BIOGRAPHICAL MEMOIRS

Shlomo Sternberg

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Elected to the NAS, 1986

A Biographical Memoir by Jeff Cheeger

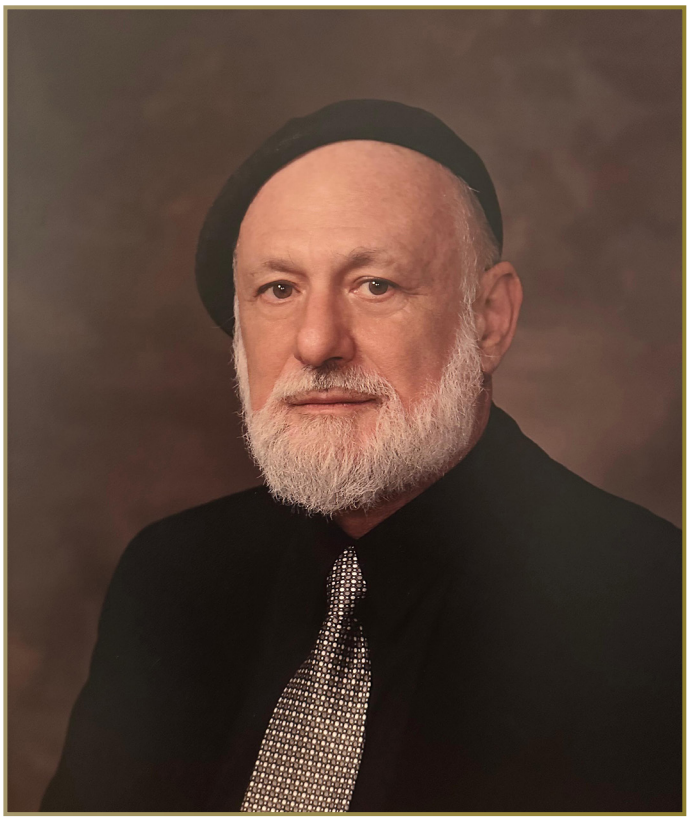

Figure 1 Shlomo Sternberg. *Photo provided by Aviva Green.*

Shlomo Sternberg was a renowned mathematician best known for his work in integrability for systems of linear partial differential equations, symplectic geometry, group representations, dynamical systems, and the relations of these subjects with physics.

Born on January 20, 1936, to second-generation American born parents, Shlomo Sternberg grew up on the Lower East Side of Manhattan. He was raised in an Orthodox Jewish home. His father, Matthias Sinai Sternberg was a practicing attorney and scholar. His mother, Bluma (Last) Sternberg was a high school English teacher. As a child, he attended the Rabbi Jacob Joseph Yeshiva Elementary School on the Lower East Side.

From a early age, Shlomo was already known locally as a prodigy in Talmud and mathematics. The following incident serves to illustrate the point. The famous Garden Cafeteria, a fixture in the neighborhood, was frequented by Isaac Bashevis Singer and other prominent Yiddish writers. Around the time that the atomic bomb was dropped on Japan in 1945, the writers asked the young Shlomo, to explain $E = mc^2$. Apparently they were satisfied with his explanation.

When Shlomo was ten, a family friend, Maurice Plotnick, headmaster of the Westchester Day School, gave him a copy of G.H. Hardy's, *A Course in Pure Mathematics*. He described it as "a remarkable book, written precisely for people my age." (Presumably, this would have been news to Hardy.) Shlomo attended the Talmudical Academy High School of Yeshiva University in Washington Heights. He often referred to the long subway commute as where he got the bulk of his high school education. He read in many subjects and augmented his studies in Talmud and mathematics. Not infrequently, he would play hooky. After getting off the F train at 50th St., he would walk to the Museum of Modern Art, where paintings and film made a big impression. These interests stayed with him throughout his life.

At the age of fifteen, Shlomo applied for admission to Columbia, but was rejected on the grounds that they did not admit anyone under sixteen. Undeterred, he applied to Johns Hopkins where he was accepted and his brilliance was soon recognized. During his time at Hopkins, the famous mathematicians Shiing-Shen Chern and Carl Ludwig

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Siegel were visiting members of the faculty. Shlomo attended a course given by Siegel and in Shlomo's words, he "attached himself to Siegel." He referred to Chern as "a great inspiration," who along with his wife, Zheng Shining, "took care of him." It was during this time that he was struck by the idea that through one's own mental efforts, one could discover new mathematics. Aurel Wintner, a difficult and demanding personality, took on Shlomo as a PhD student and devotedly nurtured him throughout his four years at Hopkins. Shlomo received his PhD under Wintner in 1956 at the age of twenty. His thesis was entitled "Some Problems in Discrete Nonlinear Transformations in One and Two Dimensions."

