BIOGRAPHICAL MEMOIRS

Garrett Birkhoff

January 19, 1911–November 22, 1996
Elected to the NAS, 1968

A Biographical Memoir by Jerry Bona and Peter Olver

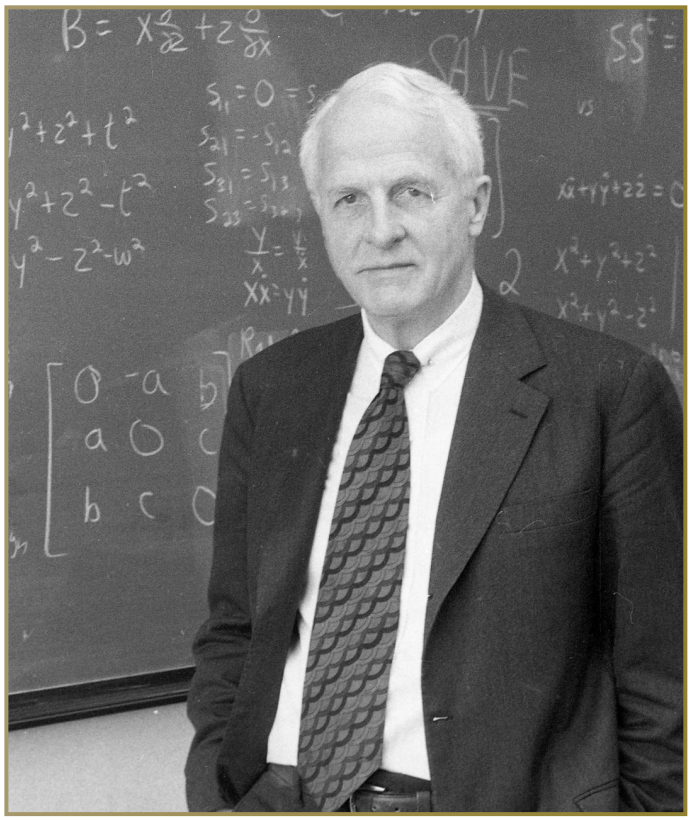

Figure 1 Garrett Birkhoff, 1979. *Courtesy of Harvard University Archives.*

Garrett Birkhoff was a leading figure in the field of abstract algebra, including the development of lattice theory and universal algebra. Later in his career, he did groundbreaking work in the early use of the electronic computer, the applications of Lie groups to differential equations, particularly those arising in fluid mechanics, and numerical methods for linear algebra and the solution of elliptic partial differential equations.

Birkhoff was born on January 19, 1911, in Princeton, New Jersey, to George David Birkhoff, among the most prominent American mathematicians of the first half of the twentieth century, and Margaret Elizabeth Graftus. He was one of three children, including sister Barbara and brother Rodney. As a consequence, Garrett grew up in a mathematical environment. He was educated at home until age eight, then went to a public grammar school, followed by enrolling in the private Browne & Nichols School at age twelve. He entered Harvard University as an undergraduate in 1928. According to an interview, he had already decided to become a mathematician, and his father advised him to study mathematical physics. Garrett took undergraduate courses on analytical mechanics, differential equations, potential theory, quantum mechanics, real analysis, and topology and was inspired by lecturers such as Oliver Kellogg, Marston Morse, and Hassler Whitney. He also discovered group theory while in the library, which particularly attracted him.

Birkhoff graduated from Harvard in 1932 and was awarded a Henry Fellowship to study at the University of Cambridge in England. With a growing interest in abstract algebra, he asked Philip Hall to serve as his advisor. Notable during his European stay was a trip to Munich, where he met Constantin Carathéodory, who pointed him towards two important texts: Bartel van der Waerden's book on abstract algebra, which was based on Emmy Noether's modern foundation of the subject, and Andreas Speiser's book on group theory. Upon his return to the United States, he was appointed as a member of Harvard's Society of Fellows and began teaching there in 1936. The following year, he married Ruth Collins, with whom he had a son, John, and two daughters, Ruth and Nancy. In the British tradition of the time, Birkhoff never received a master's or Ph.D. degree

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before launching into mathematical research. He remained at Harvard throughout his career and in 1969 was appointed the George Putnam Professor of Pure and Applied Mathematics. He held this post until he retired in 1981.

