisting mostly of cognitive-like fragments until this: Peter, stroke, puppy, whimper. It translates as "Peter strokes the puppy who whimpers," at which Grenander exclaimed, "finally a complete thought!" As was his custom, the book is full of ideas and concrete examples. Perhaps he dreamed of what he would have done if a company like Amazon had given him 100 brilliant programmers and a hundred million dollars. During these heady times of AI and the Silicon Valley economy, if he were alive and younger, this might have happened.

Grenander published more than ninety research articles and fifteen authored books and earned many honors and awards. These include being named an Arhennius Fellow (1948), a Fellow of the Institute of Mathematical Statistics (1953), a Guggenheim Fellow (1979), and an Honorary Fellow of the Royal Statistical Society, London (1989). He received the Prize of the Nordic Actuaries (1961) and the Arnberger Prize of the Royal Swedish Academy of Science (1962) and was made a member of the Royal Swedish Academy of Science (1965) and both the American Academy of Arts and Sciences (1995) and the National Academy of Sciences, U.S.A. (1998). He delivered numerous prestigious lectures, including the Rietz Lecture (1985), the Wald Lectures (1995), and the Mahalanobis Lecture (2004), and was awarded an honorary Doctor of Science degree from the University of Chicago (1993).

Grenander was a voracious reader, broadly knowledgeable in history and science, and fluent in many languages. He was a passionate sailor and a skilled do-it-yourself electrician, plumber, and carpenter. Almost singlehandedly, he built entire wings of his summer home in Västervik, which now accommodate frequent stays by his three children and six grandchildren, all of whom survive him.

Sailing in the Tjust archipelago, near summer home in Västervik.

ACKNOWLEDGMENTS

All of the direct quotations from Ulf Grenander, not including quoted emails, come from Nitis Mukhopadhyay's 2007 article "A Conversation with Ulf Grenander," *Statistical Science* 21:404–426.

REFERENCES

1. Kolmogorov, A. N. 1956. *Foundations of the Theory of Probability*. 2nd ed. Trans. Nathan Morrison. New York, NY: Chelsea Publishing Company.

2. Cramér, H. 1945. *Mathematical Methods of Statistics.* Princeton, NJ: Princeton University Press.

3. Grenander, U. 1949. Stochastic processes and integral equations. *Ark. Mat.* 1(1):67–70.

4. Grenander, U. 1950. Stochastic processes and statistical inference. *Ark. Mat.* 1(3):195–277.

5. Grenander, U. 1963. *Probabilities on Algebraic Structures*. New York, NY: Wiley.

6. Grenander, U., and M. Rosenblatt. 1957. *Statistical Analysis of Stationary Time Series.* New York, NY: Wiley.

7. Grenander, U. 1952. On Toeplitz forms and stationary processes. *Ark. Mat.* 1(6):555–571.

8. Grenander, U., and M. Rosenblatt. 1953. Statistical spectral analysis of time series arising from stationary stochastic processes. *Ann. Math. Statist.* 24(4):537–558.

9. Grenander, U., and G. Szegő. 1958. *Toeplitz Forms and Their Applications*. Berkeley: University of California Press.

10. Spitzer, F. 1959. Review: Ulf Grenander and Gabor Szegő. Toeplitz Forms and Their Applications. *Bull. Amer. Math. Soc.* 65(2):97–101.

11. Grenander, U. 1963. *Probabilities on Algebraic Structures*. New York, NY: Wiley.

12. Grenander, U. 1956. On the theory of mortality measurement. Part I. *Scand. Actuar. J.* 39:70–96.

13. Grenander, U. 1957. On the theory of mortality measurement. Part II. *Scand. Actuar. J.* 39:125–153.

14. Grenander, U. 1981. *Abstract Inference*. New York, NY: Wiley.

15. Grenander, U. 1976. *Pattern Synthesis: Lectures in Pattern Theory. Vol. 1.* Applied Mathematical Sciences Vol. 18. New York, NY: Springer-Verlag.

16. Grenander, U. 1978. *Pattern Analysis: Lectures in Pattern Theory. Vol. 2.* Applied Mathematical Sciences Vol. 24. New York, NY: Springer-Verlag.

17. Grenander, U. 1981. *Regular Structures: Lectures in Pattern Theory. Vol. 3.* Applied Mathematical Sciences Vol. 33. New York, NY: Springer-Verlag.

18. Grenander, U. 1981. *Abstract Inference*. New York, NY: Wiley.

19. Thompson, D. W. 1917. *On Growth and Form*. Cambridge, MA: Cambridge University Press.

20. Grenander, U., Y. Chow, and D. M. Keenan. 1991a. *Hands: A Pattern-Theoretic Study of Biological Shapes.* Research Notes in Neural Computing Vol. 2. New York, NY: Springer-Verlag.