Sternberg went on to a year of postdoctoral work at the Courant Institute. There, among all the luminaries, including Jürgen Moser, John Nash, and Louis Nirenberg, he was particularly inspired by Peter Lax. In light of Sternberg's subsequent well-known book, *Geometric Asymptotics*, written with Victor Guillemin, one might be tempted to guess that this had specifically to do with the Luneberg-Lax-Ludwig technique for constructing asymptotic solutions for differential equations involving a large parameter, a precursor to Fourier integral operators. But, according to Sternberg, at the time, the source of his inspiration was more general.

In several respects, 1958 was a banner year. In tandem with his mathematical studies at Hopkins, Shlomo had studied with a number of rabbinic authorities, and in that year, he received his formal Rabbinical Ordination from Rabbi Yitzhak Schneider. Later, for over 40 years, Shlomo's Talmud seminar, open to students, met regularly on Wednesdays, for two hours of intensive study.

The second major event in 1958 was the publication of the paper "On the structure of local homeomorphisms of euclidean *n*-space II" which appeared in in the *American Journal of Mathematics*. The number "II" refers to the important precursor, "Local contractions and a theorem of Poincaré," published in 1957. The 1958 paper contained Sternberg's first famous result, the "Sternberg linearization theorem," a C^∞ version of a celebrated theorem of Siegel in the real analytic case. Sternberg's theorem states that near a hyperbolic fixed point, a smooth map can be made linear by a smooth change of coordinates, provided certain non-resonance conditions hold. Unlike in the real analytic case, small divisor conditions are not required. Sternberg retained a justifiable life-long pride in this early result.

Most importantly, in 1958, Shlomo and Aviva Green were married. On one of their first dates, Shlomo's time spent playing hooky at the MoMA, had been used to good effect. Aviva, trying very hard to impress Shlomo, had taken him to her favorite place, the National Gallery in Washington, DC. Her mission, though sincere, turned out to be totally flawed. "Shlomo stood in front of a painting by Goya, a favorite of his, covering one eye then the other with his hand, moving inch by inch, periscoping across the surface and explaining the artist's intent." Aviva has described their experience as "pure joy."

Figure 2 Seated from left to right, Zemach, Shlomo, Rebecca, Aviva, Akiva. Standing in the back, Amiel, Saadya. *Photo provided by Aviva Green.*

A devoted couple, Shlomo and Aviva's marriage of sixty-six years produced four sons and a daughter, Amiel, Saadya, Akiva, Zemach, and Rebecca, and as of this writing, eight grandchildren and fifteen great grandchildren.

Aviva still remembers very well when the family photo was taken and

> "those early years of the constant hubbub of noise and tumult of young children all around us. But Shlomo, all the same, if need be, on occasion, could create a bubble, seemingly unperturbed, and find a spot for himself, sitting with pencil and yellow pad doing mathematics. He might do this too in a dark theater, a movie, a concert, without pad or pencil, his fingers moving in the air doing computations, while still enjoying the film or the music."

All of the Sternberg children, each in their own way, were profoundly influenced by their father's multi-faceted talents, interests, and personality.

When Rebecca, the youngest, was eight years old, Aviva went back to school to study art, first at the Avni Art School in Tel Aviv and then at the Bezalel Art Academy in Jerusalem. She graduated from the Boston Museum School (BSMFA) with the intention to become a painter. According to Aviva, with the all consuming demands of raising a family, the endeavor might not have succeeded absent Shlomo's crucial unwavering support. Aviva entered the New York art scene with her first important solo show in the early 1990s at the Kenkeleba Gallery in the East Village. Closing the circle, her

last New York exhibition was again at the Kenkeleba Gallery (just prior to the Sternberg's final move to Jerusalem in 2016). At that exhibition, Jim and Marilyn Simons acquired Aviva's large painting, titled "Eve." The painting now hangs at the Simons Foundation's Flatiron Institute, on the high wall prominently facing the main entrance of the front lobby.