Research Accomplishments

Initially, his research concentrated on pure mathematics. His 1935 paper, "On the Structure of Abstract Algebras," effectively founded new branches of mathematics: universal algebra and lattice theory. Many concepts in mathematics are named after him, perhaps most notably the Poincaré-Birkhoff-Witt Theorem. He also wrote papers on Lie groups, devising an example that could not be faithfully realized as a group of matrices. During this time, he published two important texts. The first was *Lattice Theory*, a field that he had developed, and the second was his famous work, *A Survey of Modern Algebra,* written jointly with Saunders MacLane, who had been at Harvard from 1934 to 1936. The text grew out of their jointly taught undergraduate course on abstract algebra and served to introduce the subject, pioneered by van der Waerden's text, to a new generation of mathematics students by making it accessible and attractive. The influence of this text on the American undergraduate mathematics curriculum continues to this day.

During World War II, Birkhoff switched his focus to applied mathematics, but with an engineering rather than physics orientation. He served on a committee that included Marston Morse and John von Neumann and that studied methods of calculating distances to a target based on radar echoes. He then joined the Ballistic Research Laboratory at the Navy's Aberdeen Proving Ground in Maryland, conducting war research that included modeling exploding shells and bazooka charges, bouncing bombs on water, shock waves around projectiles, and phenomena associated with air-launched missiles entering water.

Inspired by his war research, Birkhoff wrote two unique texts, *Hydrodynamics*, published in 1950, and *Jets, Wakes and Cavities*, the latter coauthored with Eduardo Zarantello and published in 1957. The first of these was a response to Horace Lamb's classic text, *Treatise on the Motion of Fluids*, to show that there was much more to fluid mechanics and its applications than Lamb envisioned. Of particular note is chapter VI, which promoted the application of Lie group methods. Lie groups were first formulated by Norwegian analyst and geometer Sophus Lie in the 1870s, who wanted to solve differential equations in a manner following Galois' group-theoretic approach to solving algebraic equations. Although the use of Lie groups had already made important inroads into fundamental questions arising in quantum mechanics, Birkhoff's book was the first to advocate their importance for solving the differential equations arising in classical systems such as fluids and solids, an area of research that nowadays produces thousands of published papers.

In the 1950s, in part inspired by his work as a consultant for the Westinghouse Electric Corporation and by conversations with von Neumann, Birkhoff changed research directions yet again and moved into computing and numerical linear algebra. This area was to occupy much of his attention during the latter part of his career. He supervised the thesis of David Young that introduced the method of successive overrelaxation (SOR) for solving the linear systems arising in the solution of elliptic partial differential equations. He then worked with former student Richard Varga on the design of nuclear reactors, in the process extending the results produced by Young. He was an early advocate of the use of cubic splines in computer-aided design and manufacturing. This grew out of his consulting work with the research and development team at General Motors, during which he recommended them as a means of representing automobile surfaces for numerically controlled milling machines.

He continued with textbook writing. In 1962, he and Gian-Carlo Rota wrote the extensively used undergraduate text *Ordinary Differential Equations*. In 1970, he published an update of his classic textbook, *Modern Applied Algebra*, coauthored with Thomas Bartee, that covers applications such as coding theory. Finally, in 1987, he and Robert Lynch coauthored the text *Numerical Solution of Elliptic Problems*. Many of his books are considered classics and remain in print. Following his later interests in the history and philosophy of mathematics, towards the end of his life he was working on a history of mathematics at Harvard that he unfortunately never completed.

Birkhoff received many honors, including honorary degrees from six universities worldwide and election to the National Academy of Sciences, the American Academy of Arts and Sciences, and the American Philosophical Society. He was a Guggenheim Fellow for the academic year 1948–49. He served as the president of the Society for Industrial and Applied Mathematics (SIAM) from 1966–68. In 1978, he was awarded the AMS/SIAM Birkhoff Prize, named after his father, together with Mark Kac and Clifford Truesdell. In 1981, he gave the SIAM John von Neumann Lecture. He published more than 200 papers and had more than fifty Ph.D. students, many of whom went on to very influential mathematical careers of their own. Birkhoff died on November 22, 1996.

Personal Reminiscences

Jerry Bona

After passing the qualifying exam, I looked around the Harvard Mathematics Department for a thesis supervisor. I was interested in applied mathematics; Garrett was the only

member of the department in 1968 who had similar interests. That turned out to be a good choice for me. He suggested a problem that he and the well-known French mathematician Joseph Kampé de Fériet had conceived. They advertised it as a problem connected to turbulence. I did make a little progress on a version of the problem and thereby earned my Ph.D. The results appeared in a long joint work with Garrett and Kampé de Fériet that was published some years later. At about the same time that this article appeared, I realized the mathematical problem itself had no real connection to turbulence and left that subject for others.