21. Grenander, U., U. Amit, and M. Piccioni. 1991b. Structural image restoration through deformable templates. *J. Am. Stat. Assoc.* 86(414): 376–387.

22. Grenander, U., and M. I. Miller. 1994. Representations of knowledge in complex systems. *J. Roy. Statist. Soc. Ser. B (Meth.)* 56:549–603.

23. Grenander, U., and M. I. Miller. 1998. Computational anatomy: An emerging discipline. *Q. Appl. Math. 56*(4):617–694.

24. Grenander, U., and M. I. Miller. 2007. *Pattern Theory: From Representation to Inference.* Oxford, UK: Oxford University Press.

25. Younes, L., M. Albert, and M. I. Miller. 2014. Inferring changepoint times of medial temporal lobe morphometric change in preclinical Alzheimer's disease. *NeuroImage Clin.* 5:178–187.

26. Grenander, U. 2012. *A Calculus of Ideas: A Mathematical Study of Human Thought*. Singapore: World Scientific.

SELECTED BIBLIOGRAPHY

1949 Stochastic processes and integral equations. *Ark. Mat.* 1(1):67–70.

1950 Stochastic processes and statistical inference. *Ark. Mat.* 1(3):195–277.

1952 On Toeplitz forms and stationary processes. *Ark. Mat.* 1(6):555–571.

1953 With M. Rosenblatt. Statistical spectral analysis of time series arising from stationary stochastic processes. *Ann. Math. Statist.* 24(4):537–558.

1954 With M. Rosenblatt. An extension of a theorem of G. Szegő and its application to the study of stochastic processes. *Trans. Amer. Math. Soc.* 76:112–126.

1956 On the theory of mortality measurement. Part I. *Scand. Actuar. J.* 39:70–96.

1957 On the theory of mortality measurement. Part II. *Scand. Actuar. J.* 39:125–153.

With M. Rosenblatt. *Statistical Analysis of Stationary Time Series.* New York, NY: Wiley.

1958 Some remarks on bonus systems in automobile insurance. *Scand. Actuar. J.* 40:180–197.

With G. Szegő. *Toeplitz Forms and Their Applications*. Berkeley, CA: University of California Press.

1963 *Probabilities on Algebraic Structures*. New York, NY: Wiley.

1971 With W. F. Freiberger. *A Course in Computational Probability and Statistics.* Applied Mathematical Sciences Vol. 6. New York, NY: Springer-Verlag.

1976 *Pattern Synthesis: Lectures in Pattern Theory. Vol. 1.* Applied Mathematical Sciences Vol. 18. New York, NY: Springer-Verlag.

1978 *Pattern Analysis: Lectures in Pattern Theory. Vol. 2.* Applied Mathematical Sciences Vol. 24. New York, NY: Springer-Verlag.

1981 *Regular Structures: Lectures in Pattern Theory. Vol. 3.* Applied Mathematical Sciences Vol. 33. New York, NY: Springer-Verlag.

Abstract Inference. New York, NY: Wiley.

1982 *Mathematical Experiments on the Computer.* New York, NY: Academic Press.

1991 With Y. Chow and D. M. Keenan. *Hands: A Pattern-Theoretic Study of Biological Shapes.* Research Notes in Neural Computing Vol. 2. New York, NY: Springer-Verlag.

With U. Amit and M. Piccioni. Structural image restoration through deformable templates. *J. Am. Stat. Assoc.* 86(414):376–387.

1993 *General Pattern Theory.* Oxford Mathematical Monographs. Oxford, UK: Clarendon Press.

1994 With M. I. Miller. Representations of knowledge in complex systems. *J. Roy. Statist. Soc. Ser. B (Meth.)* 56:549–603.

1996 *Elements of Pattern Theory.* Johns Hopkins Studies in the Mathematical Sciences. Baltimore, MD: Johns Hopkins University Press.

1998 With M. I. Miller. Computational anatomy: An emerging discipline. *Q. Appl. Math. 56*(4):617–694.

2007 With M. I. Miller. *Pattern Theory: From Representation to Inference.* Oxford, UK: Oxford University Press.

2012 *A Calculus of Ideas: A Mathematical Study of Human Thought*. Singapore: World Scientific.

Published since 1877, *Biographical Memoirs* are brief biographies of deceased National Academy of Sciences members, written by those who knew them or their work. These biographies provide personal and scholarly views of America's most distinguished researchers and a biographical history of U.S. science. *Biographical Memoirs* are freely available online at www.nasonline.org/memoirs.