After his year at the Courant Institute, Sternberg became an instructor at University of Chicago (1957–1959). In 1959, he joined the Mathematics Department at Harvard University where he remained throughout his career, until his retirement as George Putnam Professor of Pure and Applied Mathematics in 2017.

Soon after his arrival, in 1960, Sternberg created the highly innovative course Math 55. The present author took Math 55 as a freshman in the first year in which it was given. Admission had required a brief oral exam on which his performance was not perfect. But eventually, he heard the magic words: "Ok, I'll let you in." The course turned out to be exciting and inspiring in a manner unlike any he subsequently took.

The aim of Math 55 was to teach advanced calculus from a standpoint that exposed the students to modern concepts such as distributions, differential forms, functoriality, and so on. As an example, the general Stokes' theorem was presented in the context of integration of differential forms over cubical chains. The role of functoriality was emphasized with the comment, "You drag it around until you drag it to death!" thereby reducing the general Stokes' theorem to the fundamental theorem of calculus.

With his full black beard and characteristic beret, Sternberg was a charismatic lecturer with a dramatic style. To the present author, he created the impression that being a mathematician was something like being a member of a special order, an order into which one could hope one day to be initiated. When coming to a key point in a proof, he would draw himself up to his full height of 6'3" and declare, "I claim that. . . . " As a break from trying to come to terms with mysteries of exterior multiplication, the author and certain of his friends would amuse themselves with increasingly exaggerated renditions of "I claim that. . . ."

One aspect of the presentation which arguably was less than optimal was Sternberg's *disclaimer*, prior to starting on the Fourier inversion formula, that all formulas were only guaranteed to be correct up to factors of $1/\pi$. (Or maybe, $1/\sqrt{\pi}$.) For impressionable young minds, this may have had the effect of suggesting that real mathematicians don't concern themselves with such trivialities.

An added attraction in Math 55 was the presence of the eccentric grader, Michael Gilmartin, later known for having proved that analytic varieties are locally contractible. Gilmartin would attend all of the lectures and from time to time, was given to kibitzing. On one occasion, when Sternberg temporarily became confused and made several attempts to fix what he written on the blackboard, Gilmartin called out "Don't change it, you'll only make it worse!" At least in name, Math 55 survives and is sometimes billed on the internet as the world's hardest undergraduate math course. Sadly, the current version bears little resemblance to the original.

Early on, three of Sternberg's Harvard colleagues made an especially strong impression. Of Raul Bott, he said, "He cheered people up," while adding that he himself did not have this ability. He believed that he saw a parallel between Bott's outgoing personality and Bott as a mathematician: "He was very open to new ideas." By the same token, Sternberg's late Harvard colleague, Benedict Gross, described Shlomo as "extraordinarily brilliant with a very rare attribute, not often seen in men of such brilliance: He always kept an open mind." As with anyone who had ever encountered Andrew Gleason, Sternberg was struck by the fact that, "His mind worked very fast." Nonetheless, he wondered, "Was this good or bad?" In light of Sternberg's subsequent work, it is not surprising that the faculty member from whom he said he learned the most was George Mackey. He mentioned in particular Mackey's course, "Mathematical Foundations of Quantum Mechanics," in which the role of group representations was particularly emphasized.

Already in 1962, Sternberg's first graduate student Victor Guillemin obtained his PhD. His mentorship of Guillemin evolved into a lifelong collaboration, yielding fifty joint papers and six books. Other prominent members of Sternberg's cohort of eighteen PhD students include Hubert Goldschmidt (1967), Ravi Kulkarni (1968), Israel Sigal (1972), Eric Grinberg (1982) and Yael Karshon (1993). Shlomo loved his students. In turn, they remembered him as being very warm and generous, with a sharp sense of humor.

Sternberg's mathematical works extend well into the twenty-first century. Recurring themes in his 125 publications are integrability for systems of linear partial differential equations, symplectic geometry, group representations, dynamical systems, and the relations of these subjects with physics.