During the Summer of 1973, I spent a wonderful four months visiting Garrett at Harvard. During this very productive period, I got to know him as much more than a supervisor. Although he often appeared stern to graduate students, I discovered a soft inner core when he shared with me his grief at the death of his mother. He also shared with me some of his thoughts on the development of his research agenda. For example, he opined that about every seven years, he consciously decided to change subjects. Looking over his publications, and allowing for variable publication times and the vicissitudes of life, you can see the outline of this predilection. Once in a while, he would cycle back to an older topic, but his thought that seven years should be enough for any one subject partially guided my own early years. Garrett taught me many lessons about research and the ways of the academy, but one stands out, although it was inadvertent. Growing up in Arkansas, I did not encounter Xerox machines. That remained true during my undergraduate days in Saint Louis. At Harvard, there was a Xerox machine in the mathematics department, but graduate students did not usually use it. Garrett taught me the use of such machines in the following way: I had a handwritten draft of my Ph.D. thesis, and he generously offered to have it typed by an expert mathematical typist that he used. She produced a beautiful 100-page version of my sloppy handwritten text. Typed up like that with an IBM Selectric typewriter, it looked far more impressive than it actually was. I proudly marched over to Garett's office. He took the manuscript and offered to meet me to discuss it in the afternoon after a luncheon appointment. Later that day, when we reconvened, he allowed as how it needed a bit of work, but the main result looked fine. He then handed me back the typed manuscript, *covered* in red marks. I realized then how very useful the Xerox machine could be!

During this period, he also shared with me some of his extensive collection of preprints and reprints. I realized later that he was particularly generous with subjects in which he had once worked but did not plan to revisit. For example, I possess a handwritten draft of Leray's fundamental work on the Navier-Stokes equations. Not long after a wonderful 80th Birthday Conference held at Harvard, which both Peter and I attended, one of my colleagues at Penn State showed up in my office with a pristine copy of the 50th anniversary reprinting of *A Survey of Modern Algebra.* She knew I was friends of both Birkhoff and MacLane and wanted me to help her get her copy autographed. I sent it first to MacLane who dutifully signed it and returned it. I then sent it to Garrett, who wrote back inquiring who this gal was and could he have a picture, before agreeing to sign! I have lost track of the book, but somewhere on earth is a jointly signed copy of an anniversary version of this influential text.

Peter Olver

Garrett was not only my thesis advisor, whose guidance and inspiration was fundamental to my subsequent mathematical career, he also played an essential role in my marriage and hence my family! Here is how it happened: In 1972, the Iranian Mathematical Society hosted the Third National Mathematics Conference in Tehran. The three featured foreign speakers were Garrett Birkhoff, Paul Erdős, and Paul Halmos. Some photographs that Halmos took during the meeting, including a couple of Garrett and Ruth, can be found in the online Paul R. Halmos Photograph Collection at the University of Texas's Texas Archival Resources Online website.

My future wife, Chehrzad Shakiban, was assigned to be their local guide, because she was a star math student at her university and, moreover, had spent a year as a high school exchange student in St. Louis and thus was fluent in English. When she graduated at the top of her class, she was awarded a government fellowship for graduate study anywhere in the world. She had made particular connections with Garrett, who arranged for her to enroll as a special student in graduate mathematics at Harvard. She and I both entered Harvard in the fall of 1973 and were married in June 1976, two days after I received my Ph.D.

Regarding my choice of thesis research area—applications of Lie groups to differential equations—after he promoted the use of Lie group methods to analyze the partial differential equations of classical applied mathematics, his suggestion was only acted upon by Soviet mathematicians, particularly Lev Ovsiannikov and his team at the University of Novosibirsk. Because I already knew some Russian, Garrett saw that I could help him catch up on what they were accomplishing in this field of research and applications.

One final anecdote: we corresponded regularly after I left Harvard, and he was perpetually encouraging, often offering useful advice on references I might profit from consulting. Towards the end of his life, he mentioned that he owned his father's copy of Sophus Lie's collected works, a seven-volume set containing all his papers (but not his books), and wanted to send it to me, which I agreed to immediately and

enthusiastically. I was hoping to see G. D. Birkhoff's marginalia, or at least his signature, in the volumes. But what showed up was a pristine copy, which may be the only one remaining in the world, given that, to the best of my knowledge, the other copies reside in libraries and have been well worn by extensive consultation and use over the decades!

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