Among his works are nineteen books. According to Shlomo, "The books just seemed to pour out of me." By virtue of their attention to historical context, together with a mix of general theory complemented by a detailed treatment of interesting and highly nontrivial examples, typically, Sternberg's books have a distinctly personal quality. In some cases, even after half a century or more, the most influential of his books have no direct competitors. Of his advanced text books, notable examples are *Lectures on Differential Geometry* (1964), which played a significant role in the early mathematical education of the present author, and *Group theory and Physics*, the hundreds of other texts on this subject notwithstanding. The following four books, which could be

viewed being as somewhere between advanced texts and research monographs, were coauthored with Victor Guillemin: *Geometric Asymptotics* (1977), *Symplectic Techniques in Physics* (1984), *Symplectic Fibrations and Multiplicity Diagrams* (also with Emanuel Lerman) (1994), and *Supersymmetry and Equivariant de Rham Theory* (1999). All of these books have been highly cited.

Any selection of Sternberg's most significant works would include the following:

- During the 1960s, joint work of Sternberg and Isadore Singer provided rigorous proofs of Eli Cartan's classification of the simple transitive infinite Lie pseudogroups and related them to theory of G-structures. (A detailed introduction to the theory of G-structures is the subject of the final chapter of Sternberg's *Lectures on Differential Geometry*.) With Guillemin and Daniel Quillen, he extended the classification to the primitive infinite pseudogroups. As an application, they obtained the "integrability of characteristics" theorem for over-determined systems of partial differential equations.
- In "Hamiltonian group actions and dynamical systems of Calegaro type," published in *Communications in Pure Applied Mathematics* in 1978, Sternberg, together with David Kazhdan and Bertram Kostant, showed how one can simplify the analysis of completely integrable dynamical systems of Calegaro type, by describing them as symplectic reductions of much simpler systems.
- In "Convexity properties of the moment mapping" (1982), Guillemin and Sternberg prove that for a Hamiltonian torus action on a compact symplectic manifold, the image of the momentum map (or "moment map") is a convex polytope. This result, which was proved independently and at the same time by Michael Atiyah, led to a flood of activity, connecting symplectic geometry to the geometry of polytopes.
- In "Geometric quantization and multiplicities of group representations" (1982) Guillemin and Sternberg formulated the "Quantization commutes with reduction" principle, also known as $[Q, R] = 0$. For a compact Lie group G and a symplectic G-manifold X, the quantization $Q(X)$ should then be a linear G representation. The relation, $[Q, R] = 0$, identifies the linear subspace of $Q(X)$ that is fixed by G with the quantization of the symplectic quotient, X/G, In that paper, Guillemin and Sternberg proved $[Q, R] = 0$ rigorously when X is a Kähler manifold and $Q(X)$ is interpreted as the space of global holomorphic sections of a prequantization line bundle. In actuality, the "Quantization commutes with reduction" principle goes well beyond the Kähler case and has inspired an enormous amount of subsequent work.

Throughout his adult life, Sternberg maintained a strong connection with Israel. He and Aviva lived both in Boston and in the Old City of Jerusalem where they spent their latter years. Four of their five children permanently reside in Israel.

During his time at Harvard, Sternberg was also a member of the Tel Aviv University (TAU) Institute for Advanced Studies, as part of his long collaboration with the distinguished Israeli physicist Yuval Ne'eman. Sternberg played a major role in the early development of the Mathematics and Physics departments at TAU. Sternberg's close collaboration with Ne'eman lasted from 1962 until Ne'eman's death in 2006. According to Sternberg, they were "the best of friends." A guiding principle in their work is the use of supersymmetry to eliminate certain undetermined coefficients in the standard model of elementary particles. Their 1975 paper with Lawrence Corwin, "Graded Lie algebras in mathematics and physics (Bose–Fermi symmetry)," is extensively cited by both mathematicians and physicists. Their 2005 review paper, derives physical properties (the electroweak isospin and hypercharge) of elementary particles from representations of the superalgebra $su(m/1)$ and expresses the Higgs field as the degree zero component of a superconnection in the sense of Quillen.

Sternberg delivered the American Mathematical Society Colloquium Lectures in 1990 and the Hebrew University's Albert Einstein Memorial Lecture in 2006. He received an honorary doctorate from the University of Mannheim in 1991. In addition to his election to the National Academy of Sciences in 1986, Sternberg was an elected member of the American Academy of Arts and Sciences (1969), the Spanish Royal Academy of Sciences (1999), and the American Philosophical Society (2010).

After a long illness, Sternberg passed away in the Old City of Jerusalem on August 23, 2024.

Acknowledgment

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